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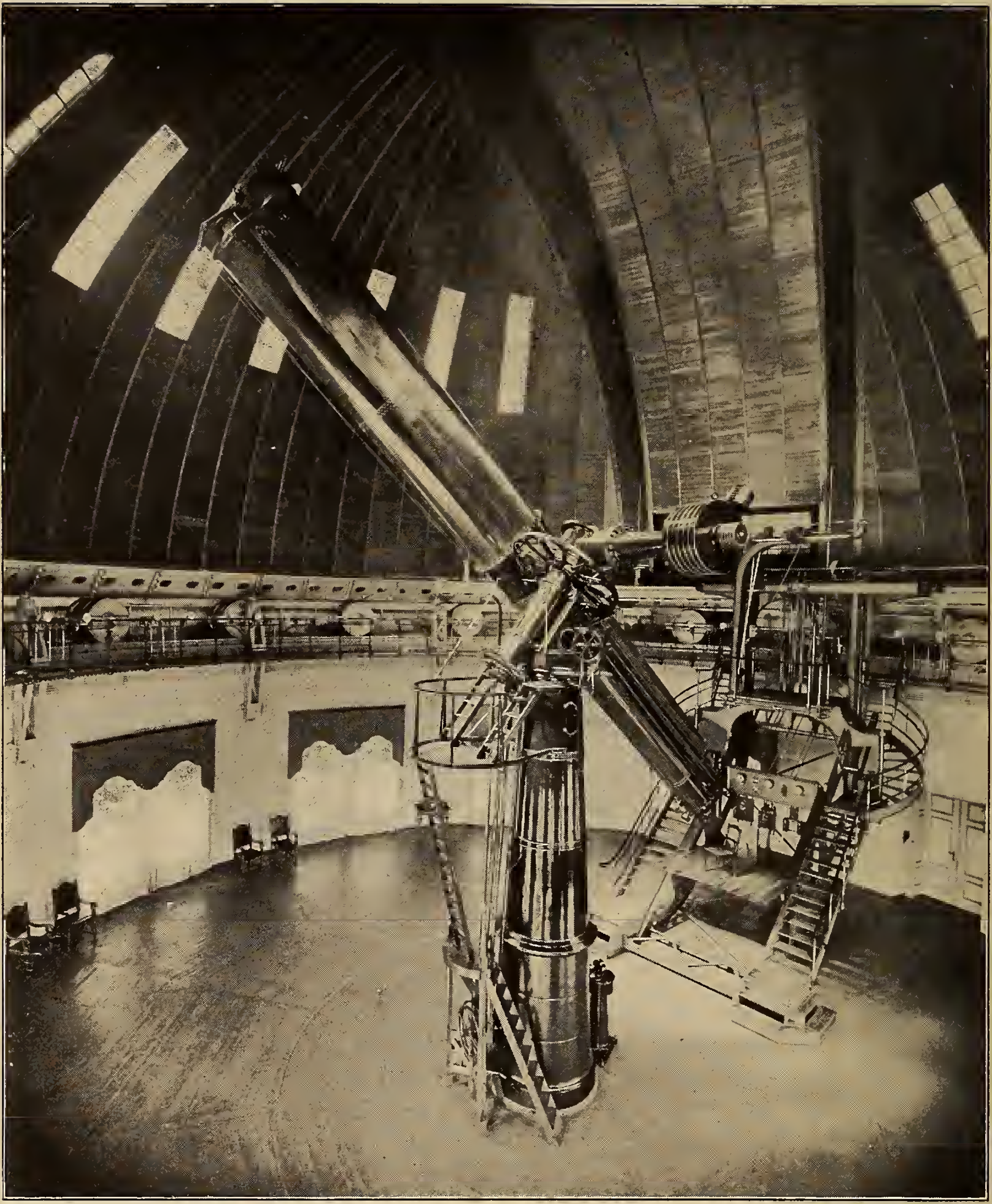

Great Double Telescope, Visual and Photographic, of the Potsdam Astrophysical Observatory. (See Sec. 51.)

MANUAL OF ASTRONOMY

A TEXT-BOOK

BY

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"THE SUN" AND OF A SERIES OF ASTRONOMICAL TEXT-BOOKS

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PREFACE

The present volume has been prepared in response to a rather pressing demand for a text-book intermediate between the author's *Elements of Astronomy* and his *General Astronomy*. The latter is found by many teachers to be too large for convenient use in the time at their disposal, while the former is not quite sufficiently extended for their purpose.

The material of the new book has naturally been derived largely from its predecessors; but everything has been carefully worked over, rearranged and rewritten where necessary, and changed and added to in order to bring it thoroughly up to date.

The writer is under great obligations to many persons who have assisted him in various ways, especially to Miss Anne S. Young, the head of the astronomical department in Mt. Holyoke College, who has carefully read and corrected all the proof. He is greatly indebted also to D. Appleton & Co. for permission to use illustrations from *The Sun*, to Warner & Swasey for photographs of astronomical instruments, and to numerous other friends who have kindly furnished material for engravings. Among these may be mentioned specially Professor Pickering of the Harvard Observatory, the lamented Keeler, and Professors Campbell and Hussey of the Lick

Observatory, and Professors Hale, Frost, and Barnard of the Yerkes, besides several others to whom acknowledgment is made in the text.

The volume speaks for itself as to the skillful care of printers and publishers in securing the most perfect mechanical execution.

C. A. YOUNG.

PRINCETON, N.J.,
April, 1902.

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MANUAL OF ASTRONOMY

INTRODUCTION

1. **Astronomy** is the science which treats of the heavenly bodies, as is indicated by the derivation of its name (ἄστρον νόμος).

It considers:

(1) Their motions, both real and apparent, and the laws which govern those motions.

(2) Their forms, dimensions, and masses.

(3) Their nature, constitution, and physical condition.

(4) The effects which they produce upon each other by their attractions, radiations, or any other ascertainable influence.

The earth is an immense ball, about 8000 miles in diameter, composed of rock and water, and covered with a thin envelope of air and cloud. Whirling on its axis, it rushes through empty space with a speed fifty times as great as that of the swiftest shot. On its surface we are wholly unconscious of the motion, because of its perfect steadiness.

The off-look from the earth.

As we look up at night we see in all directions the countless stars; and conspicuous among them, and looking like stars, though very different in their real nature, are scattered a few planets. Here and there appear faintly shining clouds of light, like the so-called Milky Way and the nebulæ, and perhaps now and then a comet. Most striking of all, if she happens to be in the heavens at the time, though really the most insignificant of all, is the moon. By day the sun alone is visible, flooding the air with its light and hiding the other bodies from the unaided eye, but not all of them from the telescope.

2. The Heavenly Bodies. — The bodies thus seen from the earth are the *heavenly bodies.* For the most part they are globes like the earth, whirling on their axes, and moving swiftly, though at such distances from us that their motions can be detected only by careful observation.

Classification of the heavenly bodies.

They may be classified as follows: First, *the solar system proper*, composed of the sun, the planets which revolve around it, and the satellites which attend the planets in their motion. The moon thus accompanies the earth. The distances between these bodies are enormous as compared with the size of the earth; and the sun, which rules them all, is a body of almost inconceivable magnitude.

Secondly, we have *the comets and the meteors*, which, while they acknowledge the sun's dominion, move in orbits of a different shape and are bodies of a very different character.

Thirdly, we have *the stars*, at distances from us immensely greater than even those which separate the planets. The *visible* stars are suns, bodies like our own sun in nature, and like it, self-luminous, while the planets and their satellites shine only by reflected sunlight. The telescope reveals millions of stars invisible to the naked eye, and there are others, possibly thousands of them, that are dark and do not shine, but manifest their existence by effects upon their neighbors.

Finally, there are *the nebulæ*, of which we know very little except that they are cloudlike masses of shining matter, and belong to the region of the stars.

Branches of astronomy.

3. Branches of Astronomy. — Astronomy is divided into many branches, some of which generally recognized are the following:

(1) *Descriptive Astronomy.* This, as its name implies, is merely an orderly statement of astronomical facts and principles.

(2) *Spherical Astronomy.* This, discarding all considerations of absolute dimensions and distances, treats the heavenly bodies simply as objects on the surface of the celestial sphere; it deals

only with angles and directions, and, strictly regarded, is merely spherical trigonometry applied to astronomy.

(3) *Practical Astronomy.* This treats of the instruments, the methods of observation, and the processes of calculation by which astronomical facts are ascertained. It is quite as much an art as a science.

(4) *Theoretical Astronomy.* This deals with the calculation of orbits and ephemerides, including the effect of perturbations.

(5) *Astronomical Mechanics.* This is simply the application of mechanical principles to explain astronomical facts, chiefly the planetary and lunar motions. It is sometimes called "gravitational astronomy," because, with few exceptions, gravitation is the only force sensibly concerned in the motions of the heavenly bodies.

Abandonment of the term "physical astronomy."

Until about 1860 this branch of the science was generally designated "physical astronomy," but the term is now objectionable because of late it has been used by some writers to denote a very different and comparatively new branch of the science, viz.:

(6) *Astronomical Physics, or Astro-Physics.* This treats of the physical characteristics of the heavenly bodies, their brightness and spectroscopic peculiarities, their temperature and radiation, the nature and condition of their atmospheres and surfaces, and all phenomena which indicate or depend on their physical condition. It is sometimes called *The New Astronomy.*

The above branches are not distinct and separate, but overlap in all directions. Valuable works exist, however, bearing the different titles indicated above, and it is important for the student to know what subjects he may expect to find discussed in each, although they do not distribute the science between them in any strictly logical and mutually exclusive manner.

4. Rank of Astronomy among the Sciences. — Astronomy is the oldest of the natural sciences. Obviously, in the very infancy of the race the rising and setting of the sun, the phases of the moon, and the progress of the seasons must have compelled the attention of even the most unobservant. Nearly the earliest of all existing records relate to astronomical subjects, such as eclipses and the positions of the planets.

Astronomy still progressive.

As astronomy is the oldest of the sciences, so also it is one of the most perfect and complete, though not in the sense that it has reached a maturity which admits no further development, for in fact it was never more vigorously alive or growing faster than at present. In certain aspects astronomy is also the noblest of the sisterhood, being the most "unselfish" of them all, cultivated not so much for material profit as for pure love of learning.

5. **Utility.** — But although bearing less directly upon the material interests of life than the more modern sciences of physics and chemistry, it is really of high utility.

Use in navigation and geodesy.

It is by means of astronomy that the latitude and longitude of points upon the earth's surface are determined, and by such determinations alone is it possible to conduct extensive navigation. Moreover, all the operations of surveying upon a large scale, such as the determination of international boundaries, depend more or less upon astronomical observations.

Use in regulation of time.

The same is true of all operations which, like the railway service, require an accurate knowledge and observance of the time; for our fundamental timekeeper is the daily revolution of the heavens, as determined by the astronomer's transit instrument.

At present, however, the end and object of astronomical study is chiefly knowledge, pure and simple. It is not likely that great inventions and new arts will grow out of its principles, such as are continually arising from chemical, physical, and biological studies; but it would be rash to say that such outgrowths are impossible.

Chief value purely intellectual.

The student of astronomy must, therefore, expect his chief profit to be intellectual, — in the widening of the range of thought and conception, in the pleasure attending the discovery of simple law working out the most far-reaching results, in the delight over the beauty and order revealed by the telescope and spectroscope in systems otherwise invisible, in the recognition of the essential unity of the material universe and of the kinship between his own mind and the Infinite Reason.

Astrology a pseudo-science.

In ancient time it was believed that human affairs of every kind, the welfare of nations, and the life history of individuals, were controlled, or at least prefigured, by the motions of the stars and planets; so that from the study of the heavens it ought to be possible to predict futurity. The *pseudo-science of astrology*, based upon this belief, supplied the motives that led to most of the astronomical observations of the ancients. As modern chemistry had its origin in alchemy, so astrology was the progenitor of astronomy, and it is remarkable how persistent a hold this baseless delusion still retains upon the credulous.

Educational value.

6. Place in Education. — Apart from the utility of astronomy in the ordinary sense of the word, the study of the science is of high value as an intellectual training. No other so operates to give us, on the one hand, just views of our real insignificance in the universe of space, matter, and time, or to teach us, on the other hand, the dignity of the human intellect as being the offspring, and measurably the counterpart, of the Divine, — able in a sense to comprehend the universe and understand its plan and meaning.

The study of the science cultivates nearly every faculty of the mind; the memory, the reasoning power, and the imagination all receive from it special exercise and development. By the precise and mathematical character of many of its discussions it enforces exactness of thought and expression, and corrects the vague indefiniteness which is apt to be the result of purely literary training; while, on the other hand, by the beauty and grandeur of the subjects which it presents, it stimulates the imagination and gratifies the poetic sense.

CHAPTER I

PRELIMINARY CONSIDERATIONS AND DEFINITIONS

Fundamental Notions and Definitions — Astronomical Coördinates and the "Doctrine of the Sphere" — The Celestial Globe

Astronomy, like all the other sciences, has a terminology of its own, and uses technical terms in the description of its facts and phenomena. In a popular work it would be proper to avoid such terms as far as possible, even at the expense of circumlocutions and occasional ambiguity; but in a text-book it is desirable that the student should be introduced to the most important of them at the very outset and be made sufficiently familiar with them to use them intelligently and accurately.

The celestial sphere conceived as infinite.

7. The Celestial Sphere.[1] — The sky appears like a hollow vault, to which the stars seem to be attached, like gilded nail-heads upon the inner surface of a dome. We cannot judge of the distance to this surface from the eye further than to perceive that it must be very far away; it is therefore natural and extremely convenient to regard the distance of the sky as everywhere the same and unlimited. The *celestial sphere*, as it is called, is conceived of as so enormous that the whole material universe of stars and planets lies in its center like a few grains of sand in the middle of the dome of the Capitol. Its diameter, in technical language, is taken as *mathematically infinite*, *i.e.*, greater than any assignable quantity.

Since the radius of the sphere is thus infinite, it follows that all the lines of any set of parallels will appear, if produced indefinitely, to pierce it at a single point, the *vanishing point* of perspective, or *the point at infinity* of projective geometry. However far apart the lines may be and whatever, therefore, may be the distances in miles between the points at which they pierce the surface of the celestial sphere, yet, seen by the observer at its infinitely distant center, the angular distance between those points is utterly insensible, and they coalesce into one. Thus the axis of the earth and all lines parallel to it pierce the heavens at one point, the celestial pole; and the plane of the earth's equator, keeping parallel to itself during her annual circuit around the sun, marks out only one celestial equator in the sky.

---

[1] The study of the celestial sphere and its circles is greatly aided by the use of a globe or armillary sphere. Without some such apparatus it is rather difficult for a beginner to get clear ideas upon the subject.

Apparent convergence of parallels to a single point on the celestial sphere.

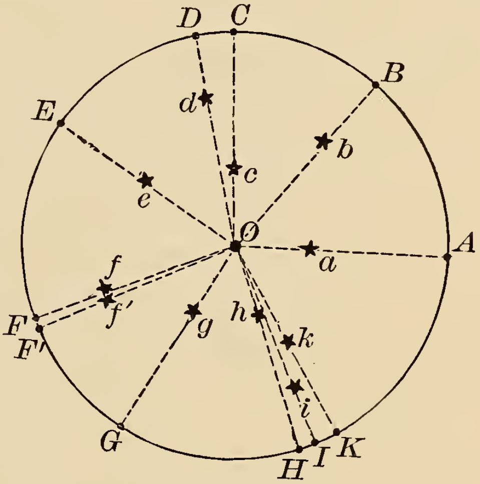


FIG. 1

8. The place of a heavenly body is simply the point where a line drawn from the observer through the body in question and continued onward pierces the celestial sphere. It depends solely upon the direction of the body and has nothing to do with its distance. Thus, in Fig. 1 *A*, *B*, *C*, etc., are the apparent places of *a*, *b*, *c*, etc., the observer being at *O*. Objects that are nearly in line with each other, as *h*, *i*, *k*, will appear close together in the sky, however great the real distance between them. The moon, for instance, often looks to us very near a star, which is always at an immeasurable distance beyond her.

Place of a heavenly body depends solely on its direction from observer.

9. Linear and Angular Dimensions and Measurement. — Linear dimensions are such as can be expressed in linear units; *i.e.*, in miles, feet, or inches; kilometers, meters, or millimeters. Angular dimensions are expressed in angular units; *i.e.*, in degrees, minutes, and seconds, or sometimes in *radians*, the radian being the angle which is measured by an arc equal in length to the radius, determined by dividing the circumference by 2π.

Value of the radian in degrees, minutes, and seconds.

The radian, therefore, equals **57°.29** (*i.e.*, $360° \div 2\pi$),
or **3437′.75** (*i.e.*, $21600' \div 2\pi$),
or **206264″.8** (*i.e.*, $1\,296000'' \div 2\pi$).

Hence, to reduce to seconds of arc an angle expressed in radians, we must multiply its value in radians by 206264.8; a relation of which we shall make frequent use.

Angular units used in expressing measurements on celestial sphere.

Obviously, angular units alone can properly be used in describing apparent distances in the sky. One cannot say correctly that the two stars known as "the pointers" are so many *feet* apart; their distance is about five *degrees*.

It is very important that the student of astronomy should accustom himself as soon as possible to estimate celestial measures in angular units. A little practice soon makes it easy, although the beginner is apt to be embarrassed by the fact that the sky appears to the eye to be not a true hemisphere, but a flattened vault, so that all estimates of angular distances for objects near the horizon are apt to be exaggerated. The moon when rising or setting looks to most persons much larger than when overhead, and the "Dipper-bowl" when underneath the pole seems to cover a much larger area than when above it.

Apparent enlargement of sun and moon near the horizon.

This illusion (for it is merely an illusion), which makes the sun and heavenly bodies when near the horizon appear larger than when high up in the sky, is probably due to the fact that in the latter case we have no intervening objects by which to estimate the distance, and it therefore is judged to be smaller than at the horizon. If we look at the sun or moon when near the horizon through a lightly smoked glass which cuts off the view of the landscape, the object immediately shrinks to its ordinary size.

Relation between distance, radius, and angular semi-diameter of a globe.

10. Relation between the Distance and Apparent Size of an Object. — Suppose a globe having a (linear) radius BC equal to r. As seen from the point A (Fig. 2) its *apparent* (*i.e.*, angular) semidiameter will be BAC or s, its distance being AC or R.

We have immediately, from trigonometry, since B is a right angle, $\sin s = r / R$, whence also $r = R \times \sin s$, and $R = r \div \sin s$.

If, as is usual in astronomy, the diameter of an object is small as compared with its distance, so that sin s practically

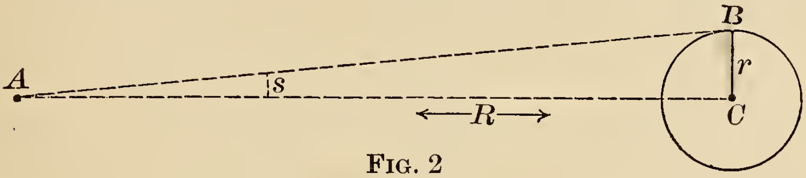


FIG. 2

equals s itself, we may write $s = r/R$, which gives s in radians (not in degrees or seconds). If we wish to have it in the ordinary angular units,

$$s^\circ = 57.3\, r/R\,;\ \text{ or } s' = 3437.7\, r/R\,;\ \text{ or } s'' = 206264.8\, r/R\,;$$
$$\text{also } R = 206264.8\, r/s'',$$

where s° means s in *degrees;* s', in *minutes* of arc; s'', in *seconds* of arc.

In either form of the equation we see that the apparent diameter *varies directly as the linear diameter and inversely as the distance.*

In the case of the moon, $R =$ about 239000 miles; and r, 1081 miles. Hence s (in radians) $= \frac{1081}{239000} = \frac{1}{221}$ of a radian, which is about $933''$, — a little more than one fourth of a degree.

Apparent distance of the surface of the celestial sphere.

It may be mentioned here as a rather curious fact that to most persons the moon, when at a considerable altitude, appears about a foot in diameter; — at least, this seems to be the average estimate. This implies that the surface of the sky appears to them only about 110 feet away, since that is the distance at which a disk one foot in diameter would have an angular diameter of $\frac{1}{110}$ of a radian, or $\frac{1}{2}^\circ$.

Probably this is connected with the physiological fact that our muscular sense enables us to judge moderate distances pretty fairly up to 80 or 100 feet, through the "binocular parallax" or convergence of the eyes upon the object looked at. Beyond that distance the convergence is too slight to be perceived. It would seem that we instinctively estimate the moon's distance as small as our senses will permit when there are no intervening objects which compel our judgment to put her further off.

POINTS AND CIRCLES OF REFERENCE AND SYSTEMS OF COÖRDINATES

In order to be able to describe intelligently the position of a heavenly body in the sky, it is convenient to suppose the inner surface of the celestial sphere to be marked off by circles traced upon it, — imaginary circles, of course, like the meridians and parallels of latitude upon the surface of the earth.

Three distinct systems of such circles are made use of in astronomy, each of which has its own peculiar adaptation for its special purposes.

A. SYSTEM DEPENDING ON THE DIRECTION OF GRAVITY AT THE POINT WHERE THE OBSERVER STANDS

Astronomical zenith and nadir defined.

11. The Zenith and Nadir. — If we suspend a plumb-line, and imagine the line extended upward to the sky, it will pierce the celestial sphere at a point directly overhead, known as the *Astronomical Zenith*, or the *Zenith* simply, unless some other qualifier is annexed.

As will be seen later (Sec. 130, *b*), the plumb-line does not point exactly to the center of the earth, because the earth rotates on its axis and is not strictly spherical. If an imaginary line be drawn from the *center of the earth* upward through the observer, and produced to the celestial sphere, it marks a different point, known as the *geocentric zenith*, which is never very far from the astronomical zenith, but must not be confounded with it.

Geocentric zenith.

For most purposes the astronomical zenith is the better practical point of reference, because its position can be determined directly by observation, which is not the case with the other.

The *Nadir* is the point opposite to the zenith, directly under foot in the invisible part of the celestial sphere.

Both "zenith" and "nadir" are derived from the Arabic, as are many other astronomical terms. It is a reminiscence of the centuries when the Arabs were the chief cultivators of science.

The horizon defined.

12. The Horizon. — If now we imagine a great circle drawn completely around the celestial sphere half-way between the zenith and nadir, and therefore 90° from each of them, it will be the *Horizon* (pronounced ho-rī′-zon, not hor′-ĭ-zon).

Since the surface of still water is always perpendicular to the direction of gravity, we may also define the horizon as the great circle in which a plane tangent to a surface of still water at the place of observation cuts the celestial sphere.

Unnecessary distinction between sensible and rational horizon.

Many writers distinguish between the *sensible* and *rational* horizons, — the former being defined by a horizontal plane drawn through the observer's eye, while the latter is defined by a plane parallel to this, but drawn through the center of the earth. These two planes, however, though 4000 miles apart, coalesce upon the infinite celestial sphere into a single *great circle* 90° from both zenith and nadir, agreeing with the first definition given above. The distinction is unnecessary.

The visible horizon.

13. Visible Horizon. — The word "horizon" (from the Greek) means literally "the boundary" — that is, the limit of the landscape, where sky meets earth or sea; and this boundary line is known in astronomy as the *visible horizon.* On land it is of no astronomical importance, being irregular; but at sea it is practically a true circle, nearly coinciding with the horizon above defined, but a little below it. When the observer's eye is at the water-level, the coincidence is exact; but if he is at an elevation above the surface, the line of sight drawn from his eye tangent to the water inclines or dips down, on account of the curvature of the earth, by a small angle known as the *dip of the horizon*, to be discussed further on (Sec. 77).

Vertical circles and the meridian.

14. Vertical Circles; the Meridian and the Prime Vertical. — *Vertical circles* are great circles drawn from the zenith at right angles to the horizon, and therefore passing through the nadir also. Their number is indefinite.

That particular vertical circle which passes north and south through the pole, to be defined hereafter, is known as the *Celestial Meridian*, and is evidently the circle traced upon the celestial sphere by the plane of the terrestrial meridian upon which the observer is located. The vertical circle at right angles to the meridian is called the *Prime Vertical.* The points where the meridian intersects the horizon are the *north* and *south points;* and the *east* and *west points* are midway between them. These are known as the *Cardinal Points.*

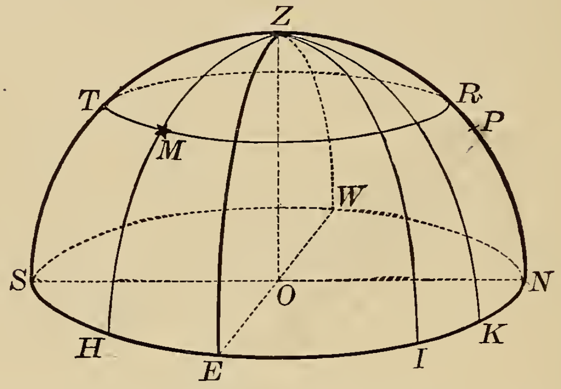


Fig. 3. — The Horizon and Vertical Circles

O, the place of the observer.
OZ, the observer's vertical.
Z, the zenith; *P*, the pole.
SWNE, the horizon.
SZPN, the meridian.
EZW, the prime vertical.

M, some star.
ZMH, arc of the star's vertical circle.
TMR, the star's almucantar.
Angle *TZM*, or arc *SH*, star's *azimuth.*
Arc *HM*, star's *altitude.*
Arc *ZM*, star's *zenith-distance.*

The parallels of altitude, or *almucantars*, are small circles of the celestial sphere drawn parallel to the horizon, sometimes called *circles of equal altitude.*

Altitude and zenith-distance defined.

15. Altitude and Zenith-Distance. — The *Altitude* of a heavenly body is its angular elevation above the horizon, *i.e.*, the number of degrees between it and the horizon, measured on a vertical circle passing through the object. In Fig. 3 the vertical circle *ZMH* passes through the body *M*. The arc *MH* is the *altitude* of *M*, and the arc *ZM* (the complement of *MH*) is its *zenith-distance*.

Azimuth defined.

16. Azimuth. — The *Azimuth* (an Arabic word) of a heavenly body is the same as its "bearing" in surveying; measured, however, from the true meridian and not from the magnetic. It may be defined as *the angle formed at the zenith between the meridian and the vertical circle which passes through the object;* or, what comes to the same thing, it is *the arc of the horizon intersected between the south point and the foot of this circle.*

Amplitude defined.

In Fig. 3 *SZM* is the azimuth of *M*, as is also the arc *SH*, which measures this angle. The distance of *H* from the east or west point of the horizon is called the *amplitude* of the body, but the term is seldom used except in describing the point where the sun or moon rises or sets.

Method of reckoning azimuth.

There are various ways of reckoning *azimuth*. Formerly it was usually expressed in the same way as the "bearing" in surveying; *i.e.*, so many degrees east or west of north or south. In the figure, the azimuth of *M* thus expressed is about S. 50° E. The more usual way at present, however, is to reckon it from the south point clear around through the west to the point of beginning, so that the arc *SWNKEH* would be the azimuth of *M*, — about 310°.

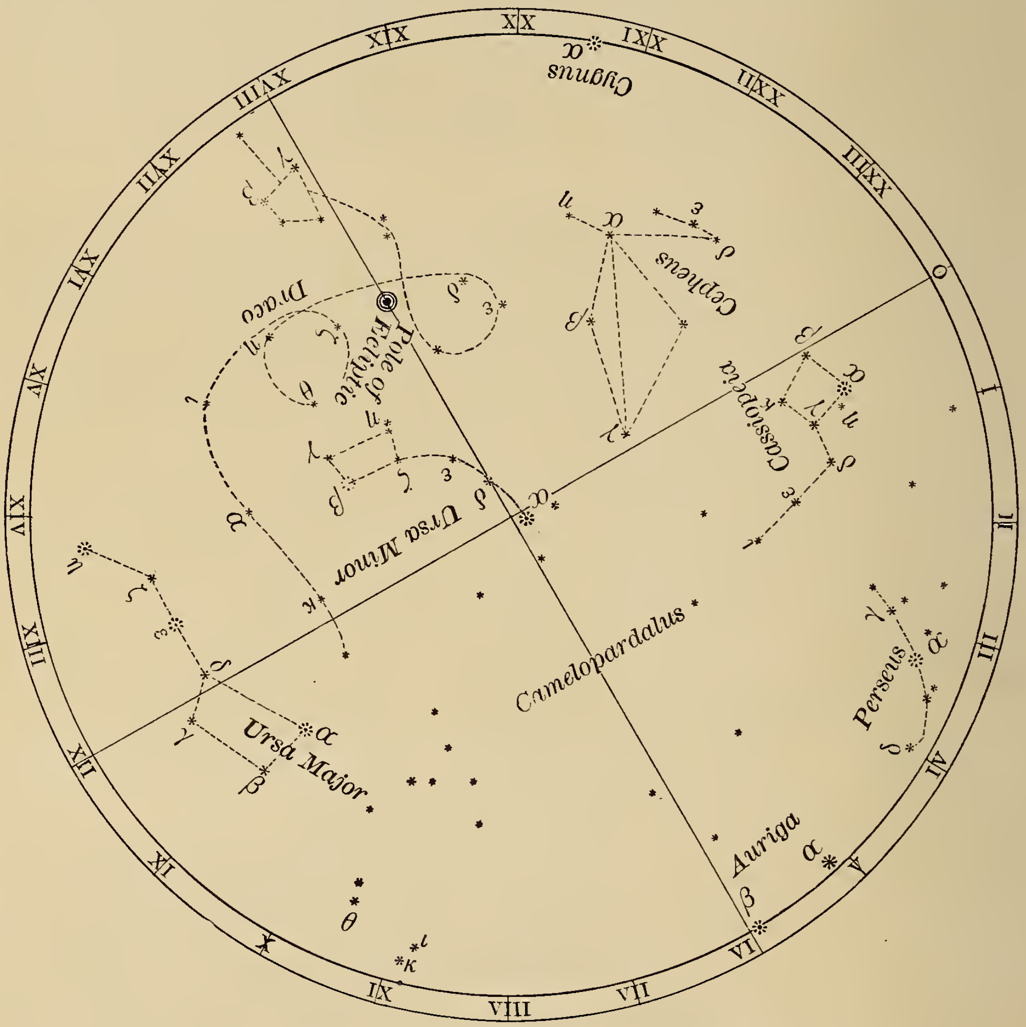
Inconvenience of altitude and azimuth.

17. Altitude and azimuth are for many purposes inconvenient, because they continually change for a celestial object. It is desirable, therefore, in defining the place of a body in the heavens, to use a different way which shall be free from this objection; and this can be done by taking as the fundamental line of our system, not the direction of gravity, which is different at any two different points on the earth's surface and is continually changing as the earth revolves, but the *direction of the earth's axis*, which is practically constant.

B. SYSTEM DEPENDING UPON THE DIRECTION OF THE EARTH'S AXIS OF ROTATION

Apparent rotation of the heavens.

18. The Apparent Diurnal Rotation of the Heavens. — If on some clear evening in the early autumn, say about eight o'clock on the 22d of September, we face the north, we shall find the appearance of that part of the heavens directly before us substantially as shown in Fig. 4. In the north is the constellation of the Great Bear (Ursa Major), characterized by the conspicuous group of seven stars, known as the Great Dipper, which lies with its handle sloping upward to the west. The two


XX
XXI
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0
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IV
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VI
VII
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XI
XII
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XIV
XV
XVI
XVII
XVIII
XIX
Cygnus
Cepheus
Draco
Pole of Ecliptic
Cassiopeia
Ursa Minor
Camelopardalus
Perseus
Ursa Major
Auriga

Fig. 4. — The Northern Circumpolar Constellations

easternmost stars of the four which form its bowl are called the *pointers*, because they point to the *pole-star*, — a solitary star not quite half-way from the horizon to the zenith (in the latitude of New York), and about as bright as the brighter of the two pointers. It is often called *Polaris*.

The pole-star and the pointers.

High up on the opposite side of the pole-star from the Great Dipper, and at nearly the same distance, is an irregular zigzag of five stars, each about as bright as the pole-star itself. This is the constellation of Cassiopeia.

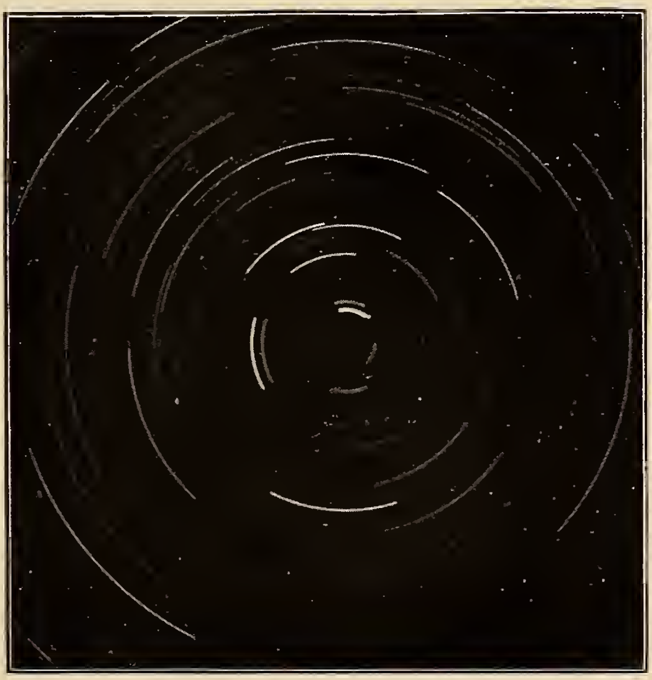

FIG. 5. — Polar Star Trails

If now we watch these stars for only a few hours, we shall find that while all the configurations remain unaltered, their places in the sky are slowly changing. The Dipper slides downward towards the north, so that by eleven o'clock the pointers are directly under *Polaris*. Cassiopeia still keeps opposite, however, rising towards the zenith; and if we were to continue to watch them the whole night, we should find that all the stars appear to be moving in circles around a point near the pole-star, revolving in the opposite direction to the hands of a watch (as we look up towards the north), with a steady motion which takes them completely around once a day, or, to be exact, once in the *sidereal day*, consisting of $23^h56^m4^s.1$ of ordinary time. They behave just as if they were attached to the inner surface of a huge revolving sphere.

Instead of watching the stars with the eye, the same result can be still better reached by photography. A camera is pointed

Polar star trails.

up towards the pole-star and remains firmly fixed while the stars, by their diurnal motion, impress their "trails" upon the plate. Fig. 5 is copied from a negative made by the author with an exposure of about three hours.

If instead of looking towards the north we now look southward, we shall find that there also the stars appear to move in the same kind of way. All that are not too near the pole-star rise somewhere in the eastern horizon, ascend not vertically but obliquely to the meridian, and descend obliquely to their setting at points on the western horizon. The motion is always in an arc of the circle, called the star's *diurnal circle*, the size of which depends upon the star's distance from the pole. Moreover, all these arcs are strictly parallel.

Diurnal circles.

The ancients accounted for these obvious facts by supposing the stars actually fixed upon a real material sphere, really turning daily in the manner indicated. According to this view there must therefore be upon the sphere two opposite, pivotal points which remain at rest, and these are the *poles.*

Definition of the poles of rotation.

19. Definition of the Poles. — The *Celestial Poles*, or *Poles of Rotation* (when it is necessary, as sometimes happens, to distinguish between these poles and the *poles of the ecliptic*), may therefore be defined as those *two points in the sky where a star would have no diurnal motion.* The exact position of either pole may be determined with proper instruments by finding the center of the small diurnal circle described by some star near it, as for instance by the pole-star.

Since the two poles are diametrically opposite in the sky, only one of them is usually visible from a given place; observers north of the equator see only the north pole, and *vice versa* in the southern hemisphere.

Mechanical definition of the pole.

Knowing as we now do that the apparent revolution of the celestial sphere is due to the real rotation of the earth on its axis, we may also define the poles as the *two points where the earth's axis of rotation* (*or any set of lines parallel to it*), *produced indefinitely, would pierce the celestial sphere.*

Definition of the celestial equator.

20. The Celestial Equator, or Equinoctial, and Hour-Circles. — *The Celestial Equator is the great circle of the celestial sphere, drawn half-way between the poles* (therefore everywhere 90° from each of them), *and is the great circle in which the plane of the earth's equator cuts the celestial sphere* (Fig. 6). It is often called the *Equinoctial.* Small circles drawn parallel to the equinoctial, like the parallels of latitude on the earth, are called *parallels of declination*, a star's parallel of declination being identical with its *diurnal circle.*

Parallels of declination identical with diurnal circles.

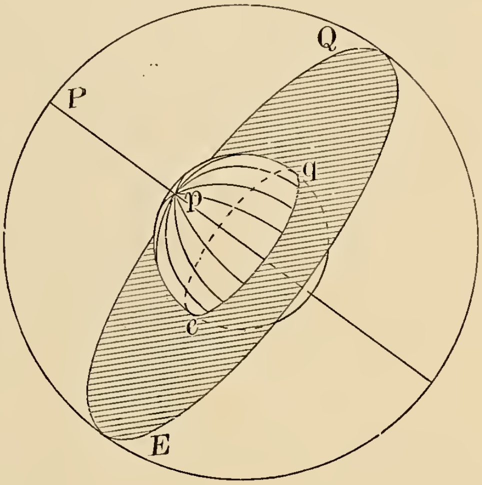

Fig. 6. — The Plane of the Earth's Equator produced to cut the Celestial Sphere

The great circles of the celestial sphere which pass through the poles, like the meridians on the earth, and are therefore perpendicular to the celestial equator, are called *Hour-Circles.* On celestial globes twenty-four of them are usually drawn, corresponding one to each of the twenty-four hours, but the real number is indefinite; an hour-circle can be drawn through any star. *That particular hour-circle which at any moment passes through the zenith of the observer, coincides with the celestial meridian*, already defined.

Hour-circles defined. The meridian as an hour-circle.

21. Declination and Hour Angle. — *The Declination of a star is its distance in degrees north or south of the celestial equator; + if north, — if south.* It corresponds precisely with the *latitude* of a place on the earth's surface, but cannot be called *celestial latitude*, because the term has been preoccupied by an entirely different quantity to be defined later (Sec. 27).

Declination defined.

The *Hour Angle* of a star at any moment *is the angle at the pole between the celestial meridian and the hour-circle of the star.* In Fig. 7, for the body *m* it is the angle *mPZ*, or the arc *QY.*

Hour angle defined.

Relation of units of time to units of angle.

This angle, or arc, may of course be measured like any other, in degrees, but since it depends upon the time which has elapsed since the body was last on the meridian, it is more usual to measure it in hours, minutes, and seconds of time. The *hour* is then equivalent to $\frac{1}{24}$ of a circumference, or 15°, and the *minute* and *second* of *time* to 15′ and 15″ of arc, respectively. Thus, an hour angle of $4^h2^m3^s$ equals 60° 30′ 45″.

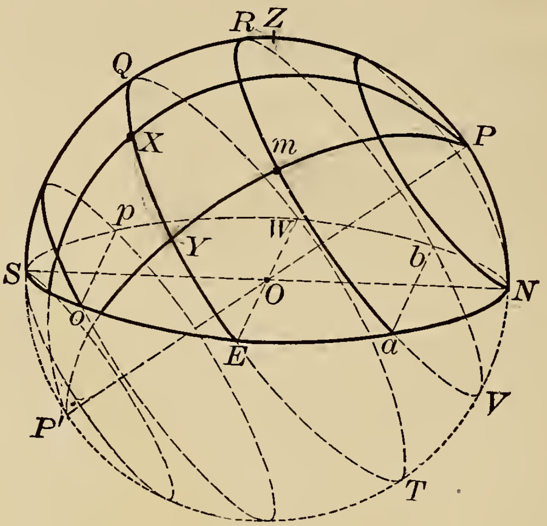

FIG. 7. — Hour-Circles, etc.

O, place of the observer; *Z*, his zenith.
SENW, the horizon.
POP′, the axis of the celestial sphere.
P and *P′*, the two poles of the heavens.
EQWT, the celestial equator, or equinoctial.
X, the vernal equinox, or "first of Aries."
PXP′, the equinoctial colure, or zero hour-circle.
m, some star.
Ym, the star's declination; *Pm*, its north-polar distance.
Angle *mPR* = arc *QY*, the star's (eastern) hour angle; = 24^h *minus* star's western hour angle.
Angle *XPm* = arc *XY*, star's right ascension.
Sidereal time at the moment = 24^h *minus* angle *XPQ*.

The position of the body *m* (Fig. 7) is, then, perfectly defined by saying that its *declination* is + 25° and its *hour angle* 40° *east* (or simply 320°, if we choose, as is usual, to reckon completely around in the direction of the diurnal motion). Instead of 40 *degrees*, we might say 2^h40^m of *time east*, or simply 21^h20^m to correspond to the 320°.

22. The declination of a star, omitting certain minutiæ for the present, remains practically unaltered even for years, but the hour angle changes continually and uniformly at the rate of 15° for every sidereal hour. This unfits it for use in ephemerides or star-catalogues. We must substitute for the meridian some other hour-circle passing through a well-defined point which participates in the diurnal rotation and so retains an unchanging position relative to the stars. Such a point, selected by astronomers nearly two thousand years ago, is the so-called *Vernal Equinox*, or *First of Aries.*

The hour angle changes continually with the time.

23. The Ecliptic, Equinoxes, Solstices, and Colures. — The sun, moon, and planets, though apparently carried by the diurnal revolution of the celestial sphere, are not, like the stars, apparently fixed upon it, but move over its surface like glowworms creeping on a whirling globe. In the course of a year, as will be explained later (Sec. 156), the sun makes a complete circuit of the heavens, traveling among the stars in a great circle called the *Ecliptic.*

The ecliptic.

The ecliptic cuts the celestial equator in two opposite points at an angle of about 23½°. These points are the *equinoxes.* The *Vernal Equinox*, or *First of Aries* (symbol ♈), *is the point where the sun crosses from the south to the north side of the equator*, on or about the 21st of March. The other is the *autumnal equinox.*

The vernal equinox.

The summer and winter *Solstices* are points on the ecliptic, midway between the two equinoxes and 90° from each, where the sun attains its maximum declination of + 23½° and − 23½° in summer and winter, respectively.

The solstices.

The hour-circles drawn from the pole (of rotation) through the equinoxes and solstices are called the equinoctial and solstitial *Colures.*

The colures.

Neglecting for the present the gradual effect of precession (Sec. 165), these points and circles are fixed with reference to the stars, and form a framework by which the places of celestial objects may be conveniently defined and catalogued.

Position of the vernal equinox.

No conspicuous star marks the position of the vernal equinox; but a line drawn from the pole-star through β Cassiopeiæ and continued 90° from the pole will strike very near it.

Definitions of right ascension.

24. Right Ascension.—The *Right Ascension* of a star may now be defined as *the angle made at the celestial pole between the hour-circle of the star and the hour-circle which passes through the vernal equinox* (called the *equinoctial colure*), or as *the arc of the celestial equator intercepted between the vernal equinox and the point where the star's hour-circle cuts the equator.* Right ascension is reckoned always *eastward* from the equinox, completely around the circle, and may be expressed either in degrees or in time units. A star one degree *west* of the equinox has a right ascension of 359°, or 23^h56^m.

Evidently the diurnal motion does not affect the right ascension of a star, but, like the declination, it remains practically unchanged for years. In Fig. 7 (Sec. 21), if X be the vernal equinox, the right ascension of m is the angle XPm, or the arc XY measured from ♈ X *eastward.*

The sidereal day.

25. Sidereal Day and Sidereal Time. — The sidereal *day* is the interval of time between two successive passages of a fixed star over a given meridian, and at any place it begins at the moment when the *vernal equinox is on the meridian;* it is about *four minutes shorter* than the solar day, and like it is divided into twenty-four (sidereal) hours with corresponding sidereal minutes and seconds, all shorter than the corresponding solar units.

Sidereal time.

The sidereal *time* at any moment is the time shown by a clock so set and regulated as to show zero hours, zero minutes, and zero seconds at the moment when the vernal equinox crosses the meridian. It is *the hour angle of the vernal equinox,* or, what is the same thing, *the right ascension of the observer's meridian.*

Definition of sidereal time.

26. Observatory Definition of Right Ascension. — The right ascension of a star may now be correctly, and for observatory purposes, most conveniently defined as *the sidereal time at the moment when the star is crossing the observer's meridian.* Since the sidereal clock indicates zero hours at sidereal noon, *i.e.*, at the moment when the vernal equinox is on the meridian, its face at any other time shows the hour angle of the equinox; and this is what has just been defined as the right ascension of all stars which may then happen to be on the meridian (common to them all since they all lie on the same hour-circle).

Observatory definition of right ascension.

C. SYSTEM DETERMINED BY THE PLANE OF THE EARTH'S ORBIT

27. Celestial Latitude and Longitude.—The ancient astronomers confined their observations mostly to the sun, moon, and planets, which are never far from the ecliptic, and for this reason the *ecliptic* (which is simply the trace of the plane of the earth's orbit upon the celestial sphere) was for them a more convenient circle of reference than the equator, — especially as they had no accurate clocks. According to their terminology, *Latitude (celestial) is the angular distance of a heavenly body north or south of the ecliptic; Longitude (celestial) is the arc of the ecliptic intercepted between the vernal equinox* (♈) *and the foot of a circle drawn from the pole of the ecliptic to the ecliptic through the object.* Longitude, like right ascension, is always reckoned *eastward* from the equinox.

Definition of celestial latitude and longitude.

Circles drawn from the poles of the ecliptic perpendicular to the ecliptic are called *secondaries to the ecliptic,* — by some writers "ecliptic meridians," and on some celestial globes are drawn instead of hour-circles.

Secondaries to the ecliptic.

The *poles of the ecliptic* are the points 90° distant from the ecliptic. The position of the north ecliptic pole is shown in Fig. 4. It is on the solstitial colure, about $23\frac{1}{2}°$ distant from the pole of rotation, in declination $66\frac{1}{2}°$ and right ascension 18^h. It is marked by no conspicuous star.

Poles of the ecliptic.

It is unfortunate, or at least confusing to beginners, that celestial latitude and longitude should not correspond with the terrestrial quantities that bear the same name. Great care must be taken to observe the distinction.

The gravity system of coördinates.

28. Recapitulation. — The *direction of gravity* at the point where the observer happens to stand determines the *zenith* and *nadir*, the *horizon* and the *almucantars*, or *parallels of altitude*, and all the *vertical circles*. One of the verticals, the *meridian*, is singled out from the rest by the circumstance *that it is the projection of the observer's terrestrial meridian upon the celestial sphere and passes through the pole*, marking the north and south points where it cuts the horizon. *Altitude* and *azimuth*, or their complements, zenith-distance and amplitude, define the position of a body by reference to the horizon and meridian.

This set of points and circles shifts its position among the stars with every change in the place of the observer and every moment of time. Each place and hour has its own zenith, its own horizon, and its own meridian.

The two systems which depend upon the rotation of the earth.

In a similar way, the *direction of the earth's axis*, which is independent of the observer's place on the earth, determines the *pole* (of rotation), the *equator*, *parallels of declination*, and the *hour-circles*. Two of these hour-circles are singled out as reference lines: one of them is the hour-circle which at any moment passes through the zenith and coincides with the meridian, — a purely local reference line; the other, the *equinoctial colure*, which passes through the vernal equinox, a point chosen from its relation to the sun's annual motion.

Declination and *hour angle* define the place of a star with reference to the equator and *meridian*, while *declination* and *right ascension* refer it to the equator and *vernal equinox*. The latter are the coördinates usually given in star-catalogues and almanacs for the purpose of defining the position of stars and planets, *and they correspond exactly to latitude and longitude on* *the earth*, by means of which geographical positions are designated. ♈ in the sky takes the place of Greenwich on the earth.

The ecliptic system.

Finally, the earth's orbit gives us the great circle of the sky known as the *ecliptic;* and *celestial latitude* and *longitude* define the position of a star with reference to the ecliptic and the vernal equinox (♈). For most purposes this pair of coördinates is practically less convenient than right ascension and declination; but it came into use centuries earlier, and has advantages in dealing with the planets and the moon.

29. The scheme given below presents in tabular form the relations of the four different systems to each other. In each case one of the two coördinates is measured along a *primary* circle, from a point selected as the *origin*, to a point where a *secondary* circle cuts it, drawn through the object perpendicular to the primary. The second coördinate is the angular distance of the object from the primary circle measured along this secondary.

Tabular exhibit of the four systems of coördinates.

System		Primary Circle, how determined	Primary Circle	Origin	Secondary Circle	Coördinates	Usual Symbol
A		Direction of gravity	Horizon	South point on horizon	Vertical circle of star	Azimuth Altitude	(A) (h)
B	1	Rotation of earth	Celestial equator	Foot of the meridian on equator	Hour-circle of star	Hour angle Declination	(θ) (δ)
	2	Rotation of earth	Celestial equator	The vernal equinox (♈)	Hour-circle of star	Right ascension Declination	(α) (δ)
C		Plane of earth's orbit	Ecliptic	The vernal equinox (♈)	Secondary to ecliptic through star	Longitude Latitude	(λ) (β)

30. Relation of the Coördinates on the Sphere. — Fig. 8 shows how these coördinates are related to each other. The reader is supposed to be looking down on the celestial sphere from above, the circle *SENWA* being the horizon.

Diagram showing the relation of the systems.

Z is the zenith; P, the north pole (of rotation); P', the pole of the ecliptic; ♈, the vernal equinox, and ♎, the autumnal; S, E, N, W are the cardinal points of the horizon. The oval W♈$MQCE$♎R is the celestial equator, and the narrower one, ♈LB♎K, is the ecliptic. The angle B♈C, measured by the arcs BC and PP', is the *obliquity* of the ecliptic, for which the usual symbol is e or ϵ.

O is some celestial object. Then the arc AO (projected as a straight line) is its *altitude* and the angle OZS its *azimuth*. OM is its declination

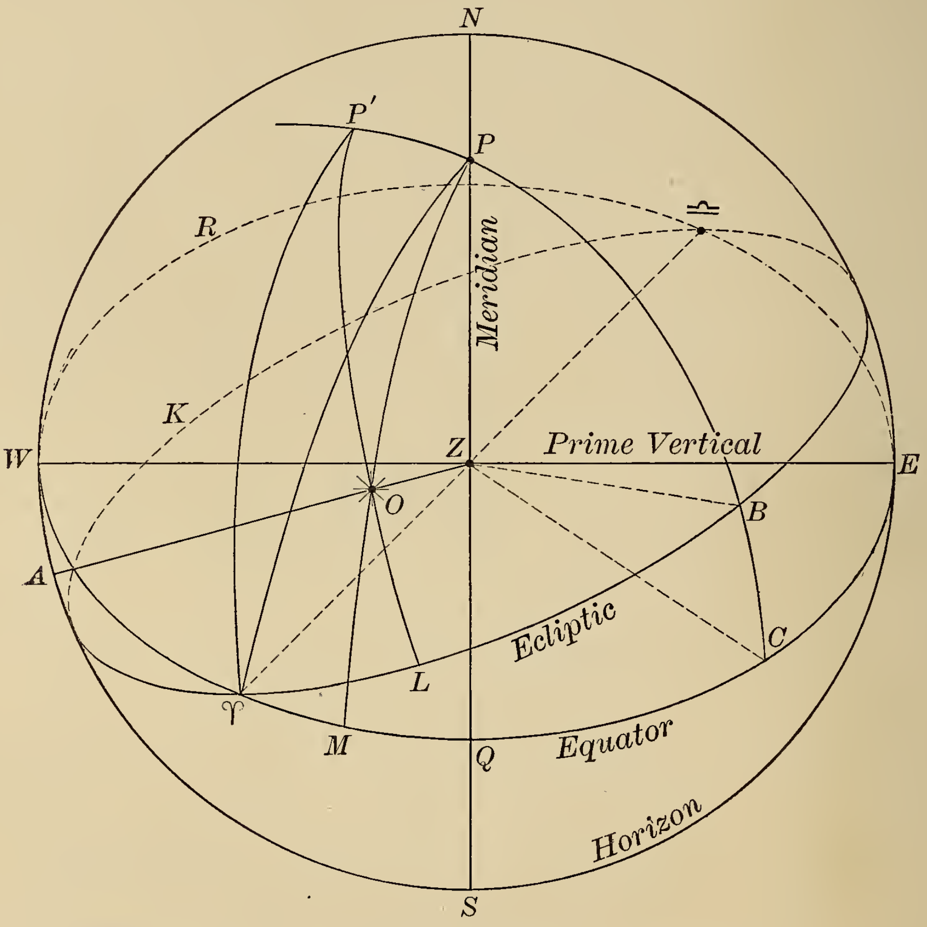

Fig. 8. — Relation of the Different Coördinates

and OPQ its *hour angle*. ♈PM is its *right ascension* = arc ♈M. OL is its *latitude* and ♈$P'L$ (= arc ♈L) is its *longitude*. ♈P is 90° of the equinoctial colure and $P'PBC$ is 90° of the solstitial colure. The angles ♈$P'B$ and ♈PC' are each 90°.

For methods and formulæ by which either set of coördinates may be "transformed" into one of the others, see Secs. 700 and 701 (Appendix).

31. The Astronomical Triangle. — The triangle *PZO* (pole-zenith-object) (Fig. 8) is often called *the astronomical triangle* because so many problems, especially of nautical astronomy, depend on its solution. Its sides and angles are all named, — *PZ* is the *colatitude of the observer*, *ZO* is the *zenith-distance of the object*, and *OP* is its *north polar distance*, or complement of its declination. The angle *P* is the *hour angle* of the object, the angle *Z* is the *supplement of its azimuth*, and, finally, the angle at *O* is called the *parallactic angle*, because it enters into the calculations of the effects of parallax and refraction upon the right ascension and declination of a body. Any three of the parts being given the others can, of course, be found.

The "astronomical triangle."

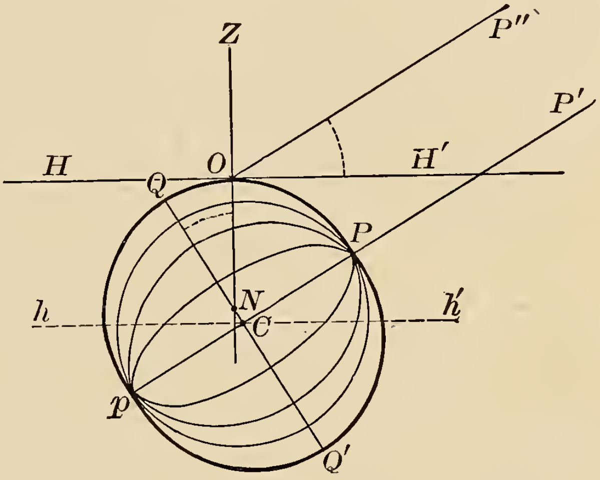

Fig. 9. — Relation of Latitude to the Elevation of the Pole

32. Relation of the Place of the Celestial Pole to the Observer's Latitude. — If an observer were at the north pole of the earth, it is clear that the pole-star would be very near his zenith, while it would be at his horizon if he were at the equator. The place of the pole in the sky, therefore, depends entirely on the observer's latitude, and in this very simple way *the altitude of the pole* (its height in degrees above the horizon) *is always equal to the latitude of the observer*. This will be clear from Fig. 9. The latitude (astronomical) of a place may be defined as *the angle between the direction of gravity at that place and the plane of the earth's equator*, — the angle *ONQ* in Fig. 9. If at *O* we draw *HH'* perpendicular to *ON*, it will be a level line, and will lie in the plane of the horizon. From *O* also draw *OP''* parallel to *CP'*, the earth's axis. *OP''* and *CP'*, being parallel, will both be directed

Position of the pole in the sky.

The altitude of the pole equals the observer's latitude.

to their "vanishing point" in the celestial sphere (Sec. 7), which is the *celestial pole.* The angle $H'OP''$ is therefore the *altitude of the pole* as seen at O; and it obviously equals ONQ. This fundamental relation, that the **altitude of the pole is identical with the observer's latitude,** cannot be too strongly emphasized.

Aspect of the heavens as seen from the earth's equator.

33. The Right Sphere. — If the observer is situated at the earth's equator, that is, in latitude *zero*, the pole will be in his horizon and the celestial equator will be a vertical circle, coinciding with the prime vertical (Sec. 14). All heavenly bodies will *rise and set vertically*, and their diurnal circles will all be bisected by the horizon, so that they will be twelve hours above and twelve hours below it; and the length of the night will always equal that of the day (neglecting refraction, Sec. 82). This aspect of the heavens is called the *right sphere.*

It is worth noting that for an observer *exactly* at the north pole the definitions of meridian and azimuth break down, since at that point the zenith coincides with the pole. Facing which direction he will, he is still looking directly *south.* If he change his place a few steps, however, his zenith will move, and everything will become definite again.

Aspect of the heavens as seen from the pole.

34. The Parallel Sphere. — If the observer is at the pole of the earth, where his latitude is 90°, the celestial pole will be at his zenith and the equator will coincide with the horizon. If at the north pole, all the stars north of the celestial equator will remain permanently above the horizon, never rising nor falling, but sailing around the sky on almucantars, or parallels of altitude. The stars in the southern hemisphere, on the other hand, will never rise to view.

The six months' day at the pole.

Since the sun and moon move among the stars in such a way that during half of the time they are north of the equator and half the time south of it, they will be half the time above the horizon and half the time below it, at least *approximately*, since this statement needs to be slightly modified to allow for the effect of refraction. The moon will be visible for about a fortnight each month and the sun for about six months each year.

Aspect of the heavens as seen from middle latitudes.

35. The Oblique Sphere. — At any station between the poles and the equator the pole will be elevated above the horizon, and the stars will rise and set in *oblique circles*, as shown in Fig. 10. Those whose distance from the elevated pole is less than PN (the latitude of the observer) will of course never set, remaining perpetually visible. The radius of this *circle of perpetual apparition*, as it is called (the shaded cap around P in the figure), is obviously just equal to the height of the pole, becoming larger as the latitude increases. On the other hand, stars within the same distance of the depressed pole will lie in the *circle of perpetual occultation*, and will never rise above the horizon. A star exactly on the celestial equator will have its diurnal circle bisected by the horizon and will be above the horizon twelve hours. A star north of the equator, if the north pole is the elevated one, will have more than half its diurnal circle above the horizon and will be visible for more than twelve hours each day; as, for instance, a star at A, rising at B and setting at B'.

The circles of perpetual apparition and occultation.

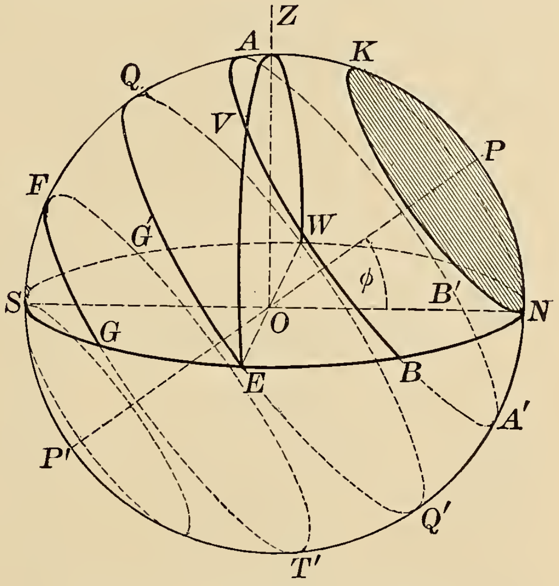

Fig. 10. — The Oblique Sphere

Whenever the sun is north of the celestial equator, the day will therefore be longer than the night for all stations in northern latitude; how much longer will depend on the latitude of the place and the sun's distance from the equator, *i.e.*, its declination.

The midnight sun.

36. The Midnight Sun. — If the latitude of the observer is such that PN in the figure is greater than the sun's polar distance or codeclination at the time when the sun is farthest north (about 66½°), the sun will come into the circle of perpetual apparition and will make a complete circuit of the heavens without setting, until its polar distance again becomes less than *PN*. This happens near the summer solstice at the North Cape and at *all stations within the Arctic circle.*

When the sun shines into north windows.

Whenever the sun is north of the equator it will in all north latitudes rise at a point north of east, as *B* in the figure, and will continue to shine upon every vertical surface that faces the north, until, as it ascends, it crosses the prime vertical *EZW* at some point *V*.

In the latitude of New York, the sun on the longest days of summer is south of the prime vertical only about eight hours of the whole fifteen during which it is above the horizon. During seven hours of the day it shines into north windows.

A celestial globe will be of great assistance in studying these diurnal phenomena. By means of this it can at once be seen what stars never set, which ones never rise, and during what part of the twenty-four hours a heavenly body at a known declination is above or below the horizon.

The celestial globe.

37. The Celestial Globe. — The celestial globe is a ball, usually of papier-mâché, upon which are drawn the circles of the celestial sphere and a map of the stars. It is mounted in a framework which represents the horizon and the meridian, in the manner shown by Fig. 11.

Its horizon and circles upon it.

The *horizon*, *HH′* in the figure, is usually a wooden ring three or four inches wide, directly supported by the pedestal. It carries upon its upper surface at the inner edge a circle marked with degrees for measuring the azimuth of any heavenly body, and outside this the so-called "zodiacal circles," which give the sun's longitude and the equation of time (Secs. 99 and 174) for every day of the year.

The meridian ring, its graduation and clamp.

The *meridian ring*, *MM′*, is a circular ring of metal which carries the bearings of the axis on which the globe revolves. Things are so arranged, or ought to be, that the mathematical axis of the globe is exactly in the same plane as the graduated face of the ring, which is divided into degrees and fractions of a degree, with zero at the equator. The meridian ring fits into two notches in the horizon circle and is held underneath the globe by a support with a clamp, which enables us to fix it securely in any desired position, the mathematical center of the globe being precisely in the planes both of the meridian ring and the horizon.

The hour index.

The *hour index* on the globe here figured is a pointer like the hour-hand of a clock, so attached to the meridian ring at the pole that it can be turned around the axis with stiffish friction, but will retain its position unchanged when the globe is made to turn under it. It points out the time on a small *time-circle* graduated usually to hours and quarters printed on the surface of the globe.

Circles drawn on surface of globe.

The *surface of the globe* is marked first with the celestial equator (Sec. 20), next with the ecliptic (Sec. 23), crossing the equator at an angle of 23½° (at X in the figure), and each of these circles is divided into degrees and fractions. The *equinoctial* and *solstitial colures* (Sec. 23) are also always represented. As to the other circles, usage differs. The ordinary way at present is to mark the globe with twenty-four *hour-circles*, fifteen degrees apart (the colures being two of them), and with *parallels of declination* ten degrees apart.

On the surface of the globe are plotted the positions of the stars and the outlines of the constellations.

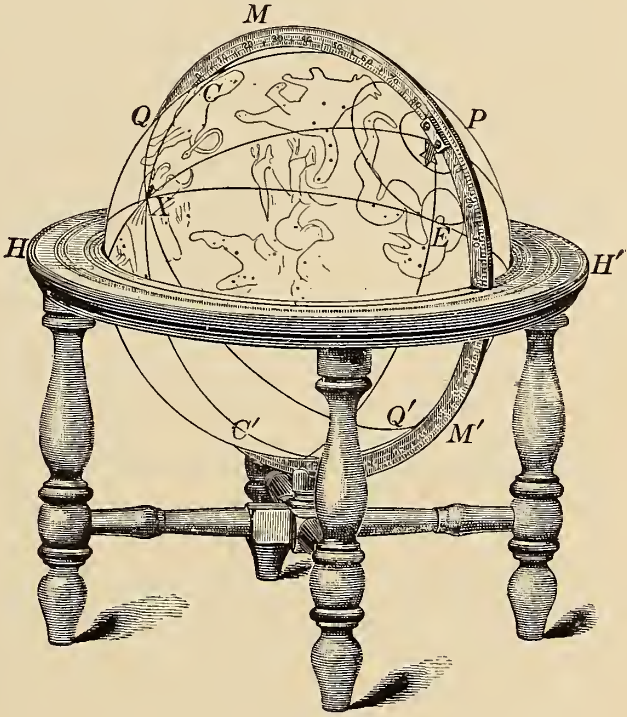

Fig. 11. — The Celestial Globe

38. To rectify a globe, — that is, to set it so as to show the aspect of the heavens at any given time, —

Setting for latitude of observer.

(1) Elevate the north pole of the globe to an angle equal to the observer's latitude by means of the graduation on the meridian ring, and clamp the ring securely.

(2) Look up the day of the month on the horizon of the globe and opposite to the day find, on the longitude circle, the sun's longitude for that day.

(3) On the ecliptic (on the surface of the globe) find the degree of longitude thus indicated and bring it to the graduated face of the meridian ring.

The globe is then set to correspond to (apparent) *noon* of the day in question. (It may be well to mark the place of the sun temporarily with a bit of moist paper applied at the proper place in the ecliptic; it can easily be wiped off after using.)

Setting for day of the year.

(4) Holding the globe fast, so as to keep the place of the sun on the meridian, turn the *hour index* until it shows on the graduated *time-circle* the local mean time of apparent noon, *i.e.*, $12^h \pm$ the equation of time given for the day on the horizon ring. (If standard time is used, the hour index must be set to the *standard* time of apparent noon.)

Setting for hour of the day.

(5) Finally, turn the globe until the hour for which it is to be set is brought to the meridian, as indicated on the hour index. The globe will then show the true aspect of the heavens.

The positions of the moon and planets are not given by this operation, since they have no fixed places in the sky and therefore cannot be put upon the globe by the maker. If one wants them represented, he must look up their right ascensions and declinations for the day in some almanac and mark the places on the globe with bits of wax or paper.

EXERCISES

1. What point in the celestial sphere has both its right ascension and declination zero?

2. What are the celestial latitude and longitude of this point?

3. What are the hour angle and azimuth of the zenith?

4. At what points does the celestial equator cut the horizon?

5. What angle does the celestial equator make with the horizon at these points, as seen by an observer in latitude 40°?

6. What if his latitude is 10°? 20°? 50°? 60°?

7. When the vernal equinox ♈ is rising on the eastern horizon, what angle does the ecliptic make with the horizon at that point for an observer in latitude 40°?

8. What angle when setting?

9. What is the angle between the ecliptic and horizon when the autumnal equinox is rising, and when setting?

10. Name the fourteen principal points on the celestial sphere (zenith, poles, equinoxes, etc.).

11. What important circles on the celestial sphere have no correlatives on the surface of the earth?

12. What are the approximate right ascension and declination (α and δ) of the sun on March 21 and September 22?

13. What is the sun's altitude at noon on March 21 for an observer in latitude 42°?

14. How far is the sun from the zenith at noon on March 21, as seen at Pulkowa, latitude 60°? How far at noon on June 21?

15. On March 21, one hour after sunset, whereabouts in the sky would be a star having a right ascension of 7 hours and declination of 40°, the observer being in latitude 40°?

16. If a star rises to-night at 10 o'clock, at what time (approximately) will it rise 30 days hence?

17. When the right ascension of the sun is 6 hours, what are its longitude (λ) and latitude (β)?

18. What, when its α is 12 hours?

19. What are the latitude and longitude of the north pole of rotation?

20. What are the right ascension and declination of the north pole of the ecliptic?

NOTE.—None of the above exercises require any calculation beyond a simple addition or subtraction.

21. What are the longitude and declination of the sun when its right ascension is 3 hours?

$$\textit{Ans.}\ \begin{cases} \text{Long.} = 47^\circ\ 27'\ 59''. \\ \text{Dec.} \ \ = 17^\circ\ 03'\ 08''. \end{cases}$$

NOTE.—This requires the solution of the spherical right angle triangle, in which the base is the given α (= 45°), the angle adjacent is ϵ (23° 27′), and the parts to be found are the hypotenuse λ and the other leg opposite ϵ, which is δ.

CHAPTER II

ASTRONOMICAL INSTRUMENTS

Telescopes, and their Accessories and Mountings — Timekeepers and Chronographs — The Transit-Instrument — The Prime Vertical Instrument — The Almucantar — The Meridian-Circle and Universal Instrument — The Micrometer — The Heliometer — The Sextant

39. Astronomical observations are of various kinds: sometimes we desire to ascertain the apparent distance between two bodies; sometimes the position which the body occupies at a given time, or the time at which it arrives at a given circle of the sky, — usually the meridian. Sometimes we wish merely to examine its surface, to measure its light, or to investigate its spectrum; and for all these purposes special instruments have been devised. We propose in this chapter to describe a few of the most important at present in use.

40. Telescopes in General. — Telescopes are of two kinds, refracting and reflecting. The former were first invented and are much more used, but the largest instruments which have ever been made are reflectors. In both the fundamental principle is identical. The large lens, or mirror, — the "objective" of the instrument — forms at its focus a "real" image of the object looked at, and this image is then examined and magnified by the eyepiece, which in principle is only a magnifying-glass.

Fundamental principle of the telescope.

41. The Simple Refracting Telescope. — This consists essentially, as shown in Fig. 12, of two convex lenses, one the object-glass *A*, of large size and long focus; the other, the eyepiece *B*, of short focus; the two being set at a distance nearly equal to the sum of their focal lengths. Recalling the optical principles

Essential elements of the refracting telescope.

of the formation of images by lenses,[1] we see that if the instrument is pointed toward the moon, for instance, all the rays that strike the object-glass from the top of the object will come to a focus at *a*, while those from the bottom will come to a focus at *b*, and similarly with rays from other points on the surface of the moon. We shall therefore get in the "focal plane" of the object-glass a small inverted "real" image of the moon, so that if a photographic plate is inserted in the focal plane at *ab* and properly exposed, we shall get a picture of the object.

Real image formed by object-glass.

The *size* of the picture will depend upon the apparent angular diameter of the object and the distance of the image *ab* from the object-glass, and is determined by the condition that, *as seen from point O* (the optical center of the object-glass), *the object and its image subtend equal angles*, since rays which pass through the point *O* suffer no sensible deviation.

Size of the image.

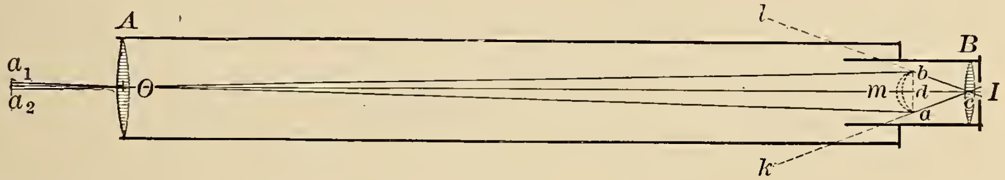

FIG. 12. — The Simple Refracting Telescope

If the focal length of the lens *A* is 10 feet, then the image of the moon formed by it will appear, when viewed from a distance of 10 feet, just as large as the moon itself; from a distance of 1 foot, the image will, of course, appear ten times as large.

With such an object-glass, therefore, even without an eyepiece, one can see the mountains of the moon and satellites of Jupiter by simply putting the eye in the line of the rays, at a distance of 10 or 12 inches back of the eyepiece hole, the eyepiece itself having been, of course, removed.

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[1] In this explanation we use the approximate theory of lenses (in which their thickness is neglected), as given in the elementary text-books. The more exact theory would require some slight modification in statements, but none of substantial importance.

42. Magnifying Power. — If we use the naked eye, one cannot, unless near-sighted, see the image distinctly from a distance much less than 10 inches; but if we use a magnifying-lens of 1-inch focus, we can view it from a distance of only an inch, and it will look correspondingly larger. Without stopping to demonstrate the principle, the magnifying power is simply equal to the *quotient obtained by dividing the focal length of the object-glass by that of the eye-lens;* or, as a formula,

Formula for the magnifying power.

$M = F/f$; that is, Od/cd in the figure.

If, for example, the focal length of the object-glass be 4 feet and that of the eye-lens one quarter of an inch, then

$$M = 48 \div \tfrac{1}{4} = 4 \times 48 = 192.$$

A magnifying power of *unity*, however, is often spoken of as "no magnifying power at all," since the image appears of the same size as the object.

The magnifying power of the telescope is changed at pleasure by simply changing the eyepiece (see Sec. 47).

Light-gathering power proportional to the square of the diameter of the object-glass.

43. Light-Gathering Power of the Telescope and Brightness of the Image. — This depends not upon the focal length of the object-glass, but upon its diameter; or, more strictly, its *area.* If we estimate the diameter of the pupil of the eye at one fifth of an inch, then (neglecting the loss in transmission through the lenses) a telescope 1 inch in diameter collects into the image of a star twenty-five times as much light as the naked eye receives; and the great Yerkes telescope of 40 inches in diameter gathers 40000 times as much, or about 35000 after allowing for the losses. The amount of light collected is proportional to the square of the diameter of the object-glass.

The apparent brightness of an object which, like the moon or a planet, shows a disk, is not, however, increased in any such ratio, because the light gathered by the object-glass is spread out by the magnifying power of the eyepiece. In fact, it can be demonstrated that no optical arrangement whatever can show

No optical arrangement can increase the intrinsic brightness of an extended surface.

an extended surface brighter than it appears to the naked eye. But the total quantity of light in the image of the object greatly exceeds that which is available for vision with the naked eye, and objects which, like the stars, are mere luminous points, have their brightness immensely increased, so that with the telescope millions otherwise invisible are brought to light. With the telescope, also, the brighter stars are easily seen in the daytime.

Chromatic aberration of a single-lens object-glass.

44. The Achromatic Telescope. — A single lens cannot bring the rays which emanate from a single point in the object to any exact focus, since the rays of different color (wave-length) are differently refracted, the blue more than the green, and this more than the red. In consequence of this so-called "chromatic aberration," the simple refracting telescope is a very poor instrument.[1]

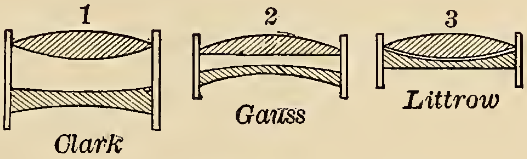

FIG. 13. — Different Forms of the Achromatic Object-Glass

The achromatic lens.

About 1760 it was discovered in England that by making the object-glass of two or more lenses of different kinds of glass the chromatic aberration can be nearly corrected. Object-glasses so made — no others are now in common use — are called *achromatic*, and they fulfil with reasonable approximation, though not perfectly, the condition of distinctness; namely, that the rays which emanate from any single point in the object should be collected to a single point in the image. In practice, only two lenses are ordinarily used in the construction of an astronomical object-glass, — a convex of crown-glass, and a concave of flint-glass, the curves of the two lenses and the distances between them being so chosen as to give the best possible correction of the spherical aberration as well as of the chromatic. Many forms of object-glass are made, three of which are shown in Fig. 13.

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[1] By making the telescope extremely long in proportion to its diameter, the distinctness of the image is considerably improved, and in the middle of the seventeenth century instruments more than 200 feet in length were used by Cassini and others. Saturn's rings and several of his satellites were discovered by Huyghens and Cassini with instruments of this kind.

Imperfect achromatism of object-glasses.

45. Secondary Spectrum. — It is not possible to obtain a perfect correction of color with the only kinds of glass which were available until very recently. Ordinary achromatic lenses, even the best of them, show around every bright object a strong purple halo, due to red and blue rays which are both brought to a focus further from the object-glass than are the yellow and green. This halo seriously injures the definition and makes it difficult to see small stars very near a bright one. It is specially obnoxious in large instruments.

More perfect object lenses from new kinds of glass.

Much is hoped from the new varieties of glass now being made at Jena in Germany. Several telescopes of considerable size have already been constructed, of which the lenses are practically *aplanatic;* that is, sensibly free from both spherical and chromatic aberration. Possibly a new era in telescope making is opening with the new century.

46. Diffraction and Spurious Disks. — Even if a lens were absolutely perfect as regards the correction of aberrations, it would still be unable to fulfil strictly the condition of distinctness.

Since light consists of waves of finite length, the image of a luminous point can never be also a point, but necessarily, on account of "diffraction," consists of a central disk of finite diameter, surrounded by a series of "interference" rings; and the image of a line is a streak and not a line. The diameter of the "spurious disk" of a star, as it is called, *varies inversely with the diameter of the object-glass;* the larger the telescope, the smaller the image of a star with a given magnifying power.

The spurious disk of a star.

With a good 4½-inch telescope and a power of about 120, the image of a small star, when the air is perfectly steady (which unfortunately seldom happens), is a clean, round disk, about 1″ in diameter, with a bright ring around it, separated from the disk by a dark space about as wide as the disk. With a 9-inch instrument the disk has a diameter of $0''.5$,—just half as great; with the Yerkes telescope, about $0''.11$. The angular diameter of a star disk in a telescope the aperture of which is a inches is, therefore, given by the following formula, due to Dawes:

Formula for diameter of spurious disk.

$$d'' = \frac{4''.5}{a}.$$

If the magnifying power is too great (more than about sixty to the inch of aperture), the disk of a star will become ill-defined at the edge; so that there is very little use with most objects in pushing the magnifying power any higher.

Superiority of large object-glasses in defining power.

This effect of "diffraction" has much to do with the superiority of large instruments in showing minute details; no increase of magnifying power on a small telescope can exhibit the object as sharply as the same power on a large one, provided, of course, that the object-glasses are equally good in workmanship and that the atmospheric conditions are satisfactory. (*But a given amount of atmospheric disturbance injures the performance of a large telescope much more than that of a small one.*)

47. Eyepieces, or "Oculars."—For some purposes the simple convex lens is the best eyepiece possible; but it performs well only for a small object, like a close double star, exactly in the center of the field of view. Generally, therefore, we employ eyepieces composed of two or more lenses, which give a larger field of view than a single lens and define fairly well over the whole extent of the field. They fall into two general classes, the *positive* and the *negative*.

Positive eyepieces.

The *positive* eyepieces are much more generally useful. They act as simple magnifying-glasses and can be taken out of the telescope and used as hand magnifiers if desired. The image of the object formed by the object-glass lies *outside* of this kind of eyepiece, between it and the object-glass.

Negative eyepieces.

In the *negative* eyepieces, on the other hand, the rays from the object are intercepted by the so-called "field lens" before reaching the focus, and the image is formed *inside* the eyepiece. It cannot therefore be used as a hand magnifier.

Fig. 14 shows the two most usual forms of eyepiece, and also the "solid eyepiece" constructed by Steinheil; but there are a multitude of various kinds. All these eyepieces show the object inverted, which is of no importance in astronomical observations.

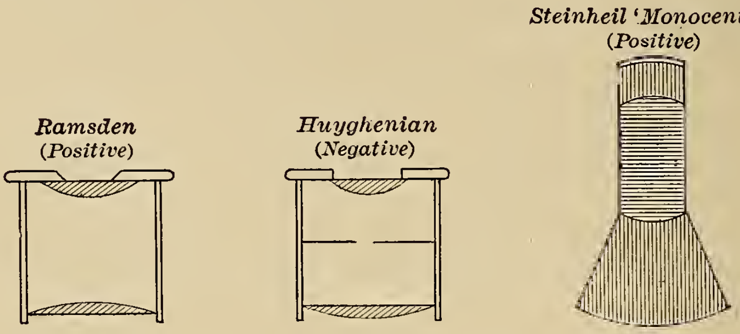


FIG. 14. — Various Forms of Telescope Eyepiece

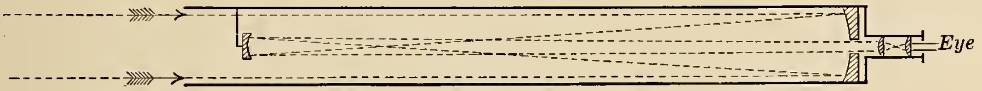
It is evident that in an achromatic telescope the object-glass is by far the most important and expensive member of the instrument. It costs, according to size, from $100 up to $65000, while the eyepieces cost only from $2 to $25 apiece, and every telescope of any pretension possesses a considerable stock, of various magnifying powers.

The reticle.

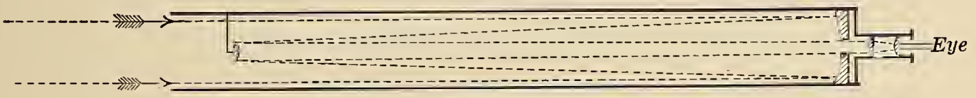
48. Reticle. — If the telescope is to be used for pointing upon an object, it must be provided with a "reticle" of some sort. The simplest is a frame with two spider-lines stretched across it at right angles to each other, their intersection being the point of reference. This reticle is placed, not at or near the object-glass, as often supposed, but *in the focal plane*, as *ab* in Fig. 12 (Sec. 41). Of course, positive eyepieces only can be used in connection with such a reticle, though in sextant telescopes a negative eyepiece is sometimes used with a pair of cross-wires placed between the two lenses of the eyepiece. Sometimes a glass plate with fine lines ruled upon it is used instead of spider-lines. In order to make the lines of the reticle visible at night, a faint light is reflected into the instrument by some one of various arrangements devised for the purpose.

The reflecting telescope.

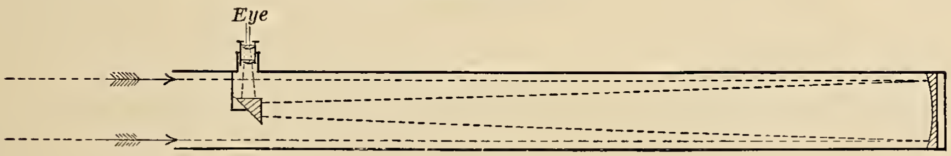
49. The Reflecting Telescope. — About 1670, when the chromatic aberration of refractors first came to be understood (in consequence of Newton's discovery of the decomposition of light), the reflecting telescope was invented. For nearly one hundred and fifty years it held its place as the chief instrument for star-gazing. There are several varieties, differing in the way in which the image formed by the mirror is brought within reach of the magnifying eyepiece. Fig. 15 illustrates three of the most common forms. The Newtonian is most used, but one or two large instruments are of the Cassegrainian form, which is exactly like the Gregorian shown in the figure (now almost obsolete), with the exception that the small mirror is convex instead of concave.



Gregorian



Cassegrainian



Newtonian

Fig. 15. — Reflecting Telescopes

Various forms of the reflector.

In the Herschelian, or "front view" form, the large mirror is slightly inclined, throwing the rays to the edge of the open end of the tube, so that the secondary mirror is dispensed with, and the observer stands with his back to the object. This is practicable only with very large instruments, since the head of the observer partly obstructs the light; the image also is somewhat distorted, and at present this construction is never used.

Mirrors of silver on glass.

Until about 1870, the large mirror (technically *speculum*) was always made of speculum-metal, a composition of copper and tin. It is now usually made of glass, silvered on the front surface by a chemical process. When new, these silvered films reflect much more light than the old speculum-metal; they tarnish rather easily, but fortunately can be easily renewed.

Superiority of the refracting telescope over the reflecting.

50. Relative Advantages of Reflectors and Refractors. — There is much earnest discussion on this point, each form of instrument having its earnest partisans. On the whole, however, the refractor is usually better. Up to a certain limit, never yet reached, it gives more light than a reflector of the same size, defines better under all ordinary conditions, has a wider field of view, is more manageable and convenient, and more permanent; the speculum of a reflector usually needs to be resilvered every few years, while a carefully used object-glass never deteriorates.

Certain advantages of the reflector.

The reflector is of course far less expensive than a refractor of the same size, and its *absolute achromatism* is a great advantage in certain lines of work, photographic and spectroscopic.

For a fuller discussion of the matter, see *General Astronomy*.

Large refractors.

51. Large Telescopes. — The largest refractors[1] at present (1901) existing are those of the Yerkes Observatory (40 inches in diameter and 65 feet long), and the telescope of the Lick Observatory, which has an aperture of 36 inches and a focal length of 56 feet. There are about fourteen others which have apertures not less than 2 feet. The object lenses of more than half of these instruments, including both of the largest, were made (that is, ground and figured) in this country by the Clarks of Cambridgeport. The glass itself was made by various firms in Europe.

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[1] No account is taken in this reckoning of the great 48-inch telescope of the Paris Exposition. It is not certain as yet how it will turn out from an astronomical point of view.

The frontispiece is the great Potsdam double telescope, — two mounted together, — one $31\frac{1}{2}$ inches in diameter for photography, the other 20 inches in diameter for visual observations; the focal length of both is about 43 feet. It was erected in 1899.

Large reflectors.

At the head of the reflectors stands the enormous instrument of Lord Rosse of Birr Castle, 6 feet in diameter and 60 feet long, made in 1842, and still used occasionally. Next in size, but probably superior in power, comes the 5-foot silver-on-glass reflector of Mr. Common, at Ealing, England, completed in 1889; and then follow a number (four or five) of 4-foot telescopes, — that of Herschel (erected in 1789, but long ago dismantled) being the first, while the instrument at Paris is the only one of this size now in active use.

In this country the only large reflector actually at work is the 3-foot instrument at the Lick Observatory (made by Mr. Common and presented to the observatory by Mr. Crossley), with which Keeler made his wonderful photographs of nebulæ, some of which are figured in the last chapter of this book. Another of 2-feet aperture has just been mounted at the Yerkes Observatory.

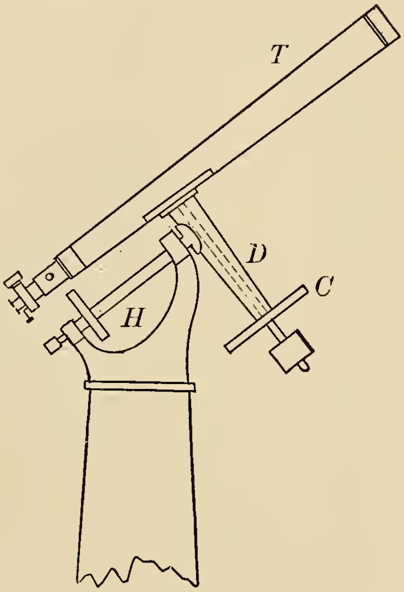

Fig. 16. — The Equatorial (Schematic)

52. Mounting of a Telescope. — A telescope, however excellent optically, is of little scientific use unless firmly and conveniently mounted.[1]

The equatorial mounting.

At present nearly all but small portable instruments are mounted as *Equatorials*. Fig. 16 represents the arrangement schematically. Its essential feature is that the "principal axis" — the one which moves in fixed bearings attached to the pier and is called the *polar axis* — is inclined so as to point towards the celestial pole. The graduated circle H attached to it is therefore parallel to the celestial equator, and is usually called the *hour-circle* of the instrument, — sometimes the *right-ascension circle.* At the upper extremity of the polar axis a sleeve is fastened, which carries the declination axis D passing through it. To one end of this declination axis is attached the telescope tube T, and at the other end the declination circle C, and a counterpoise if necessary.

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[1] We may add that it must be mounted where it can be pointed directly at the stars, *without any intervening window-glass* between it and the object. We have known purchasers of telescopes to complain bitterly because their instruments would not show Saturn well through a closed window.

Advantages of equatorial mounting.

53. The advantages of the equatorial mounting are very great. In the first place, when the telescope is once pointed upon an object it is not necessary to turn the declination axis at all in order to keep the object in view, but only to turn the *polar axis* with a perfectly uniform motion, which can be, and usually is, given by *clockwork* (not shown in the figure).

Permits use of clockwork.

Makes it easy to find objects too faint to be seen.

In the next place, it is very easy to find an object, even if invisible to the eye (like a faint comet, or a star in the daytime), provided we know its right ascension and declination *and have the sidereal time, — a sidereal clock or chronometer being an indispensable accessory of the equatorial.* We set the declination circle by its vernier to the declination of the object and then turn the polar axis until the hour-circle shows the proper *hour angle*, which is simply the difference between the right ascension of the object and the sidereal time at the moment. When the telescope has been so set the object will be found in the field of view, *provided a low-power eyepiece is used.* On account of refraction the setting does not direct the instrument precisely to the apparent place of the object, but only very near it.

Use of equatorial in determining position of planets or comets.

The equatorial does not give very accurate positions of heavenly bodies by means of the direct readings of its circles, but it can be used as explained later in Sec. 117 to determine with great precision the *difference* between the position of a known star and that of a comet or planet; and this answers the purpose as well as a direct determination.

The frontispiece shows the equatorial mounting of the great Potsdam telescope. Fig. 173 (Sec. 536) represents another form of equatorial mounting, adopted for several of the instruments of the photographic campaign. Lord Rosse's great reflector is not mounted equatorially, nor was Herschel's 4-foot reflector, but nearly all the other reflectors referred to above are equatorials.

54. Other Mountings. — With very large telescopes this mounting becomes unwieldy, notwithstanding the ingenious electrical and other arrangements by which the observer at the eyepiece is enabled to control its motions. The enormous rotating dome — that of the Yerkes Observatory is 90 feet in diameter — and the requisite elevating floor are also extremely expensive, so that at present there is among astronomers a tendency to adopt plans by which the telescope may be fixed in its position, while the light is brought to the eyepiece by one or more reflections from plane mirrors.

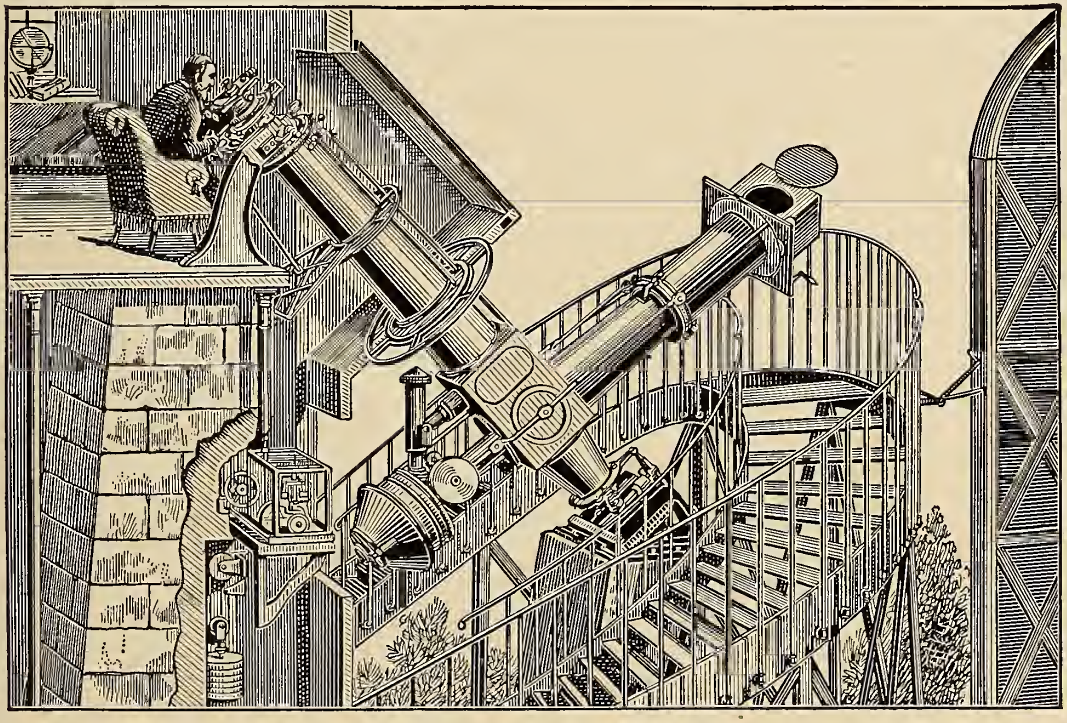


Fig. 17. — The Equatorial Coudé.

Fig. 17 represents the smaller *equatorial coudé*, or "elbowed equatorial," of the Paris Observatory. A silvered mirror at an angle of 45° in

The equatorial coudé.

the box in front of the object-glass, and another one in the cube at the center of the instrument, effect the necessary changes in the direction of the ray. The observer sits motionless, under cover, at the eyepiece, looking downward towards the south, at an angle equal to the latitude of the place. A much larger, similar instrument, since mounted at the same observatory, has an aperture of 24 inches and a focal length of about 60 feet. Three or four instruments of this sort are now in use.

The siderostat.

Another arrangement is to place the telescope horizontally, pointing towards the south, and to direct the light from the object into it by reflection from the mirror of a so-called *siderostat.* This is a simple plane mirror larger than the object-glass, properly mounted and driven by clockwork so as to send the reflected rays horizontally always in the same direction, and having connections by which its motions can be controlled from the eye end of the telescope. The great telescope of the Paris Exposition of 1900 was arranged in this way.

The cœlostat.

The *cœlostat* is a slightly different arrangement, in which the plane mirror, mounted upon a polar axis, revolves at *half* the diurnal rate, and the telescope, while retaining one fixed position for a body in a given declination, has to change its position to observe bodies in a different declination. There are still other forms in which a large reflector is used to give the rays a convenient direction.

But the use of the mirror or mirrors involves considerable loss of light; and what is worse, if the mirror is large it is extremely difficult to figure the surface with the requisite accuracy, and to prevent slight distortions by variations of temperature and changes of position. As a consequence, definition is seldom as satisfactory as with telescopes pointed directly to the heavens; still, in certain operations of astronomical photography, the siderostat and cœlostat are extremely useful.

55. Timekeepers and Recorders. — Obviously a good clock or chronometer is an essential instrument of the observatory.

Importance of Huyghens' invention of the pendulum clock.

The invention of the pendulum clock by Huyghens in 1657 was almost as important to the advancement of astronomy as that of the telescope by Galileo; and the improvement of the clock and chronometer through the invention of temperature compensation by Harrison and Graham in the eighteenth century is fully comparable with the improvement of the telescope by the achromatic object-glass.

The astronomical clock differs from any other clock only in being made with extreme care and in having a pendulum so constructed that its rate will not be sensibly affected by changes of temperature. The mercurial pendulum is most common, but other forms are also used. (See Fig. 18.)

The astronomical clock.

The pendulum usually beats seconds (rarely half seconds), and the clock face ordinarily has its second-hand, minute-hand, and hour-hand each moving on a separate center, the hour-hand making its revolution not in twelve hours, as in an ordinary clock, but in twenty-four, the hours being numbered accordingly.

In cases where the extremest accuracy of performance is required, the clock is placed in an underground chamber, where the temperature varies only slightly or not at all, and is besides inclosed in an air-tight case, within which the air is kept at a uniform pressure, since changes in the density of the air slightly affect the swing of the pendulum. Usually a clock loses about one quarter of a second a day for a rise of one inch in the barometer.

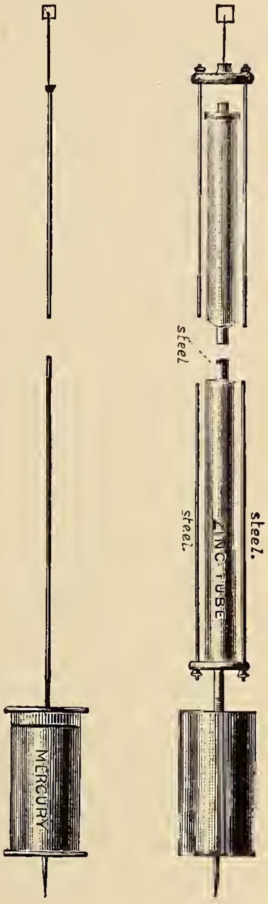


FIG. 18
Compensation Pendulums
1. Graham's Pendulum
2. Zinc-Steel Pendulum

Finally, also, the astronomical clock is usually fitted with some arrangement for making or breaking an electric circuit at every second or every other second, so that its beats can be communicated telegraphically to all parts of the observatory. The minute is usually marked either by the omission of a second or by a double tick.

The break-circuit.

56. Error and Rate. — The *error* or *correction* of a clock is *the amount which must be added (algebraically) to its face indication to give the true time;* + when slow, − when fast. The *rate* is the amount it loses or gains daily; + when losing, − when gaining. Sometimes the *hourly* rate is given instead of the

Error and rate.

daily. The *error* is adjusted by simply setting the hands; the *rate* by raising or lowering the pendulum bob, or for delicate final adjustment without stopping the clock, by adding or removing small pieces of metal on the cover of the cylindrical vessel which usually constitutes the pendulum bob.

Perfection in an astronomical clock consists in its maintaining a *constant rate*, *i.e.*, in gaining or losing precisely the same amount each day; for convenience the rate should be small, and is usually kept less than half a second daily. But this is a mere matter of adjustment.

The chronometer.

57. The Chronometer. — The pendulum clock not being portable, it is necessary to provide timekeepers that are so. The chronometer is merely a carefully made watch with a balance-wheel compensated to run, as nearly as possible, at the same rate in different temperatures, and with a peculiar escapement, which, though unsuited to ordinary usage, gives better results than any other when treated carefully.

The box chronometer used on shipboard is usually about twice the diameter of a common pocket watch, and is mounted on "gimbals" so as to remain horizontal at all times, notwithstanding the motion of the vessel. It usually beats half seconds.

It is not possible to secure in the chronometer balance as perfect a temperature correction as in the pendulum, and for this and other reasons the best chronometers cannot quite compete with the best clocks in precision; but they are sufficiently accurate for most purposes, and of course are vastly more convenient for field operations, while at sea they are simply indispensable. *Never turn the hands of a chronometer backward; it may ruin the escapement.*

Observation of time by eye and ear.

58. Eye-and-Ear Method of Observation. — The old-fashioned method of time observation consisted simply in noting by "eye and ear" the moment (in seconds and tenths of a second) when the phenomenon occurred; as, for instance, when a star passed some wire of the reticle. The tenths, of course, are merely estimated, but the skilful observer seldom errs by a whole tenth in his estimation. Skill and accuracy in this method are acquired only by long practice.

Observation by means of the chronograph.

59. Telegraphic Method; the Chronograph. — At present such observations are usually made by the help of electricity. The clock is so arranged that at every beat (or every other beat) of the pendulum an electric circuit is made or broken for an instant, and this causes a sudden sideways jerk in the

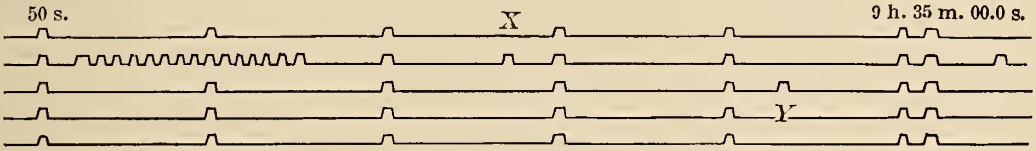


FIG. 19. — A Chronograph Record

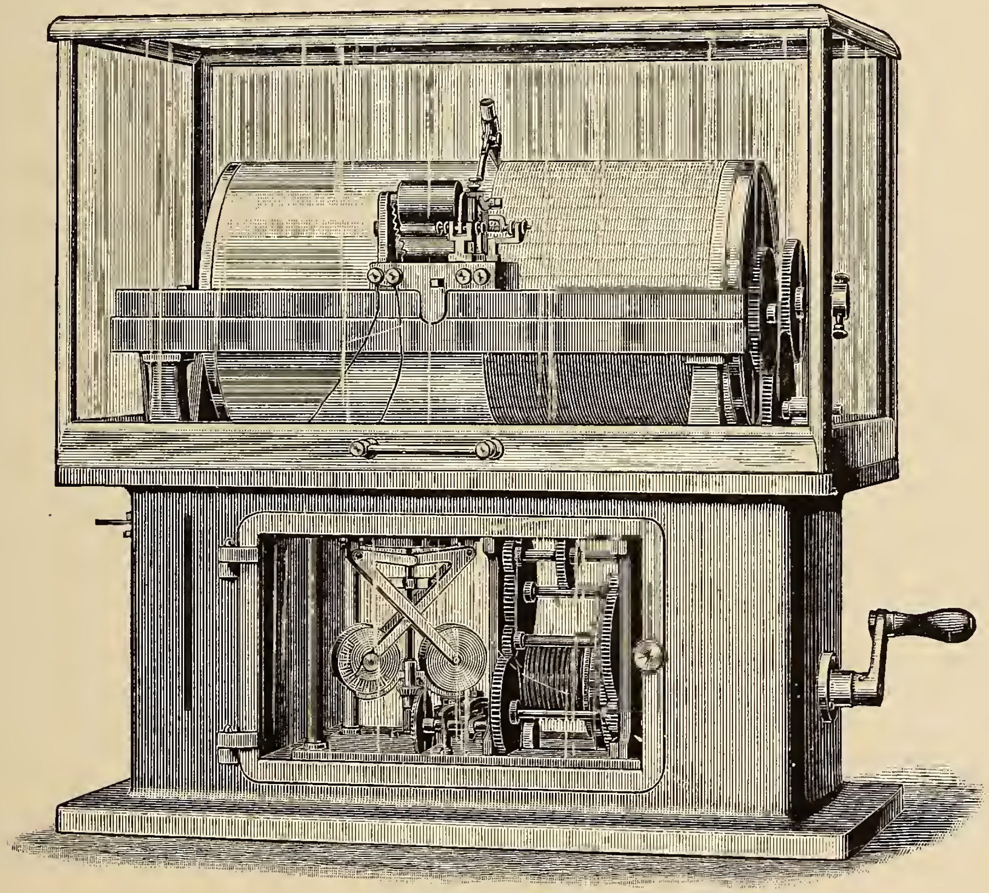


FIG. 20. — A Chronograph
By Warner & Swasey

armature of an electromagnet, like that of a telegraph sounder. This armature carries a fountain-pen, which writes upon a sheet of paper wrapped around a cylinder six or seven inches in diameter, which cylinder itself is turned uniformly by clock-work once a minute ; at the same time the pen carriage is drawn slowly along, so that the marks on the paper form a continuous helix, graduated into second or two-second spaces by the clock beats. When taken from the cylinder, the paper presents the appearance of an ordinary page crossed by parallel lines spaced off into two-second lengths, as shown in Fig. 19, which is part of an actual record.

Fig. 20 represents a chronograph of the usual American form. The observer, at the moment when a star crosses the wire, presses a "key" which he holds in his hand, and thus interpolates a mark of his own among the clock beats on the sheet; as, for instance, at *X* and *Y* in the figure. Since the beginning of each minute is indicated on the sheet in some way by the mechanism which produces the clock beats, it is very easy to read the time of *X* and *Y* by applying a suitable scale, the beginning of the mark made by the key being the moment of observation.

In the figure the initial minute marked when the chronograph was started happened to be 9^h35^m, the zero in the case of this clock being indicated by a double beat. The signal at *X*, therefore, was made at $9^h35^m55^s.45$, and that of *Y* at $9^h36^m58^s.63$. The "rattle" just preceding *X* was the signal that a star was approaching the transit wire.

In European observatories the record is usually made by a more simple but less convenient apparatus upon a long fillet or ribbon of paper drawn slowly along. At a few observatories in this country a more complicated *printing chronograph*, invented by Professor Hough of the Dearborn Observatory, is used. By this the minutes, seconds, and hundredths of a second are actually printed upon the fillet in type, like the record of sales on a stock telegraph.

The printing chronograph.

60. Meridian Observations. — A large proportion of all astronomical observations for determining the positions of the

Advantage of observations on the meridian.

heavenly bodies are made when the body is crossing the meridian or is very near it. At that time the effects of refraction and parallax (to be discussed later) are a minimum, and as they act only vertically they do not affect the *time* when a body crosses the meridian nor, consequently, its observed right ascension. In any other part of the sky both these coördinates are affected, and the calculation of the correction requires the computation of the "parallactic angle" in the astronomical triangle (Sec. 31).

61. The transit-instrument is the instrument used in connection with a sidereal clock or chronometer, and often with a chronograph, to observe the time of a star's *transit*, or passage across the meridian. If the "error" of the sidereal clock at the moment is known and allowed for, the *corrected time of the observation will be the right ascension of the star* (Sec. 26).

Vice versa, if the right ascension is known, *the error of the clock* will be the difference between the right ascension of the object and the time observed.

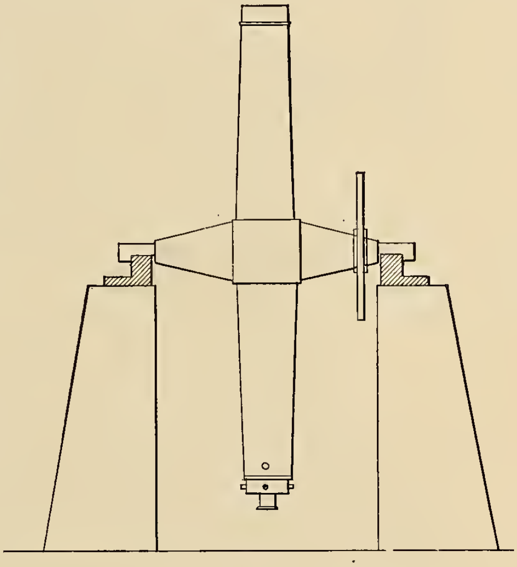


FIG. 21. — The Transit-Instrument

The transit-instrument. The level, setting-circle, and reversing apparatus.

The instrument (Fig. 21) consists essentially of a telescope carrying at the eye end a reticle and mounted on a stiff axis that turns in V-shaped bearings called "Y's," which can have their position adjusted so as to make the axis exactly perpendicular to the meridian. A delicate spirit-level, which can be placed upon the pivots of the axis to measure any slight deviation from horizontality, is an essential accessory; and it is practically necessary to have a small graduated circle attached to the instrument, in order to set it at the proper elevation for the star which is to be observed.

It is desirable, also, that the instrument should have a reversing apparatus by which the axis may be easily lifted and safely reversed in the Y's without jar or shock.

The transit reticle.

The reticle usually contains from five to fifteen "vertical wires" crossed by two horizontal ones. Fig. 22 shows the reticle of a small transit intended for observations by "eye and ear." When the chronograph is to be used, the wires are much more numerous and placed nearer together.

In order to make the wires visible at night the field must be illuminated. For this purpose one of the pivots of the instrument is pierced (sometimes both of them), so that the light from a lamp will shine through the axis upon a small reflector placed in the central cube of the instrument, where the axis and the tube are joined. This sends sufficient light towards the eye to illuminate the field, while it does not cut off any considerable portion of the rays from the object.

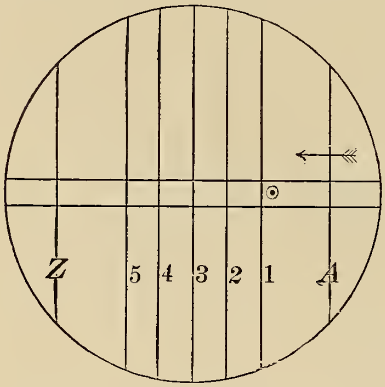


Fig. 22. — Reticle of the Transit-Instrument

The observation consists in noting the instant, as shown by the clock or chronometer, in hours, minutes, seconds, and tenths of a second, at which the star crosses each wire of the reticle.

Freedom from shakiness and flexure of parts essential.

62. The instrument must be thoroughly rigid, without any loose joints or shakiness, *especially in the mounting of the object-glass and reticle.* Moreover, the two pivots should be of the same diameter, accurately round, without taper, and precisely in line with each other; in other words, they must be *portions of one and the same geometrical cylinder.* To fulfil this condition taxes the highest skill of the mechanician.

Essentials of construction.

When exactly adjusted, the middle wire of such an instrument *always precisely coincides with the meridian,* however the instrument may be turned on its axis; and *the sidereal time when a star crosses that wire is therefore the star's right ascension.*

Another form of the instrument now much used is often called the *broken transit*, of which Fig. 23 is a representation. A reflector (usually a right-angled prism) in the central cube of the instrument directs the rays horizontally through one end of the axis where the eyepiece is placed, so that whatever may be the elevation of the star the observer looks straight forward horizontally, without needing to change his position. The instrument is very convenient, but is usually subject to rather a large error, due to flexure of the axis, which, even if it exists, produces no such effect in transits of ordinary form. The error is, however, easily determined and allowed for if the axis is not too slender.

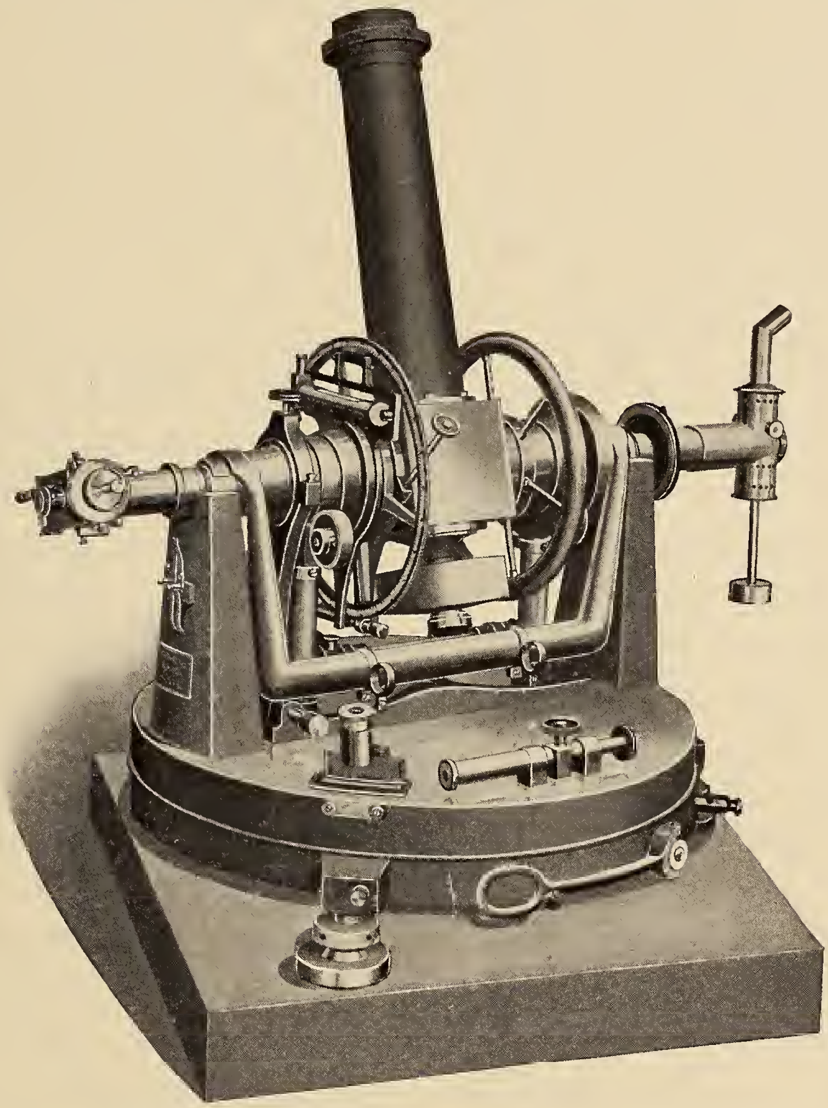


FIG. 23. — A Broken Transit
By Warner & Swasey

Necessary adjustments.

63. Adjustments of the Transit. — These are four in number:

(1) The reticle must be exactly in the *focal plane* of the object-glass and the middle wire *accurately vertical.*

(2) The *line of collimation* (*i.e.*, the line which joins the optical center of the object-glass to the middle wire) must be *exactly perpendicular to the axis of rotation.* This may be tested by pointing on a distant mark and then reversing the instrument. The middle wire must still bisect the mark after the reversal. If not, the reticle must be adjusted by the screws provided for the purpose.

(3) The axis must be *level.* This adjustment is made mechanically by the help of the spirit-level. One of the Y's has a screw by which it can be slightly raised or lowered, as may be necessary.

(4) The *azimuth* of the axis must be exactly 90°; *i.e.*, the axis must point exactly east and west. This adjustment is made by means of star observations, with the help of the sidereal clock.

Tests of adjustments.

Without going into detail, we may say that if the instrument is correctly adjusted, the time occupied by a star near the pole, in passing from its transit across the middle wire above the pole to its next transit below the pole, must be exactly twelve sidereal hours. Moreover, if two stars are observed, one near the pole and another near the equator, the difference between their times of transit ought to be precisely equal to their difference of right ascension. By utilizing these principles the astronomer can determine the error of azimuth adjustment and correct it.

Non-permanence of adjustments.

But it is to be remembered that no adjustments, however carefully made, will be absolutely exact or remain permanently correct, on account of changes in temperature which affect the instrument and the pier on which it is mounted. In cases where extreme accuracy of results is required, the slight errors which remain after the most careful adjustment must be determined from the observations themselves by means of the little discrepancies between the results obtained from stars at different distances from the pole. The methods to be used are taught in practical astronomy.

Personal equation.

64. Personal Equation. — It is found that skilled observers are in the habit of noting the passage of a star across the transit wire slightly too late or too early by an amount which is different for each observer, but nearly constant for each. This is called the observer's *personal equation*, and in some cases for eye-and-ear observation is as much as half a second. In the telegraphic method it is much less, seldom exceeding $0^s.1$. It is an extremely troublesome error, because it varies with the nature and brightness of the object and with the observer's position and physical condition.

Mechanical method of getting rid of personal equation.

Various devices have been proposed for dealing with it; either by measuring its amount, or by eliminating it by means of some apparatus which reduces the observation to the accurate *bisection* of the star disk, *made to appear to be at rest* by a clockwork motion given to the eyepiece, and carrying with it a "micrometer wire" which is under the control of the observer. When the bisection is satisfactory he touches a key which instantly stops the motion and registers the time upon the chronograph; afterwards, at his leisure, he measures the distance of his micrometer wire from the central wire of the reticle. In this way the disturbing effect of the star's motion is eliminated.

Photographic observation of transits.

65. The Photochronograph. — Another method, and one of the most promising, is by means of photography. The eyepiece of the transit is removed, and a small photographic plate, about as large as a microscope slide, is placed just back of the reticle, so arranged in the frame which holds it that it can move up and down slightly under the action of an electromagnet connected with the standard-clock circuit. When a star impresses its "trail" on the plate, the trail is broken every second (or every other second) by the clock, like the marks on a chronograph sheet, so that it consists of a row of small dashes. The image of the reticle wires is also imprinted upon the plate by holding a small lamp for an instant in front of the object-glass.

During the passage of the star some particular second is marked on the plate by cutting off the clock circuit for two or three seconds, or by making a rattle, allowing the beats to resume their regular course at some instant recorded in the note-book. After the plate is developed, its inspection and measurement under a microscope will show at what second and fraction of a second the star passed each reticle wire. But this part of the operation is laborious. On the other hand, the expensive and troublesome chronograph is dispensed with.

The prime vertical instrument.

66. The Prime Vertical Instrument. — For certain purposes a transit-instrument, provided with an apparatus for rapid reversal, is turned quarter way round and mounted with its axis *north and south*, so that the plane of rotation lies east and west instead of in the meridian. It is then called the "prime vertical instrument." It may be used for determining the latitude of the observer, the precise declination of such stars as cross the meridian between the zenith and equator, and any minute change due to "aberration" and to slight movements of the terrestrial pole. (See Sec. 94.)

Its use.

The observation consists in noting the instant when the star crosses (obliquely) the middle wire of the reticle.

The almucantar and its use.

67. The Almucantar. — This is an instrument invented about 1885 by Dr. S. C. Chandler of Cambridge, U.S., for the purpose of observing the time at which stars cross, not the meridian or any vertical circle, but some given *parallel of altitude*, usually the "almucantar" of the pole. From such observations can be determined with great accuracy the error of the clock, the declination of the stars observed, or the latitude of the observer.

It consists of a firm base carrying a tank containing mercury, on which swims a float which carries the observing telescope, its inclination being preserved absolutely constant by the principle of flotation. This dispenses with the necessity of using spirit-levels (which are always more or less unsatisfactory) for determining the inclination.

The telescope is sometimes placed horizontally on the float, while a mirror in front of its object-glass brings down the rays of the star. Two such instruments of considerable size have been built since 1899 and give promising results, — one at Cambridge, England, the other at Cleveland, Ohio.

68. The Meridian-Circle. — This is a transit-instrument of large size and most careful construction, *with the addition of a large graduated circle attached to the axis and turning with it.*

The meridian-circle: essentials of its construction.

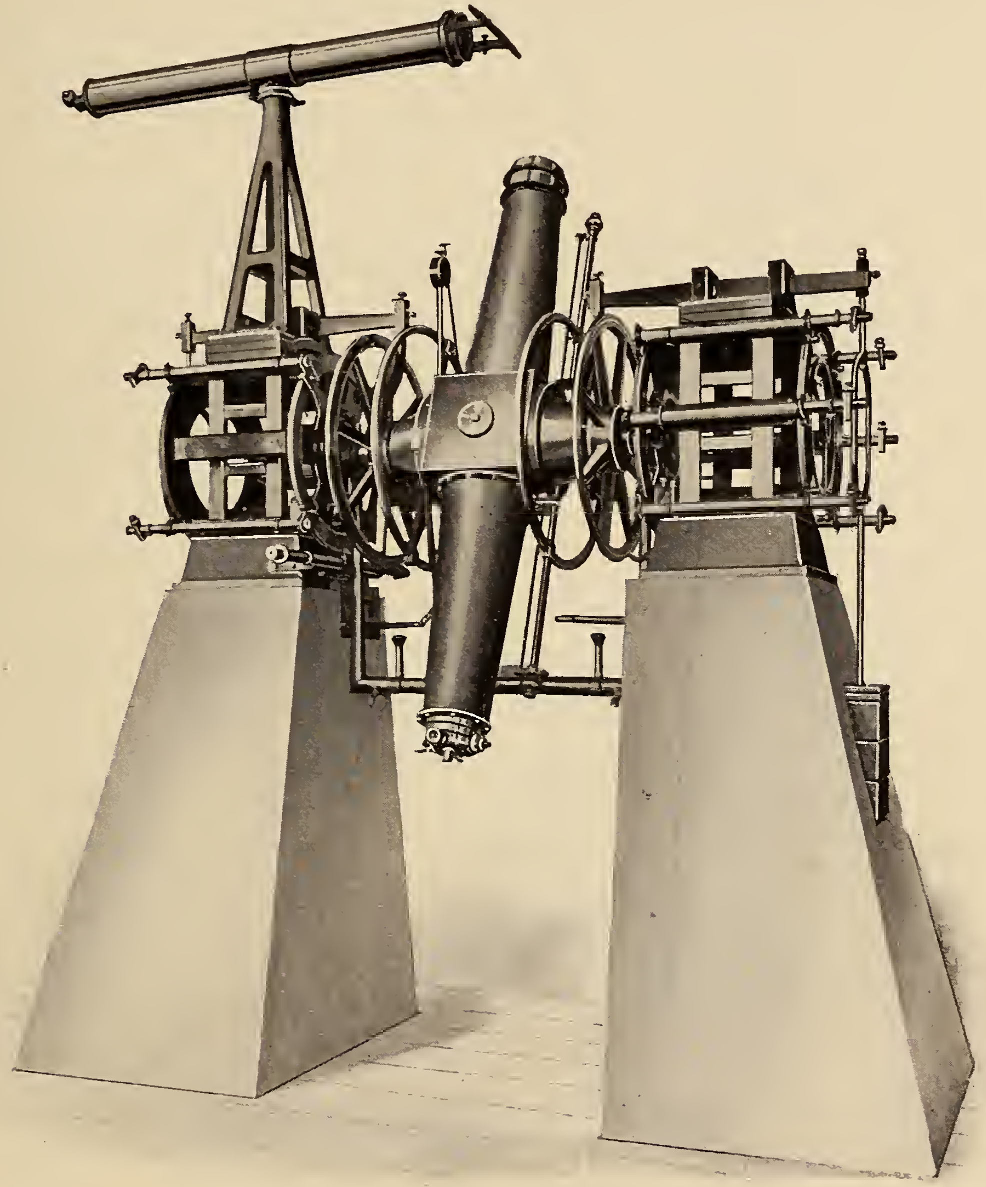


FIG. 24. — Meridian-Circle in United States Naval Observatory, Washington
By Warner & Swasey

The utmost resources of mechanical art are expended in graduating this circle with precision. The divisions are now usually made either two minutes or five minutes of arc, and the farther subdivision is effected by so-called "reading microscopes," four of which at least are always used in the case of a large instrument. (For a description of the reading microscope, the reader is referred to *General Astronomy*, Art. 64, or to Campbell's *Practical Astronomy*.) By means of these microscopes the "reading of the circle" is made in degrees, minutes, seconds, and tenths of a second of arc, the tenths being obtained by estimation.

On a circle 2 feet in diameter 1″ of arc is only about $\frac{1}{17000}$ part of an inch; an error of that amount is now very seldom made by reputable constructors in placing a graduation line, or by a good observer in reading the instrument with the microscope.

Fig. 24 represents the new meridian-circle of the United States Naval Observatory at Washington, with a 6-inch telescope and circles about 27 inches in diameter.

Its zero points.

Determination of the polar point.

69. Zero Points. — The instrument is used to measure the altitude or else the polar distance of a heavenly body at the time when it is crossing the meridian. As a preliminary we must determine some *zero point* upon the circle, — the *nadir* point or *horizontal* point, if we wish to measure altitudes or zenith-distances; the *polar* point or *equator* point, if polar distances or declinations. The polar point is determined by taking the circle reading for some star near the pole when it crosses the meridian above the pole, and then doing the same thing again twelve hours later when it crosses it below. The mean of the two readings corrected for refraction will be the reading which the circle would give when the telescope is pointed exactly to the pole, — technically, the *polar point*. The *equator point* is, of course, 90° from this.

Determination of the nadir point.

The *nadir point* is the reading of the circle when the telescope is pointed vertically downward. It is determined by the reading of the circle when the instrument is so set that the horizontal wire of the reticle coincides with its own image formed by a reflection from a basin of mercury placed on the pier below the instrument. To make this reflected image visible it is necessary to illuminate the reticle by light thrown towards the object-glass from behind the wires, — the ordinary illumination used during observation comes from the opposite direction. This peculiar illumination is effected by what is known as the "collimating eyepiece." A thin glass plate inserted at an angle of 45° between the lenses of a Ramsden eyepiece throws down sufficient light, admitted through a hole in the side of the eyepiece, and yet permits the observer to see the wires and their reflected image. The *zenith point* is, of course, just 180° from the nadir point thus determined.

The collimating eyepiece.

Obviously, the meridian-circle can be used simply as a transit, so that with this instrument and a clock the observer is in a position to determine *both the right ascension and declination* of any heavenly body that can be seen when it crosses the meridian.

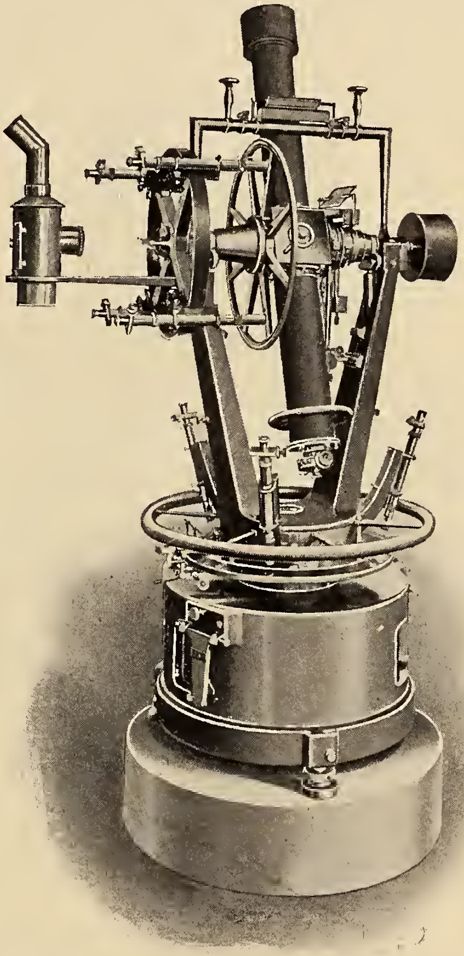


FIG. 25. — A 5-inch Altazimuth By Warner & Swasey

70. Extra-Meridian Observations. — Many objects, however, are not visible when they cross the meridian; a comet, for instance, or a planet, may be in such a part of the heavens that it transits only by daylight. To observe such objects we may employ a so-called *universal instrument*, or astronomical theodolite, which is simply an instrument with both horizontal and vertical circles like a large surveyor's theodolite and is also called an *altazimuth*. By means of this the altitude and azimuth of an object may be measured,

Extra-meridian observations.

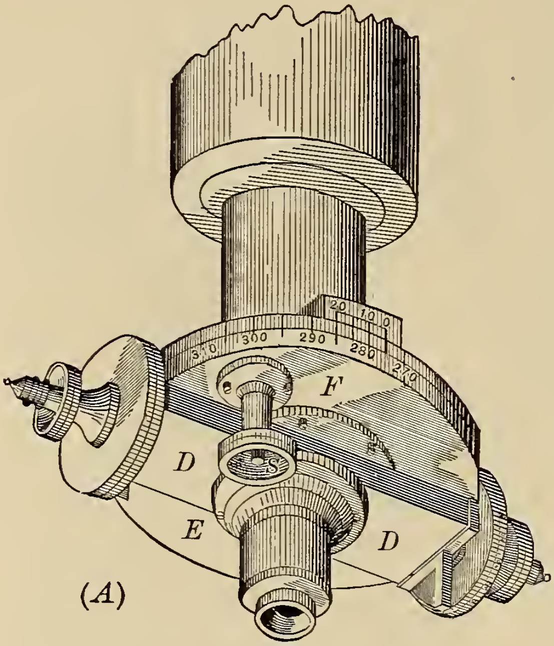
The universal instrument, or altazimuth

and, if the time is given, from these the right ascension and declination can be deduced.

Fig. 25 shows the 5-inch altazimuth of the Washington Observatory.

Extra-meridian observations with the equatorial.

More often, however, observations for the positions of bodies not on the meridian are made with the equatorial telescope already described, with which the *difference* between the right ascension and declination of the observed body and that of some star in its neighborhood is determined by means of a *micrometer* or, at present, often by photography.



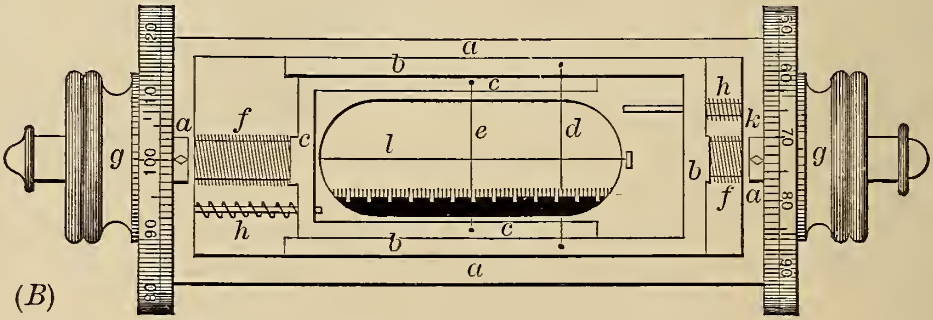


FIG. 26. — The Filar-Position Micrometer

71. The Micrometer. — There are various forms of micrometers, the most common and generally useful being that known as the *filar-position micrometer*, shown in Figs. 26 *A* and *B*. It is a comparatively small instrument which is attached at the eye end of the telescope. It usually contains a set of fixed wires, two or three of them parallel to each other (only one, *e*, is shown in *B*, which represents the internal construction of the instrument), crossed at right angles by a single line or set of lines. Under the plate which carries the fixed threads lies a fork moved by a carefully made screw with a graduated head, and this fork carries one or more wires parallel to the first set, so that the distance between the wires *e* and *d* (Fig. 26 *B*) can be varied at pleasure and read off by means of the screw-head graduation.

The box containing the wires is so arranged that it can itself be rotated around the optical axis of the telescope and set in any desired "position"; for example, so that the movable wire *d* shall be parallel to the celestial equator when the position circle *F* should read 90°. When so set that the movable wire points from one star to another in the field of view, the "position angle" (see Fig. 191, Sec. 585) can be read off on the circle *F*.

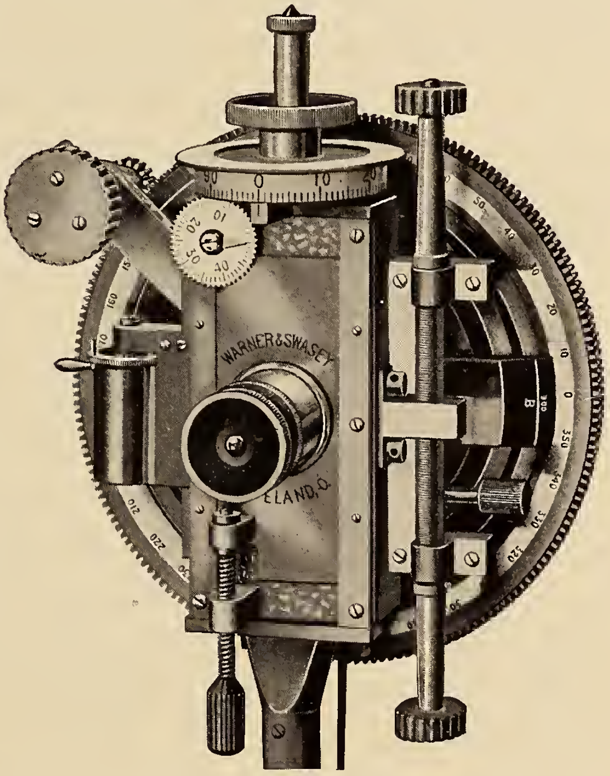


Fig. 27. — Position Micrometer
By Warner & Swasey

With such a micrometer we can measure at once the distance in seconds of arc between any two stars which are near enough to be distinctly seen in the same field of view, and can determine the position angle of the line joining them. The available range in a small telescope may reach 30′. In large telescopes, which with the same eyepieces give much higher magnifying powers, the range is correspondingly less, — not more than from 5′ to 10′. When the distance between the wires objects exceeds 2′ or 3′, the filar micrometer becomes difficult

Its use and limitations.

to use and inaccurate, because the observer cannot see both objects distinctly at the same time.

Fig. 27 is a complete micrometer, fitted with electric illumination.

The heliometer: its construction.

72. The Heliometer. — For the measurement of larger distances not exceeding two or three degrees the *heliometer* is used. This is a complete equatorially mounted telescope with its object-glass (usually from 4 to 8 inches aperture) diametrically divided into two halves which can be made to slide past each other for 3 or 4 inches (Fig. 28), the distance being measured on a delicate scale read by long microscopes which come down to the end of the instrument. The telescope tube can be rotated in its cradle so as to make the line of division of the lenses lie in any desired position.

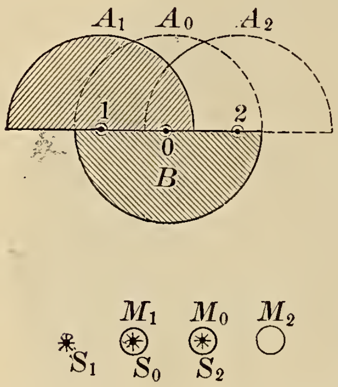


Fig. 28. — The Heliometer

When the object-glass scale is at zero, the two half lenses act as a single lens and each object in the field of view presents a single image, as S_0 and M_0 in the figure. But as soon as one of the semi-lenses is pushed past the other, two images of each object appear, and the distance and direction between them can be varied at pleasure by sliding the lenses and rotating the tube.

Method of measuring.

The distance between any two different objects is measured by *making their images coincide* (as, for instance, M_1 with S_0, or S_2 with M_0), and the observer does not have to "look two ways at once," nor is he obliged to trust to the stability of his instrument or the accuracy of the clockwork motion.

On the whole, the heliometer stands at the head of astronomical instruments for the precision of its results and is employed in the most delicate investigations, like those upon solar and stellar parallax (Secs. 467 and 550). But it is a very complicated and costly instrument, and extremely laborious to use.

The rank of the heliometer among astronomical instruments.

The only one in the United States at present is the 7-inch instrument at the Yale University Observatory.

Observations by means of photography.

At present, however, such measurements of the distance of an object from neighboring stars are very generally effected by means of *photography*. Photographs of the field of view containing the object are made and afterwards measured, and in this case the limits of distance between the object and the stars to which it is referred can be very much increased without lessening the accuracy of the determination.

The sextant: the instrument of the mariner.

73. The Sextant.—All the instruments so far mentioned, except the chronometer, require some firmly fixed support, and are therefore absolutely useless at sea. The *sextant* is the only one upon which the mariner can rely. By means of it he can measure the angular distance between two points (as, for instance, between the sun and visible horizon), not by pointing first to one and afterwards to the other, but by sighting them both *simultaneously* and in *apparent coincidence*, a "double image" measurement, in which respect the sextant is analogous to the heliometer. A skilful observer can make the measurement accurately even when he has no stable footing.

Its peculiar advantage over other instruments.

Its construction.

Fig. 29 represents the instrument. Its graduated limb is usually, as its name implies, about a sixth of a complete circle, with a radius of from 5 to 8 inches. It is graduated in *half degrees* (which are, however, *numbered as whole degrees*) and so can measure any angle not much exceeding 120°. The index arm, or "alidade" (*MN* in the figure), is pivoted at the center of the arc and carries a "vernier," which slides along the limb and can be fixed at any point by a clamp, with an attached tangent screw *T*. The reading of this vernier gives the angle measured by the instrument; the best instruments read to 10″ only, because it is impracticable to use a telescope with very much magnifying power.

Just over the center of the arc the *index-mirror* M, about 2 inches by $1\frac{1}{2}$ in size, is fastened to the index arm, moving with it and keeping always perpendicular to the plane of the limb. At H the *horizon-glass*, about an inch wide and about twice the height of the index-glass, is secured to the frame of the instrument in such a position that when the vernier reads zero the index-mirror and horizon-glass will be parallel to each other. Only half of the horizon-glass is silvered, the upper half being left transparent. E is a small telescope screwed to the frame and directed towards the horizon-glass.

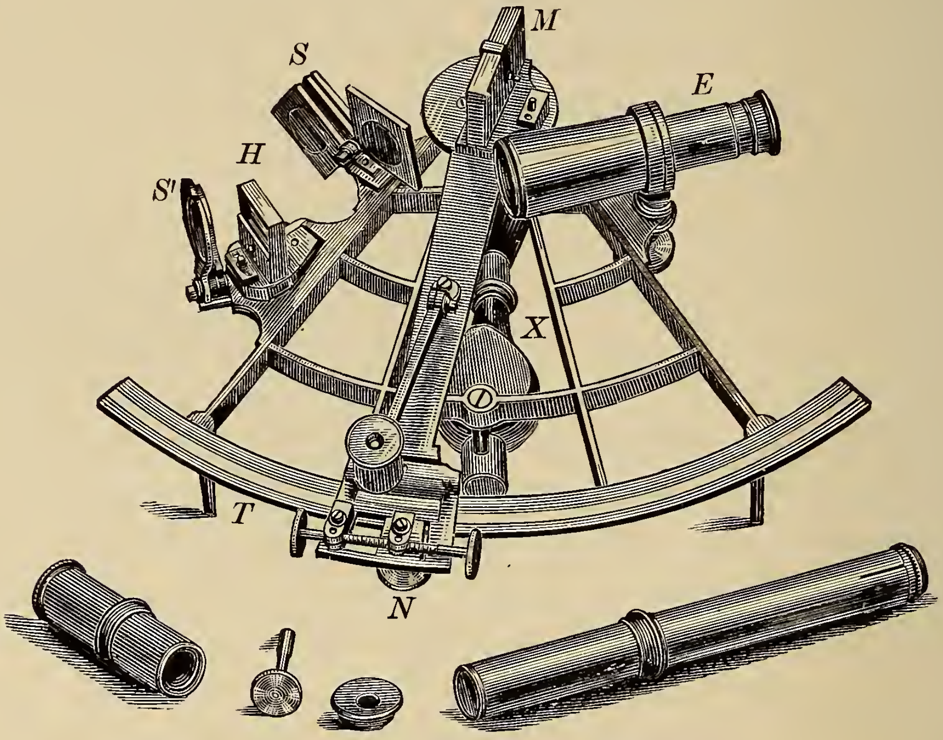


Fig. 29. — The Sextant

If the vernier stands near, but not exactly at, zero, an observer looking into the telescope will see together in the field of view two separate images of the object towards which the telescope is directed; and if he slides the vernier, he will see that one of the images remains fixed while the other moves. The fixed image is formed by the rays which reach the object-glass *directly* through the unsilvered half of the horizon-glass; the movable image, on the other hand, is produced by rays which have suffered *two reflections*, having been reflected from the index-mirror to the horizon-glass and again reflected a second time from the lower, silvered half of the horizon-glass. When the two mirrors are parallel the two images coincide, provided the object is at a considerable distance.

Double image formed by sextant.

If the vernier does not stand at or near zero, an observer looking at an object directly through the horizon-glass will see not only that object, but also, in the same telescopic field of view, whatever other object is so situated as to send its rays to the telescope by reflection from the mirrors; and *the reading of the vernier will give the angle at the instrument between the two objects whose images thus coincide*,—the angles between the planes of the two mirrors being, as easily proved, just half the angle between the two objects, and the *half* degrees on the limb being numbered as *whole ones*.

Angle between two objects whose images coincide equals half the angle between the mirrors.

74. The principal use of the instrument is in measuring the altitude of the sun. At sea the observer usually proceeds as follows: first, setting the index, loosely clamped, near zero and holding the sextant in his right hand with its plane vertical, he points the telescope towards the sun; then he slides the vernier along the arc with his left hand until he brings the reflected image of the sun down to the horizon, all the time keeping it in view in the telescope; finally, tightening the clamp and using the tangent screw, he makes the lower edge or limb of the sun just graze the horizon as he swings the sun's image back and forth by a slight motion of the instrument — it would be impossible on board ship to *hold* the image in contact with the horizon, and is not necessary. As soon as the contact is satisfactory he marks the time and afterwards reads the angle. The reading of the vernier after due corrections (see next chapter) gives the sun's true altitude at the moment.

Method of observation at sea.

On land we have recourse to an "artificial horizon." This is a shallow basin of mercury covered with a roof of glass plates having their surfaces accurately plane and parallel. In this case we measure the angle between the sun and its image reflected in the mercury. The reading of the instrument corrected for index error then gives *twice* the sun's apparent altitude.

Artificial horizon used on land.

The skilful use of the sextant requires considerable dexterity, and from the low power of the telescope the angles measured are less precise than those determined by large fixed instruments, but the portability of the instrument and its applicability at sea render it invaluable. It was invented in practical form by Godfrey of Philadelphia, in 1730, though Newton, as was discovered by Halley, had really struck upon the same idea long before.

Demonstration of the principle of the sextant.

75. The principle that the angle between the objects whose images coincide in the sextant is twice the angle between the mirrors (or between their normals) is easily demonstrated as follows:

The ray SM (Fig. 30) coming from an object, after reflection first at M (the index-mirror) and then at H (the horizon-glass), is made to coincide with the ray OH coming from the horizon.

From the law of reflection, we have the two angles SMP and PMH equal to each other, each being x. In the same way the two angles marked y are equal. From the geometric principle that the angle SMH, exterior to the triangle HME, is equal to the sum of the opposite interior angles at H and E, we get $E = 2x - 2y$. Similarly, from the triangle HMQ, $Q = x - y$; whence $E = 2Q = 2Q'$.

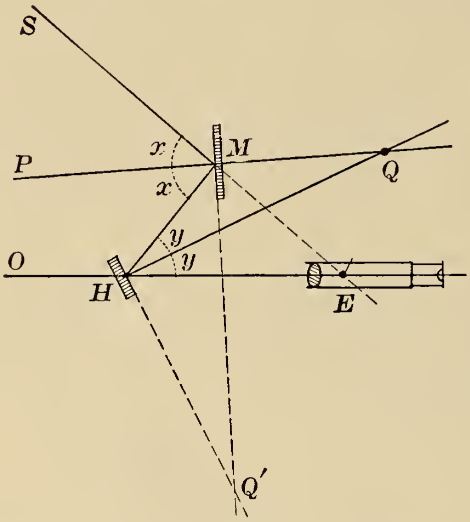


Fig. 30. — Principle of the Sextant

76. With the instruments above described all the fundamental observations required in the investigations of spherical and theoretical astronomy can be supplied, the sextant and chronometer being, however, the only ones available in nautical astronomy.

Astrophysical studies require numerous physical instruments of an entirely different character, — spectroscopes, photometers, heat-measuring instruments, and various kinds of photographic apparatus. These will be considered later, as occasion arises.

EXERCISES

1. If a firefly were to alight on the object-glass of a telescope, what would be the appearance to an observer looking through the instrument? Would he think he saw a comet?

2. When a person is looking through a telescope, if you hold your finger in front of the object-glass, will he see it?

3. If half the object-glass of a telescope pointed at the moon is covered, how will it affect the appearance of the moon as seen by the observer?

4. If a certain eyepiece gives a magnifying power of 60 when used with a telescope of 5 feet focal length, what power will it give on a telescope of 30 feet focal length?

5. What is theoretically the angular distance between the centers of two star disks which are just barely separated by a telescope of 24 inches aperture (Sec. 46)?

6. Why is it important that the two pivots of a transit-instrument should be of exactly the same diameter?

7. If the wires of a micrometer (Fig. 26) are so set that, used with a telescope of 10 feet focal length, a star moving along the right-ascension wire will occupy 15 seconds in passing from *d* to *e*, how long will it take when the micrometer is transferred to a telescope of 50 feet focus?

8. If the threads of a micrometer screw are $\frac{1}{75}$ of an inch apart, what is the angular value of one revolution of the screw when the micrometer is attached to a telescope of 30 feet focal length?

9. Does changing the eyepiece of a telescope for the purpose of altering the magnifying power affect the value of the revolution of the microscope screw?

CHAPTER III

CORRECTIONS TO ASTRONOMICAL OBSERVATIONS

Dip of the Horizon — Parallax — Semidiameter — Refraction — Twinkling or Scintillation — Twilight

Observations as actually made always require corrections before they can be used in deducing results. Those that depend on the errors or maladjustment of the instrument itself will not be considered here, but only such as are due to other causes external to the instrument and the observer.

Dip of the horizon depends on elevation of observer's eye above sea-level.

77. Dip of the Horizon. — In observations of the altitude of a heavenly body at sea, where the sextant measurement is made from the *visible* horizon, or *sea-line*, it is necessary to take into account the depression of the visible below the true astronomical horizon by a small angle called the *dip*. The amount of this dip depends upon the observer's altitude above the sea-level.

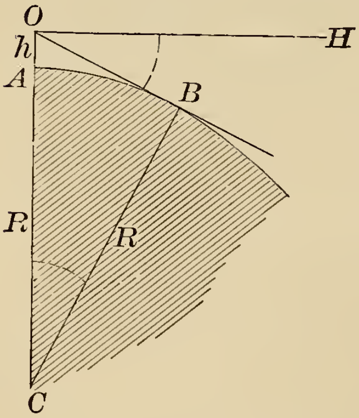


Fig. 31. — Dip of the Horizon

In Fig. 31 C is the center of the earth, AB a portion of its level surface, and O the eye of the observer at an elevation h above A. The line drawn perpendicular to OC is truly horizontal (regarding the earth as spherical), while the tangent OB is the line drawn from O to B, the visible horizon. The angle HOB is the dip, and is obviously equal to OCB.

From the triangle OCB we have

$$\cos OCB = CB/CO = R/(R+h) = \cos \Delta,$$

designating the dip by Δ.

Formulæ for the dip.

The formula in this shape is inconvenient, because it determines a small angle by means of its cosine. But since $1 - \cos {}^2\Delta = 2 \sin \frac{1}{2} \Delta$, we easily obtain the following:

$$\sin \tfrac{1}{2} \Delta = \sqrt{\frac{h}{2(R+h)}}.$$

Or, since Δ is always a small angle, and neglecting h in the denominator of the fraction as being insignificant compared with R, we get

$$\sin \Delta = 2 \sin \tfrac{1}{2} \Delta = 2 \sqrt{\frac{h}{2R}} = \sqrt{\frac{2h}{R}}.$$

This gives with quite sufficient accuracy the true depression of the sea horizon as it would be *if the line of sight were straight.* But this is not the case, owing to refraction of the rays in passing through the air, and the amount of this refraction is very uncertain and variable. Ordinarily the dip is *diminished about one eighth* of the amount computed by the formula.

Approximate formula for dip.

An approximate formula, obtained by substituting the radius of the earth (20 890000 feet) and reducing, gives Δ' (*i.e.*, in minutes of arc) $= 3438 \sqrt{\frac{2\,h\,(\text{feet})}{20\;890000\;(\text{feet})}}$ (Sec. 9), whence $\Delta' = \sqrt{h\,(\text{feet})}$ (nearly); or, in words, *the dip in minutes of arc equals the square root of the observer's elevation in feet; i.e.*, the dip is 1′ at an elevation of 1 foot, 5′ at an elevation of 25 feet, 10′ at an elevation of 100 feet, etc.

This result is generally about five per cent too large, taking into account refraction; but it is near enough for most practical purposes, since at sea the observer is seldom as much as 50 feet above the sea-level and cannot, with a sextant, measure altitudes more closely than to the nearest quarter of a minute.

The formula $\Delta' = \sqrt{3\,h\,(\text{meters})}$ agrees still more nearly with the actual value.

Formula for distance of sea horizon.

The distance OB *of the sea horizon* is easily seen, from Fig. 31, to be $R \tan \Delta$. An approximate formula is, *distance in miles* $= \sqrt{\frac{3\,h\text{ (feet)}}{2}}$. This, however, takes no account of refraction, and the actual distance is always greater.

General definition of parallax.

78. Parallax (Fig. 32). — In general the word "parallax" means the difference between the direction of a heavenly body as seen by the observer and as seen from some standard point of reference.

Annual or heliocentric parallax.

The *annual* or *heliocentric* parallax of a *star* is the difference of the star's direction as seen from the *earth* and from the *sun*. With this we have nothing to do for the present.

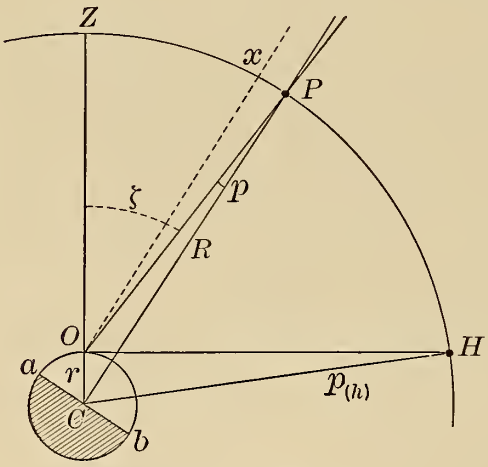


Fig. 32. — Parallax

Diurnal or geocentric parallax.

The *diurnal* or *geocentric* parallax of the sun, moon, or a planet is the difference of its direction as seen from the *center of the earth* and from the *observer's station* on the earth's surface; or, what comes to the same thing, it is the angle at the body made by two lines drawn from it, one to the observer, the other to the center of the earth. In Fig. 32 the parallax of the body P is the angle OPC, which equals xOP, and is the difference between ZOP and ZCP. Obviously this parallax is zero for a body directly overhead at Z, and a maximum for a body rising at H. Moreover, and this is to be specially noted, this parallax of a body at the horizon — the *horizontal parallax* — is simply *the angular semidiameter of the earth as seen from the body*. When we say that the moon's horizontal parallax is 57′, it is equivalent to saying that, seen from the moon, the earth has an apparent diameter of 114′.

Horizontal parallax.

79. Law of the Parallax. — From the triangle OCP we have

$$PC : OC = \sin COP : \sin CPO,$$

or, $R : r = \sin \zeta : \sin p$ (since COP is the supplement of ζ).

This gives

$$\sin p = \frac{r}{R} \sin \zeta, \qquad (a)$$

Formula embodying the laws of diurnal parallax.

or, since p is always a small angle,

$$p'' = 206265'' \frac{r}{R} \sin \zeta. \qquad (b)$$

When a body is at the horizon its zenith-distance is 90° and $\sin \zeta = 1$. Hence, the horizontal parallax, Π, of the body is given by the formula

$$\Pi = \frac{r}{R}, \quad (c); \qquad \text{and} \qquad p = \Pi \sin \zeta. \quad (d)$$

Or, in words, *the parallax at any altitude equals the horizontal parallax multiplied by the sine of the apparent zenith-distance.*

From equation (c) we have also, for finding R, the distance of the body,

Relation between distance of a body and its parallax.

$$R = \frac{r}{\sin \Pi}, \qquad \text{or} \qquad R = \frac{206265\, r}{\Pi''}, \qquad (e)$$

a relation of great importance as determining the distance of a heavenly body when its parallax is known.

80. Equatorial Parallax. — Owing to the "ellipticity," or "oblateness," of the earth, the horizontal parallax of a body varies slightly at different places, being a maximum at the equator, where the distance of an observer from the earth's center is greatest. It is agreed to take as the standard the *equatorial-horizontal-parallax*, *i.e.*, the earth's *equatorial* semi-diameter in seconds as seen from the body.

Equatorial parallax.

If the earth were exactly spherical, the parallax would act in an exactly vertical plane and would simply diminish the altitude of the body without in the least affecting its azimuth.

Effect of the earth's oblateness upon parallax in the case of the moon.

Really, however, it acts along great circles drawn from the *geocentric* zenith to the *geocentric* nadir (Sec. 11), and these circles are not identical with the vertical circles nor exactly normal to the horizon. For this reason the azimuth of the *moon*, which has a parallax of about a degree, is sensibly affected. The calculation of the parallax corrections to observations of the moon's right ascension and declination is also modified and greatly complicated. (See Campbell's *Practical Astronomy*, Sec. 26.)

In the calculation of the parallax of all other bodies it is sufficient to regard the earth as spherical.

"Augmentation" of the moon's semidiameter.

81. Semidiameter. — In the case of the sun or moon the edge, or *limb*, of the object is usually observed, and to get the true position of its center the angular semidiameter must be added or subtracted. For all objects except the moon this may be taken directly from the ephemerides, but the moon's apparent diameter increases slightly with its altitude, being about $\frac{1}{60}$ part, or about 30″, greater when in the zenith than at the horizon, because at the zenith it is about 4000 miles, or $\frac{1}{60}$ part of its whole distance from the center of the earth, nearer than at the horizon. At any observed zenith-distance, OP (Fig. 32), the apparent or "augmented" semidiameter (s'), as seen from O, is greater than the semidiameter (s) given in the ephemeris as seen from C, in the ratio of PC to PO. From the triangle POC we obtain, therefore,

Formula for the augmented semidiameter.

$$s' : s :: PC : PO :: \sin POC : \sin PCO :: \sin \zeta : \sin (\zeta - p)$$

(ζ being the apparent zenith-distance).

Whence
$$s' = s \frac{\sin \zeta}{\sin (\zeta - p)}.$$

This "augmentation" of the moon's diameter, amounting to about 30″ near the zenith, has, of course, nothing whatever to do with the optical illusion already referred to which makes the moon seem larger when near the horizon.

Astronomical refraction.

82. Refraction. — As the rays of light from a star enter our atmosphere, unless they strike perpendicularly they are bent downwards by refraction and follow a curved path, as illustrated in Fig. 33.

Its effect to increase the apparent altitude of a body.

Since the object is seen in the direction from which the rays enter the eye, the effect is to make the *apparent* altitude of the object greater than the *true.*

Refraction, like parallax, is zero at the zenith and a maximum at the horizon, where under average conditions it lifts an object about 35′, leaving the azimuth, however, unchanged. But the *law* of refraction is very different from that of parallax.

Its amount depends upon the density of the air (which is determined by the barometric pressure and temperature) as well as the altitude of the object, but is independent of its distance.

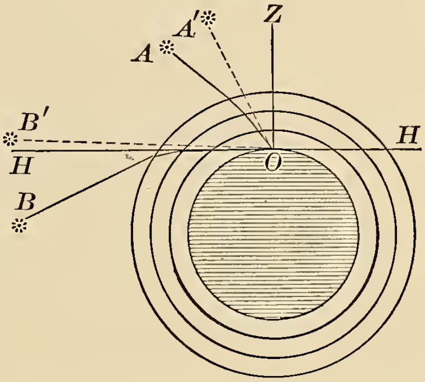


Fig. 33. — Atmospheric Refraction

The theory of refraction is too complicated to be discussed here, and the reader is referred to Campbell's or Chauvenet's *Practical Astronomy.*

Affected by temperature and barometric pressure.

The computation of the correction when precision is required is made by means of elaborate tables provided for the purpose and given in works on practical astronomy, the data being the observed altitude of the object, the temperature, and the height of the barometer. Increase of atmospheric pressure slightly increases the refraction, and increase of temperature diminishes it.

Approximate formulæ for bodies above 15° altitude.

For altitudes exceeding 25° the following approximate formula, corresponding to a temperature of zero Centigrade (32° Fahrenheit) and a barometric pressure of 30 inches, may be used, and will generally give results correct within a few seconds, viz., $r'' = 60''.7 \tan \zeta$, in which ζ is the *apparent* zenith-distance.

The following formula (due to Professor Comstock) is a little more complicated, but much more accurate, viz.,

$$r'' = \frac{983\,b}{460 + t} \tan \zeta,$$

in which b is the height of the barometer *in inches* and t is the temperature on *Fahrenheit's scale.* For altitudes above 15° this formula will seldom be over $1''$ in error.

Refraction table in Appendix.

The little Table VIII (Appendix) gives by inspection pretty accurately the refraction under the circumstances stated in its heading; and by applying the approximate corrections for barometer and thermometer indicated in the note below it, the results will seldom be more than $2''$ in error.

It is hardly necessary to add that this refraction correction, required by most astronomical observations of position, is very troublesome, and usually involves more or less uncertainty and error from the continually changing and unknown condition of the atmosphere along the path followed by the rays of light.

For methods by which the amount of the refraction *is determined by observation*, the reader is referred to works on practical astronomy, or to the author's *General Astronomy*, Art. 94.

Effect of refraction to increase length of the day at expense of the night.

83. Effect of Refraction near the Horizon. — The horizontal refraction, ranging as it does from 32′ to 40′, according to meteorological conditions, is always somewhat greater than the diameter of either the sun or the moon. At the moment, therefore, when the sun's lower limb appears to be just rising or setting, the whole disk is really below the plane of the horizon; and the time of sunrise in our latitudes is thus accelerated from two to four minutes, according to the inclination of the sun's diurnal circle to the horizon, which varies with the time of the year. Of course, sunset is delayed by the same amount, and thus at both ends the day is lengthened at the expense of the night.

Effect of refraction upon the form of the disks of sun and moon when very near the horizon.

Near the horizon the refraction changes very rapidly; while under ordinary summer temperature it is about 35′ at the horizon, it is only 29′ at an elevation of half a degree, so that as the sun or moon rises the bottom of the disk is lifted 6′ more than the top and the vertical diameter is thus made apparently about one-fifth part shorter than the horizontal. This quite notably distorts the disk into the form of an oval flattened on the under side. In cold weather the effect is much more marked.

Two other semi-astronomical effects, the twinkling of the stars and twilight, are due to the action of our atmosphere, and may be treated in this connection, though in no other way connected with the principal subject of the chapter.

Scintillation an atmospheric phenomenon.

84. Twinkling or Scintillation of the Stars. — This is a purely atmospheric phenomenon, usually conspicuous near the horizon, where it is often accompanied by marked changes of color. Near the zenith it generally disappears, and at other altitudes it differs greatly on different nights. As a rule only the *stars* twinkle strongly; the *planets*, Mercury excepted, usually shine with an almost steady light.

Authorities differ as to the details of explanation, but probably scintillation is mainly due to two coöperating causes, both depending on the fact that the air is generally full of streaks and wavelets of unequal density carried by the wind.

Unequal refractions by drifting atmospheric wavelets of unequal density.

(1) Light coming through such a medium is concentrated in some places and diverted from others by simple refraction, like light from an electric lamp shining through an ordinary window-pane upon the opposite wall. If the light of a star were strong enough, a white surface illuminated by it would be covered by bright and dark mottlings, drifting with the wind; and as such mottlings pass the eye the star appears to fade and brighten by turns. Looked at in the telescope, it also "dances," being slightly displaced back and forth by the irregular refraction.

Supplementary action of optical "interference."

(2) The other cause of twinkling is optical *interference.* Pencils of light coming from a star (optically a mere luminous *point*) reach the observer's eye by routes differing only slightly, and are just in a condition to "interfere." The result is the temporary destruction of rays of certain wave-lengths and the reinforcement of others. Accordingly, the "spectrum" (Sec. 569) of a twinkling star is traversed by dark bands in the different colors, oscillating back and forth, but, on the whole, when the star is rising, progressing from the blue towards the red, and *vice versa* when the star is near the setting.

Effect upon the spectrum of a star.

Why planets do not twinkle.

The *planets* do not twinkle, because they are not luminous *points*, but have disks made up of a congeries of such points; while each point twinkles like a star, the twinklings do not synchronize with each other, and so the general sum of light remains practically uniform. When very near the horizon, however, the irregular refraction is sometimes sufficiently violent to make them dance and change color. Since the disk of Mercury is very small, and the planet is never seen except near the horizon, it usually behaves like a star.

Cause of twilight.

85. Twilight. — This is caused by the *reflection* of sunlight from the upper portion of the earth's atmosphere, perhaps from the air itself, perhaps from the minute solid particles in the air, — authorities differ. After the sun has set, its rays, passing over the observer's head, still continue to shine through the air above him, and twilight continues as long as any portion of the illuminated air remains in sight from where he stands. It is considered to end when stars of the sixth magnitude become visible near the zenith, which does not occur until the sun is about 18° below the horizon; but this varies considerably for different places, according to the purity of the air.

Duration of twilight.

The length of time required by the sun after setting to reach this depth varies with the season and with the observer's latitude. In latitude 40° it is about ninety minutes on March 1 and October 12, but more than two hours at the summer solstice. In latitudes above 50°, when the days are longest, twilight never disappears even at midnight. On the mountains of Peru, on the other hand, it is said never to last more than half an hour, probably because the upper air in that region is practically clear from dust particles.

Height of the earth's atmosphere.

From the fact that twilight lasts until the sun is 18° below the horizon, the height of the twilight-producing atmosphere can easily be computed, and comes out about 50 miles. This, however, is not the real limit of the atmosphere. The phenomena of meteors show that at an elevation of 100 miles there is still air enough to resist their motion and cause their incandescence.

The twilight bow.

Soon after the sun has set, the *twilight bow* appears rising in the east, — a dark blue segment, bounded by a faintly reddish arc. It is the shadow of the earth upon the air, and as it rises the arc becomes rapidly diffuse and indistinct and is lost long before it reaches the zenith.

EXERCISES

1. What is the approximate dip of the horizon from a hill 900 feet high (Sec. 77)?

2. How high must a mountain be in order that the dip of the horizon from its summit may be 2°?

3. What is the distance of the horizon in miles, as seen from the summit of this mountain (Sec. 77)?

4. Assuming the horizontal parallax of the sun at $8''.8$, what is the horizontal parallax of Mars when nearest us, at a distance of 0.378 astronomical units? (The astronomical unit is the distance from the earth to the sun.)

5. What is the greatest apparent diameter of the earth as seen from Mars?

6. What is the horizontal parallax of Jupiter when at a distance of 6 astronomical units?

7. Does atmospheric refraction increase or decrease the apparent size of the sun's disk when it is near the horizon?

8. What is the lowest latitude where twilight can last all night? Can it do so at New York? at London? at Edinburgh?

CHAPTER IV

FUNDAMENTAL PROBLEMS OF PRACTICAL ASTRONOMY

Latitude — Time — Longitude — Azimuth — The Right Ascension and Declination of a Heavenly Body

Fundamental problems of observation.

86. There are certain problems of practical astronomy which are encountered at the very threshold of all investigations respecting the heavenly bodies, the earth included. The student must know how to determine his *position on the surface of the earth*, that is, his latitude and longitude; how to ascertain the *exact time at which an observation is made;* and how to observe the *precise position of a heavenly body* and fix its right ascension and declination.

87. Definitions of the Observer's Latitude. — In geography the latitude of a place is usually defined simply as its distance north or south of the equator, *measured in degrees.* This is not explicit enough unless it is stated how the degrees themselves are to be measured. If the earth were a perfect sphere there would be no difficulty, but since the earth is sensibly flattened at its poles the geographical degrees have somewhat different lengths in different parts of the earth. The fundamental definition of astronomical latitude has already been given (Sec. 32) as *the angle between the direction of gravity where the observer stands and the plane of the equator.* The angle between gravity and the *earth's axis* is the *colatitude* of the place. Other equivalent definitions of the latitude are the *altitude of the pole* and the *declination of the zenith*, which is the same as the altitude of the pole, as is clear from Fig. 34, where ZQ obviously equals NP.

Definitions of astronomical latitude.

The problem, then, is to determine by observation of the heavenly bodies *either the angle of elevation of the celestial pole, or the distance in degrees between the zenith and the celestial equator.*

Latitude by observation of circumpolar stars.

88. First Method: by Observation of Circumpolar Stars. — The most obvious method (already referred to) is by observing with a suitable instrument the *altitude* of some star near the pole at the moment when it is crossing the meridian above the pole, and again twelve sidereal hours later when it is once more on the meridian but below the pole. In the first case its altitude is the greatest possible; in the second, the least. The *mean of the two altitudes* (each corrected for atmospheric refraction) is the *altitude of the pole* or the *latitude* of the observer.

Advantages and disadvantages of the method.

The method has the great advantage that it is an *independent* one; that is, the observer is not obliged to depend upon his predecessors for any of his data. But the method fails for stations very near the equator, because there the pole is so near the horizon that the necessary observations cannot be made.

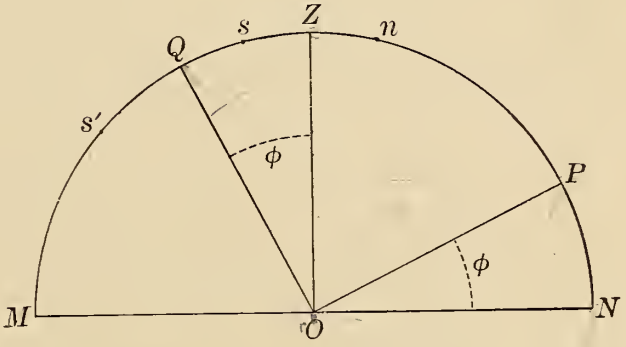


Fig. 34

At an observatory the observations are usually made with the meridian-circle, and the mean of a great number of observations is necessary in order to eliminate the slight errors in the computed refraction corrections due to varying atmospheric conditions. Where the meridian-circle is not available, the observations may also be made with a sextant or theodolite, but the results are much less precise.

89. Second Method: by the Meridian Altitude or Zenith-Distance of a Body whose Declination is accurately known. — In Fig. 34 the circle *MQPN* is the meridian, *Q* and *P* being

Latitude by meridian altitude of object of known declination.

respectively the equator and the pole and Z the zenith. QZ is the declination of the zenith, or the latitude of the observer. If, when the star s crosses the meridian, we observe its zenith-distance, ζ_s (Zs in the figure), its declination, Qs or δ_s being known, then evidently QZ equals Qs plus sZ; that is, the *latitude equals the declination of the star plus its zenith-distance.* If the star were at s', south of the equator, the same equation would still hold *algebraically*, because the declination Qs' is then a negative quantity; and if the star were at n between the zenith and pole, we should have its *north* zenith-distance, ζ_n, a negative quantity. In all cases, therefore, we may write $\phi = \delta + \zeta$.

Formula for latitude in this case.

If we use the meridian-circle in making our observations, we can always select stars that pass near the zenith, where the refraction is small, which is in itself a great advantage. Moreover, we can select them in such a way that some will be as much north of the zenith as others are south, and this will practically *eliminate* even the slight refraction errors that remain. On the other hand, in using this method we have to obtain our star declinations from the catalogues made by previous observers, so that the method is not an "independent" one.

Advantages and disadvantages of this method.

Latitude at sea by observation of the sun.

90. *At sea* the latitude is usually obtained by observing with the sextant the *sun's* maximum altitude, which occurs, of course, at noon. Since at sea one seldom knows beforehand precisely the moment of *local* noon, the observer takes care to begin his observations some minutes earlier, repeating his measure of the sun's altitude every minute or two. At first the altitude will keep increasing, but immediately after noon occurs it will begin to decrease. The observer uses, therefore, the maximum[1] altitude obtained, which, corrected for refraction, parallax, semidiameter, and dip of the horizon, will give him the true meridian altitude of the sun. The Nautical Almanac gives him its declination.

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[1] On account of the sun's motion in declination and the northward or southward motion of the ship itself, the sun's maximum altitude is usually attained not precisely on the meridian, but a short time earlier or later. This requires a slight correction to the deduced latitude, the calculation of which is explained in books on navigation.

91. Third Method: by Circummeridian Altitudes. — If the observer knows his time with reasonable accuracy, he can obtain his latitude from observations of the altitude of a heavenly body made when it is *near* the meridian with practically the same precision as at the moment of meridian passage. It lies beyond our scope to discuss the method of reduction, which is explained, with the necessary tables, in all works on practical astronomy.

Latitude by circum-meridian altitudes.

The great advantage of the method is that the observer is not restricted to a single observation at each meridian passage of the sun or of the selected star, but can utilize the half-hours preceding and following that moment. The meridian-circle, of course, cannot be used. Usually the sextant, or a so-called "universal instrument" (Sec. 70), is employed.

92. Fourth Method[1]: by the Zenith-Telescope. — The essential characteristic of the method is the measurement with a micrometer of *the difference between the nearly equal zenith-distances of two stars* which pass the meridian within a few minutes of each other, one north and the other south of the zenith, and not very far from it; such pairs of stars can now always be found in our star-catalogues.

Latitude by the zenith-telescope — the most accurate method.

A special instrument, known as the *zenith-telescope*, is generally employed, though a simple transit-instrument, provided with reversing apparatus, a delicate level attached to the telescope, and a declination micrometer is now often used.

Fig. 35 shows the very complete zenith-telescope of the Flower Observatory near Philadelphia.

At the Georgetown Observatory a photographic zenith-telescope is used, having a photographic plate in place of the eyepiece.

The telescope is set at the proper altitude for the star which first comes to the meridian and the "latitude level," as it is called, — which is attached to the telescope — is set horizontal;

Method of observation.

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[1] Known as the "American method," because first practically introduced by Captain Talcott, of the United States Engineers, in a boundary survey in 1845. It is now very generally adopted and considered the best.

as the star passes through the field of view its distance north or south of the central horizontal wire is measured by the micrometer. The instrument is then reversed so that the telescope points towards the north (if it was south before), and the telescope so readjusted, if necessary, that the level is again horizontal, — taking great care, however, *not to disturb the angle between the level and the telescope itself.* The telescope is then evidently elevated at exactly the same angle as before, but on the opposite side of the zenith. As the second star passes through the field, we measure with the micrometer its distance north or south of the central wire. The comparison of the two measures gives the difference of the two zenith-distances with great accuracy and *without the necessity of depending upon any graduated circle.*

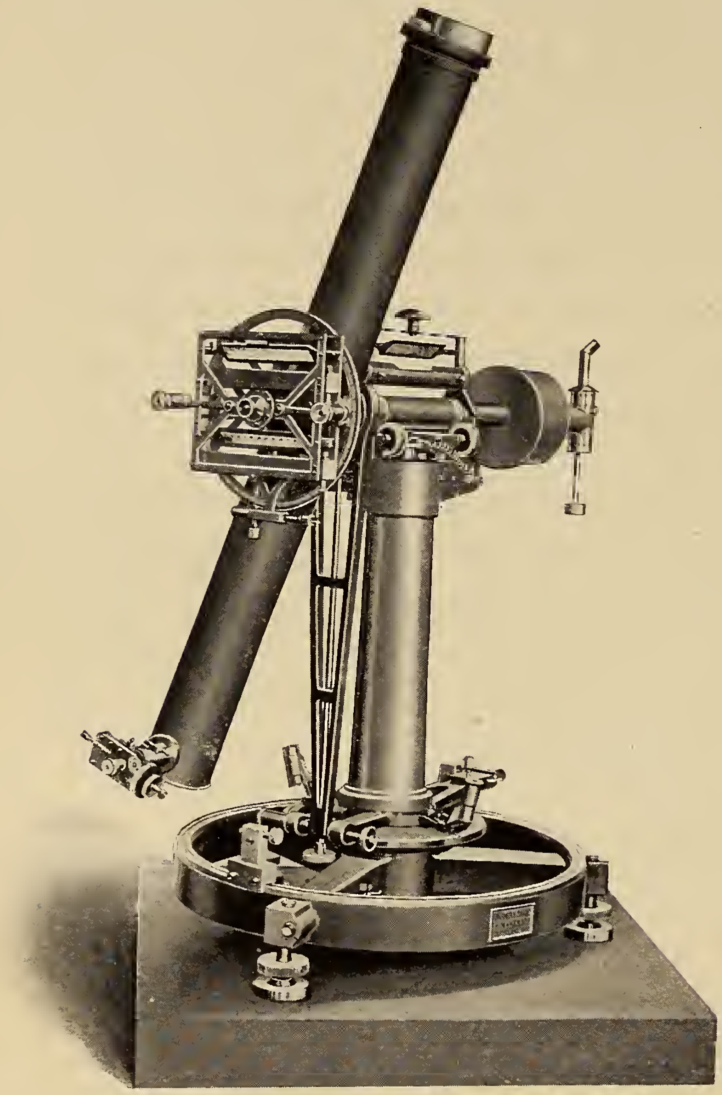

FIG. 35. — A Zenith-Telescope
By Warner & Swasey

Advantage in dispensing with a graduated circle.

In field operations like those of geodesy this is an enormous advantage, both as regards the portability of the instrument and the attainable precision of results.

From Fig. 34 we have

for star *south* of zenith, $\phi = \delta_s + \zeta_s$;
for star *north* of zenith, $\phi = \delta_n - \zeta_n$.

Adding the two equations and dividing by 2, we have

$$\phi = \left(\frac{\delta_s + \delta_n}{2}\right) + \left(\frac{\zeta_s - \zeta_n}{2}\right).$$

Formula for the latitude.

The star-catalogue gives us the declinations of the two stars $(\delta_s + \delta_n)$; and the difference of the zenith-distances $(\zeta_s - \zeta_n)$ is determined by the micrometer measures.

When the method was first introduced it was difficult to find pairs of stars whose declination was known with sufficient precision. At present our star-catalogues are so extensive and exact that this difficulty has practically disappeared.

Refraction eliminated.

Refraction is almost eliminated, because the two stars of each pair are at very nearly the same zenith-distance.

Evidently the accuracy depends ultimately upon the exactness with which the level measures the slight but inevitable difference between the inclinations of the instrument when pointed on the two stars.

In Dr. Chandler's *Almucantar* (Sec. 67) the telescope preserves its constant declination *automatically*, by being mounted upon a base which floats in mercury, thus dispensing with the level.

There are numerous other methods for obtaining the latitude. In Chauvenet's *Practical Astronomy* over forty are given, some of which can fairly compete in precision with those named above.

Ancient method of determining the latitude by the gnomon.

93. The Gnomon. — The ancients could not use any of the preceding methods for finding the latitude. They were, however, able to make a very respectable approximation by means of the simplest of all astronomical instruments, the *gnomon*. This is merely a vertical shaft or column of known height erected on a perfectly horizontal plane, and the observation consists in noting the length of the shadow cast at noon at certain times of the year. Suppose, for instance, *that on the day of the summer solstice*, at noon, the length of the shadow is *AC* (Fig. 36). The height *AB* being given, we can easily compute in the right-angled triangle the angle *ABC*, which equals *SBZ*, the sun's zenith-distance when farthest north.

Again, observe the length *AD* of the shadow *at noon of the shortest day in winter* and compute the angle *ABD*, which is the sun's corresponding zenith-distance when farthest south. Now, since the sun travels equal distances north and south of the celestial equator, the mean of the two zenith-distances will give the angular distance between the equator and the zenith, *i.e.*, the *declination of the zenith*, which is the latitude of the place.

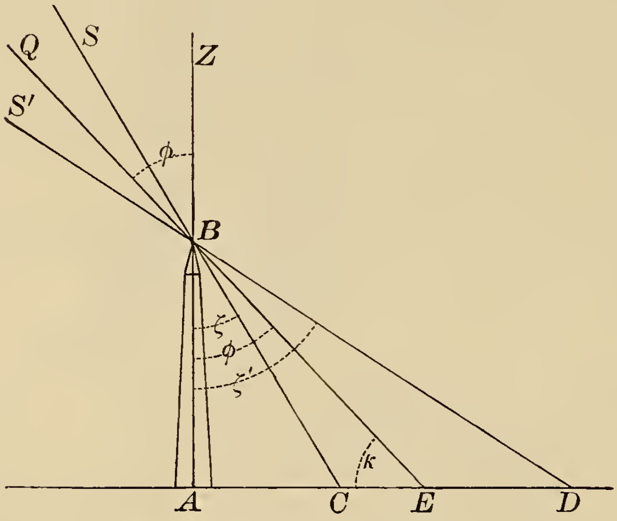

FIG. 36.—Latitude by the Gnomon

The method is an independent one, like that of the observation of circumpolar stars, requiring no data except those which the observer determines for himself. It does not admit of much accuracy, however, since the penumbra at the end of the shadow makes it impossible to measure its length very precisely.

"Climate."

It should be noted that the ancients, instead of designating the position of a place by means of its latitude, used its *climate*; the climate (from κλίμα) being the slope of the plane of the celestial equator, the angle *AEB*, which is the *colatitude*.

For the use of the gnomon in determining the obliquity of the ecliptic and the length of the year, see Secs. 164 (2) and 182. Many of the Egyptian obelisks are known to have been used for astronomical observations, and perhaps were erected mainly for that purpose.

No evidence of any considerable changes in the position of the earth's axis.

94. Variation of Latitude and Motion of the Poles of the Earth. — It has long been doubted whether latitudes are strictly constant. They cannot be so if the axis of the earth shifts its position within the globe. Some have supposed that in the past there have been great changes of this kind, seeking thus to explain certain geological epochs, as, for instance, the glacial and the carboniferous. But thus far no evidence of any considerable displacement has appeared, nor is there any satisfactory proof of certain slow, continuous "secular" changes, which have been strongly suspected.

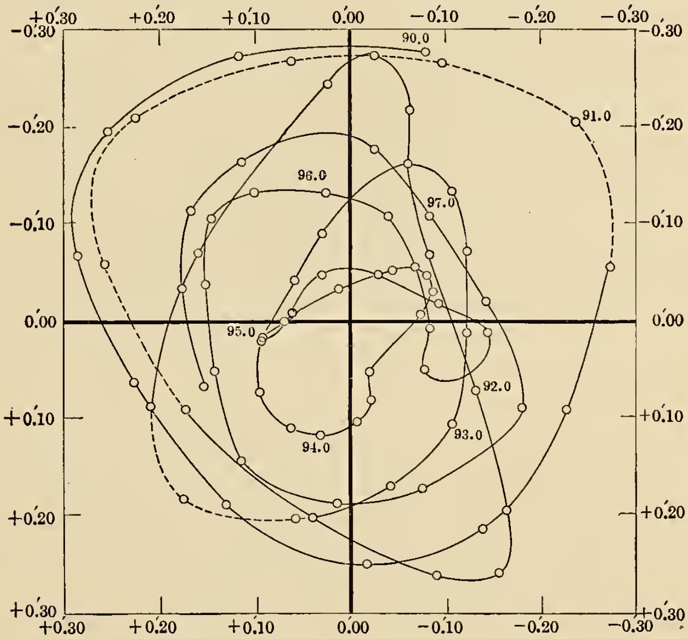

Fig. 37

Minute changes theoretically must occur.

Theoretically, however, any alteration in the arrangement of the matter of the earth, by elevation, subsidence, transportation, or denudation, must necessarily disturb the axis and change the latitudes to some extent. The question is merely whether our observations can be made sufficiently accurate to detect the change. Since 1889 the limit has been reached, and we now have conclusive proof of such effects.

First observational evidence obtained in 1888.

The first satisfactory evidence of the fact was obtained at Berlin by Küstner, and at other German stations in 1888 and 1889, and the result has since been abundantly confirmed by observations at many other stations. Moreover, Dr. S. C. Chandler of Cambridge, U.S., by a brilliant and laborious series of investigations, finds the same variations clearly exhibited in almost every extended body of reliable observations made since

1750. From the whole mass of evidence he concludes that the movement of the pole at present is composed of two motions, — one an *annual* revolution in an ellipse about 30 feet long, but varying in width and position, the other a revolution in a circle about 26 feet in diameter and having *a period of about 428 days,* — both revolutions being *counter-clockwise.* The resultant motion presents a very irregular appearance and changes greatly from year to year.

Nature of the periodic motion of the pole.

Fig. 37 represents the actual motion from 1890 to 1898 as deduced by Albrecht from all available observations.

The annual component of this polar motion is very likely due to meteorological causes which follow the seasons, such as the deposit of rain, snow, and ice. The explanation of the 428-day component is not yet entirely clear, and its discussion would take us too far.

It is likely also that irregular disturbances, due to various causes — for instance, perhaps, earthquakes — may modify the regular periodic motions.

Time defined as measured duration.

95. Different Kinds of Time. — Time is usually defined as *measured duration.* From the beginning the apparent diurnal rotation of the heavens has been accepted as the standard unit, and to it we refer all artificial measures of time, such as clocks and watches.

Determination of time.

In practice the accurate determination of time consists in finding *the Hour Angle* (Sec. 21) *of the object or point which has been selected to mark the beginning of the day by its passage across the meridian.*

The three kinds of time.

In astronomy three kinds of time are now recognized: *sidereal time*, *apparent solar time*, and *mean solar time*, — the last being the time of civil life and ordinary business, while the first is used for astronomical purposes exclusively. Apparent solar time (formerly called *true* time) has now practically fallen out of use, except in countries where watches and clocks are scarce or unknown and sun-dials are the ordinary timekeepers.

96. Sidereal Time. — The celestial object which determines sidereal time by its position in the sky at any moment is, it will be remembered, the *vernal equinox* or *first of Aries* (symbol, ♈), *i.e.*, the point where the sun crosses the celestial equator in the spring, about *March 21* every year.

Sidereal time — the hour angle of the vernal equinox.

As already stated (Sec. 25), the local sidereal *day*[1] begins at the moment when the first of Aries crosses the observer's meridian, and the sidereal *time* at any moment is *the hour angle of the vernal equinox; i.e.*, it is the time marked by a clock so set and adjusted as to show *sidereal noon* ($0^h0^m0^s$) at each transit of the first of Aries.

The equinoctial point is, of course, invisible; but its position among the stars is always known, so that its hour angle at any moment can be determined by observing the stars.

97. Apparent Solar Time. — Just as sidereal time is the hour angle of the vernal equinox, so *apparent solar time* at any moment is *the hour angle of the sun.* It is the *time shown by the sun-dial*, and its noon occurs at the moment when the sun's center crosses the meridian.

Apparent solar time — the hour angle of the sun: identical with sun-dial time.

On account of the earth's orbital motion (explained more fully in Chapter VI), the sun appears to move eastward along the ecliptic, completing its circuit in a year. Each noon, therefore, it occupies a place among the stars about a degree farther east than it did the noon before, and so comes to the meridian about four minutes *later*, if time is reckoned by a *sidereal clock.* In other words, the solar day is about four minutes *longer* than the sidereal, the difference amounting to *exactly one day* each year, which contains 366¼ sidereal days.

But the sun's eastward motion is not uniform, for several

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[1] On account of the precession of the equinoxes (to be discussed later), the sidereal day thus defined is slightly shorter than it would be if defined as the interval between successive transits of some given *star*; the difference being a little less than $\frac{1}{100}$ of a second, or one day in 25800 years, — too little to be worth taking into account in any ordinary calculation.

Apparent solar days vary in length. Hence apparent time is unsatisfactory.

reasons, and the apparent solar days therefore vary in length. December 23, for instance, is about fifty-one seconds longer from sun-dial noon to noon again (by a sidereal clock) than September 16. For this reason apparent solar or sun-dial time is unsatisfactory for scientific use and cannot be kept by any simple mechanical arrangement in clocks and watches. At present it is practically discarded in favor of mean solar time.

The fictitious sun.

98. Mean Solar Time.—A *fictitious sun* is, therefore, imagined, moving *uniformly eastward in the celestial equator* and completing its annual course in exactly the same time as that in which the actual sun makes the circuit of the *ecliptic*. This fictitious sun is made the timekeeper for mean solar time. It is *mean noon* when its center crosses the meridian, and at any moment the *hour angle of the fictitious sun* is the mean time for that moment. The mean solar days are, therefore, all of exactly the same length and equal to the length of the *average* apparent solar day, the mean solar day being *longer* than the *sidereal* by $3^m55^s.91$ (*mean solar* minutes and seconds) and the *sidereal* day *shorter* than the solar by $3^m56^s.55$ (*sidereal* minutes and seconds).

Mean solar time: the hour angle of the fictitious sun.

Sidereal time unsuitable for ordinary use.

99. Sidereal time will not answer for business purposes, because its noon (the transit of the vernal equinox) occurs at all hours of the day and night in different seasons of the year: on September 22, for instance, it comes at midnight. Apparent solar time is unsatisfactory because of the variation in the length of its days and hours. Yet we have to live by the sun: its rising and setting, daylight and night, control our actions.

Difficulty with apparent solar time.

Advantages of mean solar time.

Mean solar time furnishes a satisfactory compromise. It has a time unit which is invariable, and it can be kept by clocks and watches, while it agrees nearly enough with sun-dial time for convenience. It is the time now used for all purposes except in some kinds of astronomical work.

The difference between apparent time and mean time (never amounting to more than about a quarter of an hour) is called the *equation of time* and will be discussed hereafter in connection with the earth's orbital motion (Sec. 174).

Equation of time.

Relation between the number of sidereal and mean solar seconds in a given time interval.

Since there are 365.2421 *solar* days in a year (Sec. 182) and one more *sidereal* day, we have the following fundamental relation:—*the number of* **sidereal** *seconds in any time interval : the number of mean* **solar** *seconds in the same interval : : 366.2421 : 365.2421.*

Reduction of a solar time interval to sidereal, and *vice versa.*

From this it follows at once that to reduce a *solar* time interval to sidereal, we must divide the number of seconds it contains by ***365.2421***, and *add* the quotient to the number of solar seconds. To reduce a *sidereal* interval to solar, divide by ***366.2421***, and *subtract* the quotient from the number of sidereal seconds.

The Nautical Almanac gives the sidereal time of mean solar noon for every day of the year, with tables by means of which mean solar time can be accurately deduced from the corresponding sidereal time, or *vice versa*, by a very brief[1] calculation.

The astronomical and civil days.

100. The Civil Day and the Astronomical Day.— The *astronomical* day begins at mean noon; the *civil* day, twelve hours earlier at midnight. Astronomical mean time is reckoned around through the whole twenty-four hours instead of being counted in two series of twelve hours each: thus, 10 A.M. of Wednesday, February *27*, *civil* reckoning, is Tuesday, February *26*, 22 o'clock, by *astronomical* reckoning. This must be borne in mind in using the Almanac.[2]

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[1] The *approximate* relation between sidereal time and mean solar time is very simple. Assuming that on March 22 the two times agree, after that day the sidereal time gains *two hours each month.* On April 22, therefore, the sidereal clock is two hours in advance, on June 22, six hours in advance, and so on. On account of the differing length of months, this reckoning is slightly erroneous in some parts of the year, but is usually correct within four or five minutes. March *22* is taken as the starting-point because it distributes the errors better than the 21st. For the odd days the gain may be taken as four minutes daily.

[2] The astronomical day is made to begin at noon because astronomers are "night-birds," and would find it inconvenient to have to change dates at midnight in the middle of their work.

DETERMINATION OF TIME

Determination of time consists in ascertaining the error of a timepiece.

In practice the problem of determining time always consists in ascertaining the *error* or *correction* of a timepiece, *i.e.*, *the amount by which the clock or chronometer is faster or slower than the time it ought to indicate.*

Determination of time by the transit-instrument.

101. Determination of Time by the Transit-Instrument.—The method most employed by astronomers is by observations with the transit-instrument (Sec. 61). We observe the time shown by the sidereal clock at which a star of *known right ascension* crosses each wire of the reticle. The mean is taken as the instant of crossing the instrumental meridian, and when the instrument is in perfect adjustment the difference between the star's right ascension and the observed clock time will be the clock's "error"; or, as a formula, $\Delta t = a - t$,—Δt being the usual symbol for the clock error, and t the observed time.

Almanac stars.

The Almanac supplies a list of several hundred stars whose right ascension and declination are accurately given for every tenth day of the year, so that the observer at night has no difficulty in finding a suitable star at almost any time. In the daytime he is, of course, limited to the brighter stars.

Necessary to observe a number of stars in order to attain high precision.

The observation of a single star with an instrument in ordinary adjustment will usually give the error of the clock within half a second; but it is much better and usual to observe a number of stars, reversing the instrument upon its Y's once at least during the operation. This will enable him to determine and allow for the faults of instrumental adjustment, so that with a good instrument a skilled observer can thus determine his clock error within about a thirtieth of a second of time, provided proper correction is applied for his "personal equation" (Sec. 64).

If instead of observing a star we observe the *sun* with this instrument, the time as shown by the mean solar clock ought to be twelve hours *plus or minus the equation of time* as given in the Almanac. But for various reasons transit observations of the sun are less accurate than those of the stars, and it is far better to deduce the mean solar time from the sidereal time by means of the almanac data.

Solar time usually now deduced from sidereal.

102. The Method of Equal Altitudes.—If we observe the time shown by the chronometer or the clock when a *star* attains a certain altitude and then the time when it attains the same altitude on the other side of the meridian, the mean of the two times will be the time of the star's transit across the meridian, provided, of course, that the chronometer runs uniformly during the interval.

Method of equal altitudes.

We may also use stars of slightly differing declination, one on one side of the meridian, and the other observed a few minutes later on the other side; and by a somewhat tedious calculation it is possible to determine the error of the clock with practically the same accuracy as if both observations had been made on the same star, and much more quickly.

Modification of the method in observing stars of slightly different declination.

If we observe the *sun* in this manner in the morning, and again in the afternoon, the moment of apparent noon will seldom be exactly half-way between the two observed times, and proper correction must be made for the sun's slight motion in declination during the interval,—a correction easily computed by tables furnished for the purpose.

Correction required in deducing time from equal altitudes of the sun.

The advantage of this method is that the *errors of graduation of the instrument have no effect*, nor is it necessary for the observer to know his latitude except approximately.

On the other hand, there is, of course, danger that the second observation may be interfered with by clouds. Moreover, both observations must be made *at the same place*.

103. Marine Method: by a Single Altitude of the Sun, the Observer's Latitude being known.—Since neither of the preceding methods can be used at sea, the following is the method usually practised. The altitude of the sun, *at some time when it is rapidly rising or falling* (*i.e.*, *not* near noon), is measured

Observation to determine time at sea.

with the sextant, and the corresponding time shown by the chronometer accurately noted.

Computation of the time from the observation.

We then compute the hour angle of the sun, P, from the triangle PZS (Fig. 38), and this hour angle, corrected for the equation of time, gives the *mean solar time* at the observed moment. The difference between this time and that shown by the chronometer is the error of the chronometer on local time.

In the triangle ZPS (which is the same as SPO in Fig. 8) all three of the sides are given: PZ is the complement of the latitude ϕ, which is supposed to be known; PS is the complement of the sun's declination δ, which is *found in the Almanac*, as is also the *equation of time;* while ZS or ζ is given by observation, being the complement of the sun's altitude as measured by the sextant and corrected for dip, semidiameter, refraction, and parallax. The formula ordinarily used is

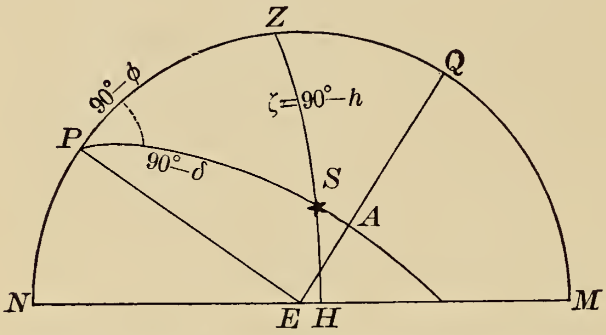

FIG. 38. — Determination of Time by the Sun's Altitude.

$$\sin \tfrac{1}{2} P = \sqrt{\frac{\sin \frac{1}{2}[\zeta + (\phi - \delta)] \sin \frac{1}{2}[\zeta - (\phi - \delta)]}{\cos \phi \cos \delta}}.$$

Time when observation should be made.

In order to insure accuracy it is desirable that the sun should be on the prime vertical, or as near it as practicable. It *should* NOT *be near the meridian*, for at that time the sun is rising or falling very slowly, and the slightest error in the measured altitude would make an enormous difference in the computed hour angle. If the sun is exactly east or west at the time of observation, an error of even several minutes of arc in the assumed latitude produces no sensible effect upon the result.

The disadvantage of the method is that any error of graduation of the sextant vitiates the result, and no sextant is perfect. But with ordinary care and good instruments the sea-captain is able to get his time correct within three or four seconds.

Disadvantage of the method and limit of accuracy.

When a number of altitude observations have been made for time, and it is desired to reduce them separately, so as to test their agreement and determine their probable error, there is an advantage in using the formula

$$\cos P = \frac{\cos \zeta}{\cos \phi \cos \delta} - \tan \phi \tan \delta,$$

employing the "Gaussian logarithms" in the computation. The second term of the formula and the denominator of the first term remain constant through the whole series, saving much labor in reduction.

104. To compute the Time of Sunrise or Sunset. — To solve this problem we have precisely the same data as in finding the time by a single altitude of the sun. The zenith-distance of the sun's center at the moment when its upper edge is rising equals 90° 51′, — made up of 90° plus 16′ (the mean semidiameter of the sun) plus 35′ (the mean refraction at the horizon). The resulting hour angle, corrected for the equation of time, gives the mean local time at which the sun's upper limb reaches the horizon under average circumstances of temperature and barometric pressure. If the sun rises or sets over the sea horizon and the observer's eye is at any considerable elevation above sea-level, the *dip of the horizon* must also be added to 90° 51′ before making the computation.

Calculation of time of sunrise and sunset.

The beginning and end of *twilight* may be computed in the same way by merely substituting 108°, *i.e.*, 90° + 18°, for 90° 51′.

DETERMINATION OF LONGITUDE

Having now the means of finding the true local time at any place, we can take up the problem of the longitude, the most important of all the economic problems of astronomy. The great observatories at Greenwich and Paris were established expressly for the purpose of furnishing the observations which could be utilized for its accurate determination at sea.

Definition of longitude.

105. *The longitude* of a place on the earth may be defined as *the angle at the pole of the earth between the standard meridian and the meridian of the place;* and this angle is measured by, and equal to, the *arc of the equator intercepted between the two meridians.*

As to the standard meridian there is some variation of usage. At sea nearly all nations at present reckon from the meridian of Greenwich, except the French, who insist on Paris.

Difference of longitude equals difference of local times.

Since the earth turns on its axis at a uniform rate, the angle at the pole is strictly proportional to the *time* required for the earth to turn through that angle; so that longitude may be, and now usually is, expressed in *time units*, — *i.e.*, in hours, minutes, and seconds, rather than degrees, etc., — and is simply the difference between *the local times at Greenwich and at the place where the longitude is to be determined.*

The knot of the problem.

Since the observer can determine his own local time by the methods already given, the *knot* of the problem is to find the Greenwich local time corresponding to his own, without leaving his place.

Telegraphic method.

106. First Method: by Telegraphic Comparison between his Own Clock and that of Some Station whose Longitude from Greenwich is known. — The difference between the two clocks will be the difference of longitude between the two stations after the proper corrections for *clock errors*, *personal equation*, *and time occupied by the transmission of the electric signals* have been applied or eliminated.

Details of process.

The process usually employed is as follows: The observers, after ascertaining that they both have clear weather, proceed early in the evening to determine the local time at each station by an extensive series of star observations with the transit-instrument. Then at an hour agreed upon the observer at the eastern station, A, "switches his clock" into the telegraphic circuit, so that its beats are communicated along the line and received upon the chronograph of the western station. After the eastern clock has thus sent its signals, say for two minutes, it is "switched out" and the *western* observer puts his clock into the circuit, so that its beats are received upon the eastern chronograph. Sometimes the signals are communicated both ways simultaneously, so that the beats of both clocks appear upon both chronograph sheets at the same time. The operation is closed by another series of transit observations by each observer.

Elimination of error due to transmission time and personal equation.

We have now upon each chronometer sheet an accurate comparison of the two clocks, showing the amount by which the western clock is slow of the eastern, and if the transmission of electric signals were instantaneous, the difference shown upon the two chronometer sheets would be identical on both. Practically, however, there will always be a discrepancy of some hundredths of a second, amounting to *twice the time* occupied in the transmission of the signals; but the mean of the two differences *after correcting for the carefully determined clock errors* will be the true difference of longitude between the places. *Especial care must be taken to determine with accuracy the personal equations of the observers*, or else *to eliminate them*, which may be done by causing the observers to change places.

In cases where the highest accuracy is required, it is customary to make observations of this kind on not less than five or six evenings.

Limit of attainable accuracy.

The astronomical difference of longitude between two places can thus be determined within about $\frac{1}{50}$ of a second of time, *i.e.*, within about 20 feet in the latitude of the United States.

The chronometric method of determining longitude at sea.

107. Second Method: by the Chronometer. — This method is available at sea. The chronometer is set to indicate Greenwich time before the ship leaves port, its "rate" having been carefully determined by observation for several days. In order to find the longitude by the chronometer, the sailor must determine its "error" upon *local time* by an observation of the altitude of the sun when near the prime vertical (Sec. 103). If the chronometer indicates true Greenwich time, its "error" deduced from the observation will be the longitude. Usually, however, the indication of the chronometer face must be corrected for the gain or loss of the chronometer since leaving port, in order to give the true Greenwich time at the moment.

Several chronometers needed to check each other.

Chronometers are only imperfect instruments, and it is important, therefore, that several of them should be carried by the vessel to check each other. This requires *three* at least, because if only two chronometers are carried, and they disagree, there is nothing to indicate which is the delinquent.

Failure of the method for long voyages.

Moreover, in the course of months, chronometers generally change their rates *progressively*, so that they cannot be depended on for very long intervals of time; and the error accumulates much more rapidly than in proportion to the time. If, therefore, a ship is to be at sea more than three or four months without making port, the method becomes untrustworthy. For voyages of less than a month it is now practically all that could be desired.

Lunar method, the moon being regarded as a clock hand showing Greenwich time.

108. Third Method: by the Moon regarded as a Clock Hand, with Stars for Dial Figures. — Before the days of reliable chronometers, navigators and astronomers were generally obliged to depend upon the *moon* for their Greenwich time. The laws of her motion are now fairly well known, so that the right ascension and declination of the moon are now computed and published in the Nautical Almanac, three years in advance, for every Greenwich hour of every day in the year. It is therefore possible to deduce the Greenwich time at any moment when the moon is visible by making some observation which will accurately determine her place among the stars.

Lunar methods available on land.

On land it may be:

(*a*) The direct transit-instrument observation of her *right ascension* as she crosses the meridian.

(*b*) The observation at the moment when she *occults a star* (incomparably the most accurate of all lunar methods) or makes contact with the sun in a solar eclipse.

(*c*) The observation of the *moon's azimuth* with the universal instrument at an accurately determined time.

Lunar distances at sea.

At sea the only practicable observation is to measure with a sextant a *lunar distance*, *i.e.*, the distance of the moon from some star or planet nearly in her path.

Since, however, the almanac place of the moon is the place she would apparently occupy if seen from the center of the earth, most lunar observations require complicated and laborious reductions before they can be used for longitude. Moreover, the motion of the moon is so slow (she requires a month to make the circuit of 360°) that any error in the observation of her place produces nearly thirty times as great an error in the corresponding Greenwich time and the deduced longitude. It is as if one should try to read accurate time from a watch that had only an hour-hand.

Inferiority of the lunar methods, occultations excepted.

109. Other Methods: Eclipses of the Moon and Jupiter's Satellites. — A rough longitude can be obtained from the observation of these eclipses, since they occur at the same moment of absolute time wherever observed. By comparing the local times of observation with the Greenwich time obtained by correspondence after the event, or from the Almanac, the difference of longitude at once comes out. The difficulty with this method is that the eclipses are *gradual* phenomena, presenting no well-marked instant for observation.

Longitude by eclipses of the moon and Jupiter's satellites.

On the same principle *artificial signals*, such as flashes of powder and explosion of rockets, can be used between two stations so situated that both can see the flashes. Early in the century the difference of longitude between the Black Sea and the Atlantic was determined by means of a chain of such signal stations on the mountain tops; so also, later, the difference of longitude between the eastern and western extremities of the northern boundary of Mexico. This method is now superseded by the telegraph.

Longitude by artificial signals.

110. Local and Standard Time. — Until recently it has been always customary to use *local* time, each city determining its own time by its own observations. Before the days of the telegraph, and while traveling was comparatively slow and infrequent, this was best. At present it has been found better for many reasons to give up the system of local times in favor of a system of *standard time*. This facilitates all railway and telegraphic business in a remarkable degree, and makes it practically easy for every one to keep accurate time, since it can be daily wired from some observatory (as Washington) to every telegraph office in the country. According to the system now established in North America, there are five such standard times in use, — the *colonial*, the *eastern*, the *central*, the *mountain*, and the *Pacific*, — which are slower than Greenwich time by

Local and standard time.

Advantages of standard time.

American systems of standard time.

exactly four, five, six, seven, and eight hours, respectively. The minutes and seconds are everywhere identical.

At most places only one of these standard times is employed; but in cities where different systems join each other, as, for instance, at Atlanta and Pittsburg, two standard times are in use, differing from each other by exactly one hour, and from the local time by about half an hour. In some such places the *local* time also maintains itself.

The same system has now been practically adopted in almost all civilized countries.

Standard time in foreign countries.

On the continent of Europe, except in Russia, the time used is one hour fast from Greenwich; in West Australia, eight hours fast; in Japan and South Australia, nine hours; in Victoria, Queensland, and Tasmania, ten hours. Cape Colony uses time one and one-half hours fast from Greenwich time, and New Zealand eleven and one-half hours fast.

In order to determine the standard time by observation it is only necessary to determine the local time by one of the methods given, correcting it by first adding the observer's longitude west from Greenwich, and then deducting the necessary integral number of hours.

The beginning of the day at the 180th meridian.

111. Where the Day begins. — It is evident that if a traveler were to start from Greenwich on Monday noon and were by some means able to travel westward along the parallel of latitude as fast as the earth turns eastward beneath his feet, he would keep the sun exactly upon the meridian all day long and have continual noon. But what noon? It was Monday noon when he started, and when he gets back to London twenty-four hours later he will find it to be Tuesday noon there. Yet it has been noon all the time. When did Monday noon become Tuesday noon?

It is agreed among mariners to *make the change of date at the 180th meridian from Greenwich*, which passes over the Pacific hardly anywhere touching the land.

Ships crossing this line *from the east skip one day* in so doing. If it is Monday afternoon when a ship reaches the line, it becomes Tuesday afternoon the moment she passes it, the intervening twenty-four hours being dropped from the reckoning on the log-book. *Vice versa*, when a vessel crosses the line from the *western* side, it counts the same day *twice over*, passing from Tuesday back to Monday, and having to do Tuesday over again.

Loss or gain of a day by vessels passing the date-line.

There is considerable irregularity in the date actually used on the different islands in the Pacific, as will be seen by looking at the so-called *date-line* as given in the Century Atlas of the World. Those islands which received their earliest European inhabitants *via* the Cape of Good Hope have adopted the Asiatic date, even if they really lie east of the 180th meridian; while those that were first approached *via* the American side have the American date. When Alaska was transferred from Russia to the United States, it was necessary to drop one day of the week from the official dates.

The date-line.

PLACE OF A SHIP AT SEA

112. Determination of the Position of a Ship. — The determination of the place of a ship at sea is commercially of such importance that, notwithstanding a little repetition, we collect here the different methods available for the purpose. They are necessarily such that the requisite observations can be made with the sextant and chronometer, the only instruments available on shipboard.

The *latitude* is usually obtained by observations of the sun's altitude at noon, according to the method explained in Sec. 90.

The *longitude* is usually found by determining the error upon local time of the chronometer, which carries Greenwich time. (See Secs. 103 and 107.)

Determination of a ship's latitude and longitude.

In case of long voyages, or when the chronometer has for any reason failed, the longitude may also be obtained by measuring lunar distances and comparing them with the data of the Nautical Almanac.

These methods require separate observations for the latitude and for the longitude.

Sumner's method.

113. Sumner's Method. — At present a method known as *Sumner's Method*, because first proposed by Captain Sumner of Boston, in 1843, has come largely into use. It is based on the principle that any single observation of the sun's altitude, giving, of course, its *zenith-distance* at the time, determines the so-called *circle of position* on which the ship is situated. The center of this circle of position on the earth's surface is the point directly under the sun at the moment of observation. The *longitude* of this point is the *Greenwich apparent time* at the moment of observation as determined by the chronometer, and its *latitude* is the sun's *declination.* The radius of the circle of position (reckoned in degrees of a great circle from this center) is the observed *zenith-distance* of the sun.

The circle of position. Its center and radius at any time.

Position of ship determined by the intersection of two circles of position.

A second observation made some hours later will give a second circle of position, and if the ship has not moved meanwhile the intersection of the two circles will give the place of the ship.

The circles intersect at *two* points, of course, but at which one the ship is situated is never doubtful, because the approximate *azimuth* of the sun, observed simply as a compass bearing, tells roughly on what part of the circle the ship is placed. If, for instance, the sun is in the southeast at the first observation, the ship must be on the northwestern part of the corresponding circle of position.

If the ship has moved between the two observations, as of course is usual, its motion as determined by log and compass can be allowed for with very little difficulty.

Practical application of the method.

114. Usually the matter is treated as follows: The latitude of the vessel is practically always known within a degree or so, from the "dead reckoning" since the last observation. Suppose the latitude is known to be about 51°; then, from the first (morning) observation of the sun's altitude and the chronometer time, the navigator computes the *longitude,* *assuming the latitude to be 50°*, and finds it to be, say, 40° 52′. Again, assuming the latitude to be 52°, he gets 43° 20′, and marks the two computed longitudes at *A* and *B* on the chart (Fig. 39). A line drawn through these points will be very nearly a part of the vessel's *circle of position* at the time of that observation.

From the second (afternoon) observation the points *C* and *D* are computed in the same way, giving a piece of the second circle of position.

Suppose now that in the interval the ship has moved 60 miles on a course north 60° west. From the points *A* and *B* lay off 60 miles on the chart in the proper direction to the points *a* and *b*, and join *ab* by a line. *S′*, the intersection of this line with the line *CD*, will be the position of the ship at the time of the second observation with all the approximation necessary for the navigator's purpose; and if we reckon back 60 miles from *S′*, we shall find *S*, the ship's position in the morning. There are, however, extended tables which greatly reduce the labor of computations and make the result more accurate than that derived from the chart.

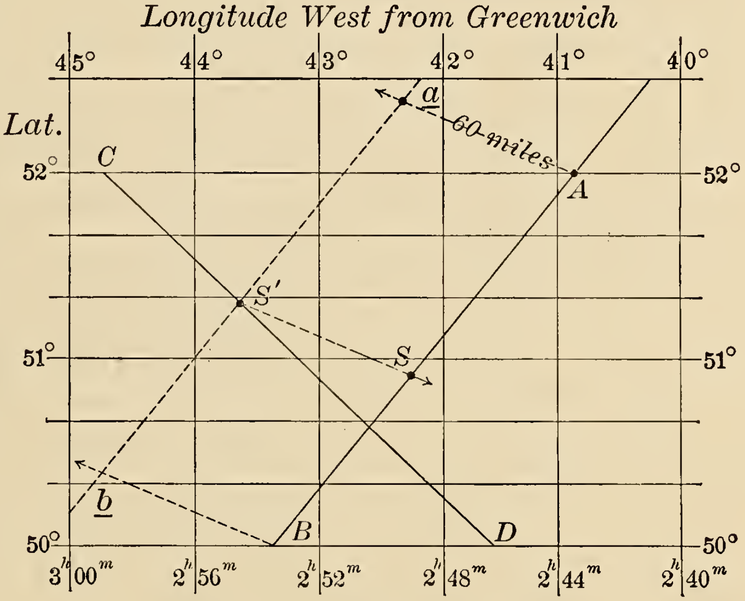

Fig. 39. — Sumner's Method

The peculiar advantage of the method is that each observation is used for all it is worth, giving accurately the position of a line upon which the vessel is somewhere situated, and approximately (by the sun's azimuth) its position on that line. Very often this knowledge is all that the navigator needs to give him the knowledge of his distance from land, even when he fails in getting the second observation necessary to determine his precise location. *Everything, however, depends upon the correctness of the Greenwich time given by the chronometer*, just as in the ordinary method of longitude determination.

Its peculiar advantage.

Must have Greenwich time.

L. of C.

Determination of azimuth by observations of pole-star.

115. Determination of "Azimuth." — A problem, important, though not so often encountered as that of latitude and longitude determinations, is that of determining the "azimuth," or "true bearing," of a line upon the earth's surface.

With a theodolite having an accurately graduated horizontal circle the observer points alternately upon the pole-star and upon a distant signal erected for the purpose at a distance of say half a mile or more, — usually an "artificial star" consisting of a small hole in a plate of metal, with a lantern behind it. At each pointing he notes the time by a sidereal chronometer. The theodolite must be carefully adjusted for collimation, and especial pains must be taken to have the axis of the telescope perfectly level.

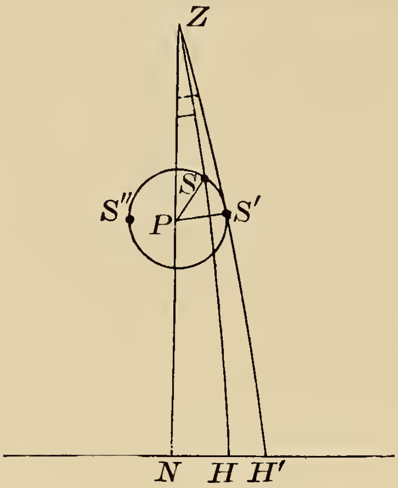

Fig. 40. — Determination of Azimuth

The next morning by daylight the observer measures the angle or angles between the night signal and the objects whose azimuth is required.

If the pole-star were exactly at the pole, the mere difference between the two readings of the circle, obtained when the telescope is pointed on the star and on the signal, would directly give the azimuth of the signal. As this is not the case, the azimuth of the star must be computed for the moment of each observation, which is easily done, as the right ascension and declination of the star are given in the Almanac for every day of the year.

Referring to Fig. 40, N being the north point of the horizon, P the pole, and NZ the meridian, we see that PS is the polar distance of the star, or complement of its declination, the side PZ is the complement of the observer's latitude, while the angle at P is the hour angle of the star, *i.e.*, the difference between the right ascension of the star and the sidereal time of observation. This hour angle must, of course, be reduced to *degrees* before making the computation. We thus have two sides of the triangle, viz., PS and PZ, with the included angle at P, from which to compute the angle Z at the zenith. This is the star's azimuth.

The pole-star is used rather than any other because, being so near the pole, any slight error in the assumed latitude of the place or in the time of the observation will produce hardly any effect upon the result, especially if the star be observed near its greatest elongation east or west of the pole.

The sun, or any other heavenly body whose position is given in the Almanac, can also be used as a reference point in the same way when near the horizon, provided sufficient care is taken to secure an *accurate observation of the time* at the instant when the pointing is made. But the results are usually rough compared with those obtained from the pole-star.

Azimuth by the sun.

DETERMINATION OF THE POSITION OF A HEAVENLY BODY

116. The "position" of a heavenly body is defined by its right ascension and declination. These may be determined:

Direct determination of both coördinates of a body's position by the meridian-circle

(1) *By the meridian-circle*, provided the body is bright enough to be seen by the instrument and crosses the meridian at night. If the instrument is in exact adjustment, the *sidereal time when the object crosses the middle wire of the reticle of the instrument is directly the right ascension of the object.* Corrections are necessary only on account of errors of the clock, errors of adjustment of the instrument, and personal equation of the observer. Parallax and refraction do not enter into the result.

The *reading of the circle* of the instrument, corrected for refraction, and for parallax if necessary, gives the *polar distance* of the object if the polar point of the circle has been determined, or it gives the zenith-distance of the object if the nadir point has been determined (Sec. 69). In either case the *declination* can be immediately deduced. A single complete observation, therefore, with the meridian-circle, determines both the right ascension and declination of the object. In order to secure accuracy, however, it is desirable that the observations should be repeated many times.

Differential use of the meridian-circle.

It is often better to use the instrument "differentially," *i.e.*, to observe some neighboring standard star or stars of accurately known position, soon before or after the object whose place is to be determined. We thus obtain the *difference* between the right ascension and declination of the object observed and others which are accurately known; and in this case slight errors in the graduation and adjustment of the instrument affect the final result very little.

When a body (a comet, for instance) is too faint to be observed by the telescope of the meridian-circle, which is seldom very powerful, or when it does not come to the meridian during the night, we must accomplish our observation with some instrument that can pursue the object to any part of the heavens. At present the equatorial is almost exclusively used for the purpose.

Determination of position by the equatorial and micrometer.

117. (2) *By the equatorial.* With this we determine the position of a body by measuring the *difference of right ascension and declination* between it and some neighboring star whose place is given in a star-catalogue, and, of course, has been accurately determined by the meridian-circle of some observatory.

In measuring this difference of right ascension and declination we usually employ a micrometer (Sec. 71) fitted with wires like the reticle of a meridian-circle. It carries a number of fixed wires which are set accurately north and south in the field of view, and these are crossed at right angles by one or more wires which can be moved by the micrometer screw. The difference of *right ascension* between the star and the object to be determined is measured by clamping the telescope firmly and simply observing and recording upon the chronograph the transits of the two objects across the wires that run north and south; the difference of declination, by bisecting each object by one of the micrometer wires as it crosses the middle of the field of view. The difference of the two micrometer readings gives the difference of declination.

The observed differences must be corrected for refraction and for the motion of the body during the time of observation.

The measurement may also be made with the position micrometer by measuring the angle of position and distance between the object and the star of comparison, as it is called.

Instead of using a micrometer we may employ photography. For this purpose the telescope is fitted with a plate-holder in place of the eyepiece, and is accurately driven by clockwork. On the sensitive plate a photograph is obtained of all the stars in the field, and also of the object; and the position of the object is afterwards determined by measuring the plate. It is found that determinations of extreme accuracy can be made in this way, and the method is rapidly coming into extensive use.

Determination of position by photography.

EXERCISES

In cases where corrections for refraction are required they are to be taken from Table VIII (Appendix), taking into account the temperature and barometric pressure, if given among the data. If preferred, the student may also use Comstock's formula (Sec. 82). The results for example 1 have their corrections computed by the regular refraction tables, and the approximate results obtained by the student may differ from them by a considerable fraction of a second.

1. Given the following meridian-circle observations on β Ursæ Minoris at its upper and lower culminations, respectively, viz.:

Altitude 55° 48′ 06″.0, temperature 30° F., barometer 30.1 inches.
24° 58′ 56″.4, " 25° F., " 30.1 "

The nadir reading (Sec. 69) was 270° 01′ 06″.8 in both cases. Required the latitude of the place and the declination of the star.

Ans. Lat. 40° 20′ 57″.8.
Dec. 74° 34′ 40″.1.

2. Given the meridian altitude of the sun's lower limb 62° 24′ 45″, the height of the observer's eye above the sea-level being 16 feet (Sec. 77).

The sun's declination was + 20° 55′ 10″ and its semidiameter 15′ 47″. Its parallax at the observed altitude was 5″ and the mean refraction from Table VIII may be used. Required the latitude of the ship.

Ans. + 48° 19′ 3″.

3. The sun's meridian altitude on a ship at sea is observed to be 30° 15′ (after being duly corrected); the sun's declination at the time is 19° 25′ *south.* What is the ship's latitude?

4. How much will a sidereal clock gain on a mean solar clock in 10 hours and 30 minutes?

Ans. $1^{m}43.5^{s}$.

5. How many times will the second-hand of a sidereal clock overtake that of a solar clock in a solar day if they start together?

Ans. 236 times.

6. At what intervals do the coincidences occur?

Ans. $6^{m}5.242^{s}$.

7. Reduce 10 hours 40 minutes and 25 seconds of mean time to sidereal time. (See Sec. 99.)

8. Reduce 10 hours 40 minutes and 25 seconds of sidereal time to solar time.

9. What is the approximate sidereal time on July 30 at 10 P.M.?

Solution by note to Sec. 99. July 22, noon sid. time $= 8^{h}00^{m}$

8 days gain $\underline{32}$

Sid. time at noon $8^{h}32^{m}$

10 hours = sid. $\underline{10\quad 1\tfrac{2}{3}}$

Sid. time at 10 P.M. $18^{h}33\tfrac{2}{3}^{m}$

10. What is the approximate sidereal time on October 4 at 7 A.M. civil reckoning?

11. In determining longitudes by telegraph, will it or will it not make any difference whether sidereal or solar clocks are used by the observers, provided both use the same?

12. A ship leaving San Francisco on Tuesday morning, October 12, reaches Yokohama after a passage of exactly 16 days. On what day of the month and of the week does she arrive?

13. Returning, the same vessel leaves Yokohama on Saturday, November 6, and reaches San Francisco on Tuesday, November 23. How many days was she on the voyage?

CHAPTER V

THE EARTH AS AN ASTRONOMICAL BODY

Its Form, Rotation, and Dimensions — Mass, Weight, and Gravitation — The Earth's Mass and Density

118. In a science which deals with the heavenly bodies it might seem at first that the earth has no place; but certain facts relating to it are similar to those we have to study in the case of sister planets, are ascertained by astronomical methods, and a knowledge of them is essential as a basis of all astronomical observations. In fact astronomy, like charity, "begins at home," and it is impossible to go far in the study of the bodies which are strictly "celestial" until one has acquired some accurate knowledge of the dimensions and motions of the earth itself.

The earth an astronomical body in many respects.

119. The astronomical facts relating to the earth are broadly these:

Leading astronomical facts relating to the earth.

(1) *The earth is a great ball about 7920 miles in diameter.*

(2) *It rotates on its axis once in twenty-four sidereal hours.*

(3) *It is not exactly spherical, but is flattened at the poles, the polar diameter being nearly 27 miles, or probably a little more than one three-hundredth part less than the equatorial.*

(4) *Its mean density is between 5.5 and 5.6 as great as that of water, and its mass is represented in tons by 6 with twenty-one ciphers following (six thousand millions of millions of millions of tons).*

(5) *It is flying through space in its orbit around the sun with a velocity of about 18½ miles a second, or nearly 100000 feet a second,—about thirty-three times as fast as the swiftest modern projectile.*

I. ROTUNDITY AND SIZE OF THE EARTH

Rotundity of the earth; its shadow always circular.

120. The Earth's Approximate Form. — It is not necessary to dwell on the ordinary familiar proofs of the earth's globularity. One, first quoted by Galileo as absolutely conclusive, is that the outline of the earth's shadow seen upon the moon during a lunar eclipse is such as only a sphere could cast.

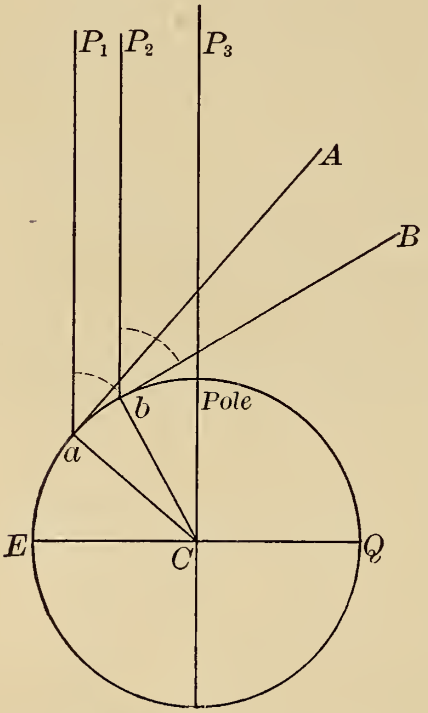

Fig. 41. — Measuring the Earth's Diameter

We may add, as to the smoothness and roundness of the earth, that if represented by an 18-inch globe, the difference between its greatest and least diameters would be only about $\frac{1}{16}$ of an inch, the highest mountains would project only about $\frac{1}{75}$ of an inch, and the average elevation of continents and depths of the ocean would be hardly greater on that scale than the thickness of a film of varnish. Relatively, the earth is much smoother and rounder than most of the balls in a bowling-alley.

Determination of its diameter by measuring an arc of meridian.

121. The Approximate Measure of the Diameter of the Earth regarded as a Sphere. — (1) *By an arc of meridian.* There are various ways of determining the diameter of the earth. The simplest and best is by measuring *the length of an arc.* It consists essentially in *astronomical measurements* which determine the distance between two selected stations (several hundred miles apart) in *degrees* of the earth's circumference, combined with *geodetic* measurements giving their exact distance in miles or kilometers.

The astronomical determination is most easily made if the two stations are on the same terrestrial meridian. Then, as is clear from Fig. 41, the distance *ab* in degrees is simply the difference of latitude between *a* and *b*. The latitudes are best determined by zenith-telescope observations (Sec. 92), but any accurate method may be used.

Astronomical work consists in measuring the latitudes of the terminal stations.

The *linear distance* (in feet or meters) is measured by a geodetic process called *triangulation*. It is not practicable to measure it with sufficient accuracy directly, as by simple "chaining."

Geodetic work consists in measuring distance in miles by triangulation.

Between the two terminal stations (*A* and *H*, Fig. 42) others are selected, such that the lines joining them form a complete chain of triangles, each station being visible from at least two others. The angles at each station are carefully measured, and the length of one of the sides, called the *base*, is also measured with all possible precision. It can be done, and is done, with an error not exceeding an inch in 10 miles. (*BU* is the base in the figure.) Having the length of the base and all the angles, it is then possible to calculate every other line in the chain of triangles and to deduce the exact *north and south distance* (*Ha*) between *H* and *A*. An error of more than three feet in a hundred miles would be unpardonable.

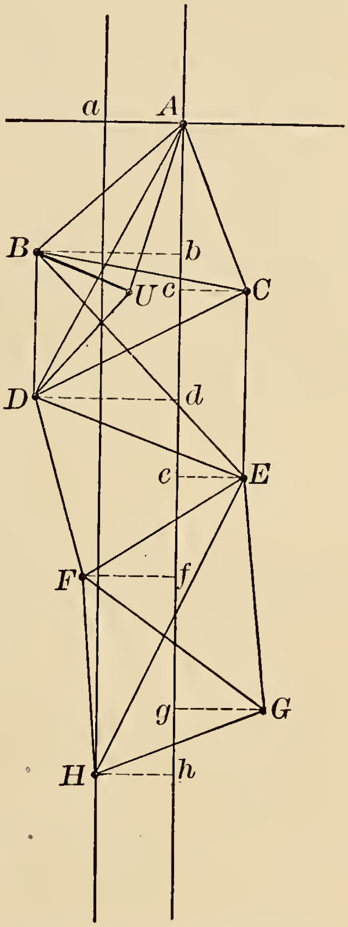

FIG. 42. — Triangulation

In this way many arcs of meridian have been measured the average of which (for they differ, because the earth, as we shall see, is not quite spherical) makes the length of a degree 69.1 miles, the mean circumference 24875 miles, and the mean diameter 7918 miles.

Result.

122. The ancients understood the principle of the operation perfectly. Their best known attempt at a measurement of the sort was made by Eratosthenes of Alexandria about 250 B.C., his two stations being Alexandria and Syene in Upper Egypt.

Method of Eratosthenes.

At Syene he observed that at noon of the longest day in summer there was no shadow in the bottom of a well, the sun being then vertically overhead. On the other hand, the gnomon at Alexandria on the same day, by the length of the shadow, gave him $\frac{1}{50}$ of a circumference (7° 15′) as the distance of the sun from the zenith at that place. This $\frac{1}{50}$ of a circumference is, therefore, the difference of latitude between Alexandria and Syene, and the circumference of the earth must be fifty times the *linear distance* between those two stations.

The weak place in his work was the measurement of this linear distance between the two places. He states it as 5000 stadia, without telling how it was measured, thus making the circumference of the earth 250000 stadia, which may be exactly right; for we do not know the length of his stadium, nor does he give any account of the means by which he measured the distance, if he measured it at all. (There seem to have been as many different stadia among the ancient nations as there were kinds of "feet" in Europe at the beginning of the century.)

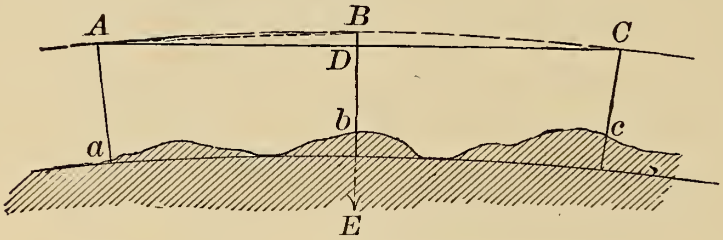

Fig. 43. — Curvature of the Earth's Surface

The first really valuable measure of an arc of the meridian was that made by Picard in Northern France in 1671, — the measure which served Newton so well in his verification of the idea of gravitation.

Experimental exhibition of curvature of earth's surface.

123. (2) The curvature of the earth's surface is easily demonstrated, and an approximate estimate of its diameter obtained, by the following method. Erect upon a reasonably level plane three rods in line, a mile apart, and cut off their tops at the same level, carefully determined by a surveyor's leveling-instrument. It will then be found that the line AC (Fig. 43), joining the extremities of the two terminal rods, *when corrected for refraction*, passes about 8 inches below B, the top of the middle rod.

(On account of refraction, however, which curves the line of sight between A and C, the result cannot be made exact. The *observed* value of BD ranges all the way from 4.5 to 6.5 inches, according to the temperature.)

Effect of refraction in lessening the apparent curvature.

Suppose the circle ABC completed, and that E is the point on the circumference opposite B, so that BE equals the diameter of the earth ($= 2R$).

Computation of earth's diameter from such observations.

By geometry, $BD : BA = BA : BE$, or $2R$,

whence $$R = \frac{BA^2}{2\,BD}.$$

Now BA is 1 mile, and $BD = 8$ inches, or $\frac{2}{3}$ of a foot, or $\frac{1}{7920}$ of a mile.

Hence, $2R$ (the earth's diameter) $= 7920$ miles, — a very fair approximation.

II. THE ROTATION OF THE EARTH

124. Probable Evidence of the Earth's Rotation. — At the time of Copernicus the only argument he could bring in favor of the earth's rotation was that this hypothesis was much more probable than the older one that the great heavens themselves revolved. All the phenomena *then known* would be sensibly the same on either supposition. The apparent diurnal motion of the heavenly bodies can be fully accounted for within the limits of observation *then* possible, either by supposing that the stars are actually attached to an immense celestial sphere which turns around daily, or that the earth itself rotates upon an axis; and for a long time the latter hypothesis seemed to most people less probable than the older and more obvious one.

Probability of the earth's rotation.

A little later, after the invention of the telescope, *analogy* could be adduced; for with the telescope we can see that the sun, moon, and many of the planets are rotating globes.

Within the last century it has become possible to adduce experimental proofs which go still further and absolutely *demonstrate* the earth's rotation. Some of them even make it visible.

Experimental proof of the earth's rotation by the Foucault pendulum.

125. Foucault's Pendulum Experiment.—Among these experimental proofs the most impressive is the pendulum experiment, devised and first executed by Foucault in 1851. From the dome of the Panthéon in Paris he hung a heavy iron ball about a foot in diameter by a wire more than 200 feet long (Fig. 44). A circular rail some 12 feet across, with a little ridge of sand built upon it, was placed in such a way that a pin attached to the swinging ball would just scrape the sand and leave a mark at each vibration. To put the ball in motion it was drawn aside by a cotton cord and left for hours to come absolutely to rest; then the cord was *burned* and the pendulum started without jar to swing in a true plane.

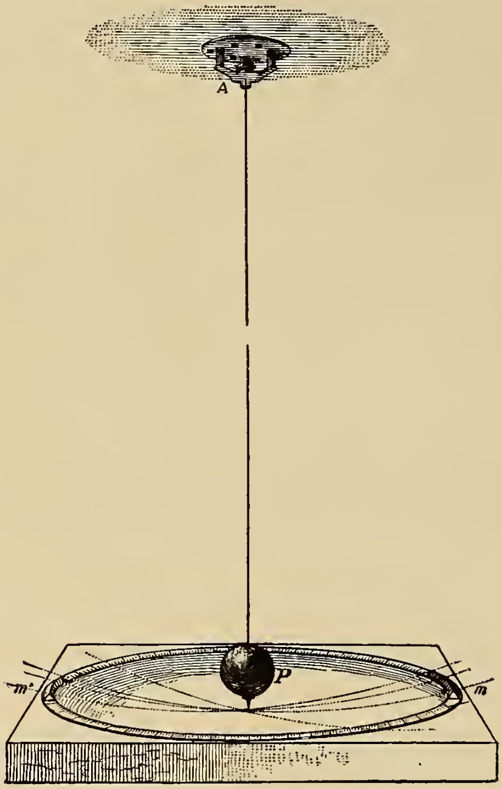

Fig. 44.—Foucault's Pendulum Experiment

But this plane at once began apparently to *deviate slowly towards the right*, in the direction of the hands of a watch, and the pin on the pendulum ball cut the sand ridge in a new place at each swing, shifting at a rate which would carry it completely around in about thirty-two hours if the pendulum did not first come to rest. In fact, the floor of the Panthéon was really and visibly turning under the plane of the vibrating pendulum.

Fig. 45 is copied from a newspaper of that date and shows the actual appearance of the apparatus and its surroundings.

The experiment created great enthusiasm at the time and has since been frequently repeated.

126. Explanation of the Foucault Experiment. — The approximate theory of the experiment is very simple. A swinging pendulum, suspended so as to be *equally free to swing in any plane* (unlike the common clock pendulum in this freedom), if set up at the pole of the earth, would appear to shift completely around in twenty-four hours. Really in this case the plane of vibration remains unchanged while the earth turns under it. This can easily be understood by setting up on a table a similar apparatus, consisting of a ball hung from a frame by a thread, and then, while the ball is swinging, turning the table around upon its casters with as little jar as possible. The plane of the swing will remain unchanged by the motion of the table.

Why the free pendulum appears to shift its plane. Rate of shift at the pole.

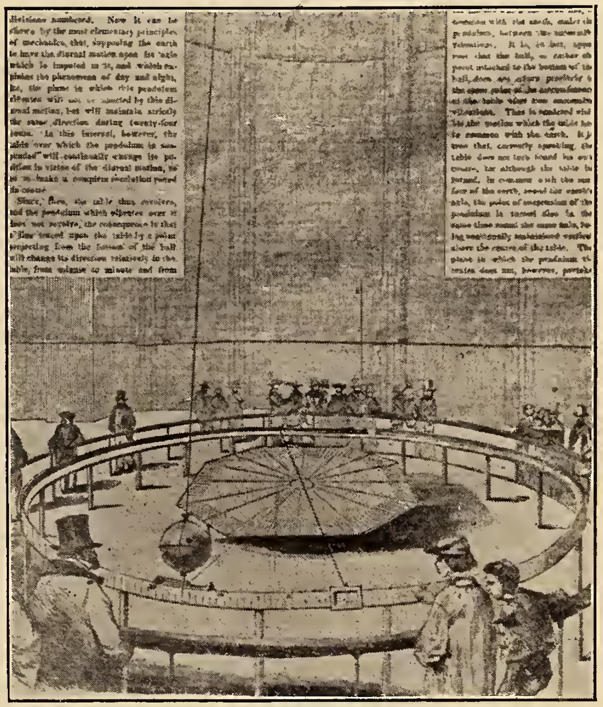

Fig. 45. — Foucault's Pendulum in the Panthéon

It is easy to see, moreover, that at the earth's equator there will be no such tendency to shift; while in any other latitude the effect will be intermediate and the time for the pendulum to complete the revolution in its plane will be longer than at the pole.

No shift at the equator.

The northern edge of the floor of a room in the northern hemisphere is nearer the axis of the earth than is its southern edge, and therefore is carried more slowly eastward by the earth's rotation. Hence, the floor must "skew" around continually, like a postage-stamp gummed upon a whirling globe, anywhere except at the globe's equator. A line drawn on the floor, therefore, continually shifts its direction, and a free

Effect at intermediate latitudes.

pendulum, set at first to swing along such a line, must *apparently* deviate at the same rate in the opposite direction.

Hourly deviation equals 15° × sin ϕ.

It can be proved (see *General Astronomy*, Arts. 140, 141) that the hourly deviation of a Foucault pendulum equals 15° multiplied by the *sine* of the latitude. In the latitude of New York it is not quite 10° an hour.

In the northern hemisphere the plane of vibration, as already stated, moves around with the hands of a watch. In the southern the motion is reversed.

Other phenomena which exhibit the rotation of the earth.

127. There are various other demonstrations of the earth's rotation which we merely mention, referring to the author's *General Astronomy* for their discussion:

(*a*) *By the gyroscope*, an experiment also due to Foucault.

(*b*) By the slight *eastward deviation of bodies in falling from a height.* This deviation is, of course, zero at the pole and a maximum at the equator; it varies as the *cosine* of the latitude, other things being equal, and amounts to about one inch in a fall of 500 feet for a station in latitude 50°. The idea of the experiment is due to Newton, but its execution has been carried out only during the past century, by several European observers.

If the earth were strictly spherical, and if gravity were directed to its center, there would also be a slight deviation towards the equator. But Laplace has shown that, as things are, no sensible deviation of that kind takes place.

(*c*) *By the deviation of projectiles* to the *right* in the northern hemisphere, to the *left* in the southern.

(*d*) By various phenomena of meteorology and physical geography, — such as the direction of the trade and anti-trade-winds and of the great ocean currents, and the counter-clockwise revolution of cyclones in the northern hemisphere, reversed in the southern.

It might at first seem that the rotation of the earth once a day is not a very rapid motion, but a point on the equator travels nearly 1000 miles an hour, or about 1500 feet a second.

128. Invariability of the Earth's Rotation. — It is a question of great importance whether the day ever changes its length, for if it does our time unit is not a constant. Theoretically, some change is almost inevitable. The friction of the tides and the deposit of meteoric matter on the earth's surface both tend to *retard* its rotation; on the other hand, the earth's loss of heat by radiation, and its consequent shrinkage, tend to *accelerate* it and to shorten the day. Then geological causes act, some one way and some the other.

Question as to variability of earth's rotation.

At present we can only say that the change which may have occurred since astronomy has been accurate is too small to be surely detected. The day is *certainly* not longer or shorter by $\frac{1}{100}$ part of a second than it was in the days of Ptolemy, and *probably* it has not altered by $\frac{1}{1000}$ of a second. The test is found in comparing the times at which celestial phenomena, such as eclipses, transits of Mercury, etc., have occurred during the range of astronomical history.

Certain that change, if any, has been extremely small.

Professor Newcomb's investigations in this line make it highly probable, however, that the length of the day has not been quite constant during the last 150 years. There are suspicious indications that Greenwich noon has, at irregular intervals of from thirty to fifty years, sometimes come too early by as much as four or five seconds, and at other times fallen as much behind. Astronomers are somewhat anxious on the subject, because if the earth's rotation should turn out to be capriciously changeable in any sensible degree, it would compel us to look for some new and independent unit of time.

Some indications of slight accelerations and retardations.

EFFECTS OF THE EARTH'S ROTATION UPON GRAVITY ON THE EARTH'S SURFACE AND UPON THE EARTH'S FORM

129. Centrifugal Force due to the Earth's Rotation. — As the earth rotates on its axis every particle of its surface is subjected to a "centrifugal force" directed perpendicularly away

Centrifugal force due to earth's rotation at the equator equals $\frac{1}{289}$ of gravity.

from the axis, and this force, C, depends upon the *velocity* of the particle and the *radius* of the circle in which it moves.

According to a familiar formula, this centrifugal force, C, equals $\frac{V^2}{R}$ (*Physics*, p. 74). An equivalent formula, often more convenient, is $C = \frac{4\,\pi^2 R}{T^2}$, since V equals $2\,\pi R$, the circumference of the circle, divided by T, the time of revolution. This gives C as an "acceleration" (in feet per second), just as gravity is given by g.

The equatorial radius of the earth being 20 926202 feet, and the time of revolution, or the sidereal day, being 86167 mean solar seconds, we find $C = 0.1112$ *feet* per second, or 1.334 inches per second. This is $\frac{1}{289}$ part of g, which is 386 inches per second. At the earth's equator, therefore, C equals $\frac{1}{289}\,g$.

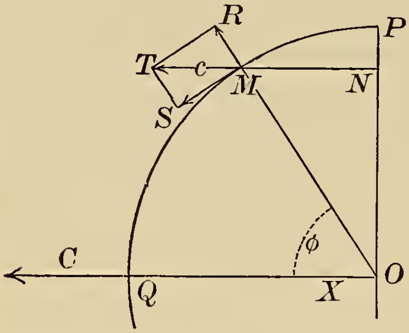

FIG. 46. — Centrifugal Force Caused by Earth's Rotation

A rotation seventeen times as rapid would cause bodies to fly off from the earth's surface.

Since centrifugal force varies with the square of the velocity, and 289 is the square of 17, it appears that if the earth revolved seventeen times as swiftly, keeping its present size and form (an impossible supposition), bodies at the equator would lose all their weight; and if the speed were increased beyond that point, everything on the surface there would fly off unless fastened down.

Centrifugal force at any latitude is proportional to the cosine of the latitude.

At any other latitude, assuming the earth to be spherical, which is sufficiently accurate for our present purpose, the radius of the circle described by M (Fig. 46) is MN, which equals R cos ϕ; *i.e.*, at any latitude the centrifugal force c equals $C \cos \phi = \frac{1}{289}\,g \times \cos \phi$, becoming, therefore, zero at the pole.

130. Effects of Centrifugal Force on Gravity. — At the equator the whole centrifugal force is opposed to gravity, so that bodies

Centrifugal force resolvable into two components.

there weigh on this account $\frac{1}{289}$ less (weighed by a *spring-balance*) than they would if the earth did not rotate; but the direction of gravity is not altered. Elsewhere, except at the pole itself, both the amount and direction of gravity are affected.

Vertical component opposed to gravity.

(*a*) *Diminution of Gravity.* Referring to the figure, we see that the centrifugal force MT, or c, is resolvable into two components, of which MR acts radially in direct opposition to gravity and equals $MT \times \cos TMR = c \cos \phi = C \cos^2 \phi$.

Horizontal component affects direction of gravity.

(*b*) *Change of Direction of Gravity.* MS, the other component of c, acts horizontally towards the equator and equals $C \cos \phi \sin \phi = \frac{1}{2} C \sin 2\phi$. It acts to make the plumb-line hang away from the radius towards a point between O and Q.

Maximum at 45° latitude.

In latitude 45° this horizontal force has its maximum and is about $\frac{1}{578}$ part of the whole force of gravity, causing the plumb-line to deviate towards the equator about 11′ from the radius.

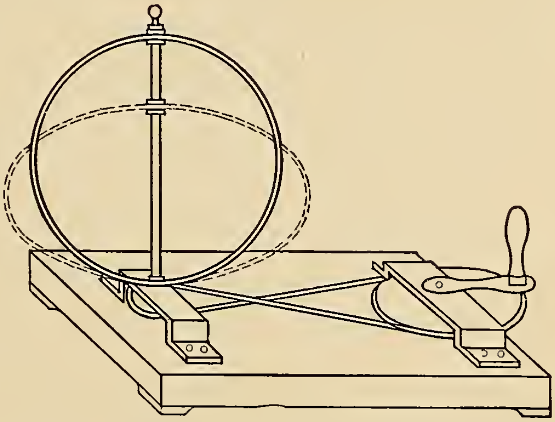

FIG. 47.—Effect of Earth's Rotation on its Form

If the earth's surface were spherical, this horizontal force would make every loose particle tend to slide towards the equator, and the water of the ocean would so move. As things actually are, the surface of the earth has already arranged itself accordingly, and the earth bulges at the equator just enough to correct this sliding tendency.

This effect of the earth's rotation on its form is well illustrated by the familiar little piece of apparatus shown in Fig. 47.

If the earth's rotation were to cease, the Mississippi River would at once have its course reversed, since its mouth is several thousand feet farther from the center of the earth than are its sources.

Distinction between gravity and gravitation.

131. Gravity is not simply *gravitation*, — the attraction of the earth for a body upon its surface, — but is the *resultant of this attraction combined with the centrifugal force at the point of observation*, as above explained. It is this resultant force which determines the *weight* of a body at rest or its velocity and direction when falling. Only at the equator and poles is gravity directed towards the center of the earth. Surfaces of *level* are, on hydrostatic principles, necessarily everywhere perpendicular to gravity, and are therefore not spheres around the earth's center, but spheroids flattened at the poles.

Classification of methods for determining accurately the form of the earth.

132. The Earth's Form. — There are three ways of determining the form of the earth: First, by *geodetic measurement of distances upon its surface* in *connection with the astronomical determination of the points of observation.* This gives not only the form but also the linear dimensions in miles or kilometers.

Second, by observing the varying *force of gravity* at points in different latitudes, — observations which are made by means of *a pendulum apparatus* of some kind and determine *only the form* but not the size of the earth.

Third, by means of *purely astronomical phenomena*, known as *precession* and *nutation* (to be treated of hereafter), and by *certain irregularities in the motion of the moon.* Observations of the occultations of stars at widely distant stations can also be utilized for the same purpose. These methods, like the pendulum method, *give only the form* of the earth.

Definition of oblateness, or ellipticity.

It is usual to characterize the form of the earth by its *oblateness*, or *ellipticity*, though the latter term is rather objectionable on account of the danger of confounding it with eccentricity.

The oblateness (Ω) is the fraction obtained by dividing the difference between the polar and equatorial semidiameters by the equatorial, *i.e.*, $\Omega = \frac{A - B}{A}$. In the case of the earth this is about $\frac{1}{300}$, but determinations by different methods range all the way from about $\frac{1}{290}$ to $\frac{1}{305}$.

The *eccentricity* of an ellipse is $\sqrt{\frac{A^2 - B^2}{A}}$, and is always a much larger quantity than the oblateness, or ellipticity. In the case of the earth's meridian it is about $\frac{1}{12\frac{1}{4}}$. Its symbol in astronomy is usually e.

Geodetic method of determining the earth's dimensions and form.

133. Geodetic Method, by which Dimensions of the Earth as well as its Form are determined. — The method in its most convenient shape consists essentially in the measurement of *two* (or more) *arcs of meridian in widely different latitudes.* These measurements are effected by the same combination of astronomical and geodetic operations already described for the measurement of a single arc (Sec. 121). More than twenty have thus far been measured in various parts of the earth. The two longest are the great Russo-Scandinavian arc, extending from Hammerfest to the mouth of the Danube, and the Indian arc of practically equal length, reaching from the Himalayas to the southern extremity of the great peninsula. These are both between 25° and 30° long; few of the others exceed 10°.

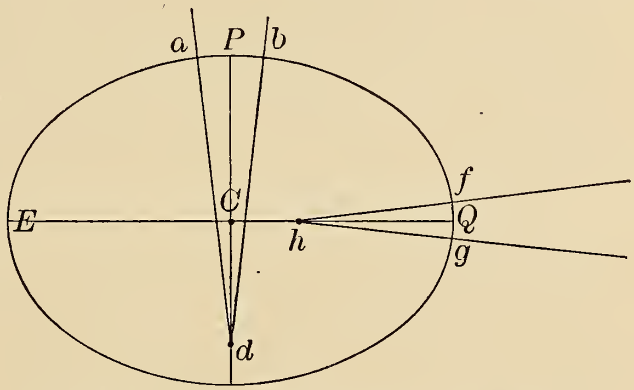

Fig. 48. — Length of Degrees in Different Latitudes

Length of degrees of astronomical latitude greatest near the pole.

From these measures it appears in a general way that the *higher* the latitude the *greater* the length of each astronomical degree. Thus, near the equator a degree has been found to be 362800 feet in round numbers, while in Northern Sweden, in latitude 66°, it is 365800 feet. In other words, the earth's surface is *flatter near the poles*, as illustrated by Fig. 48. It is necessary to travel about 3000 feet farther in Sweden than in India to increase the latitude by one degree, as measured by the elevation of the celestial pole.

The following little table gives the length of a degree of the meridian in certain latitudes:

At the equator	one	degree	= 68.704	miles.
At lat. 20°	"	"	= 68.786	"
" " 40°	"	"	= 68.993	"
" " 45°	"	"	= 69.054	"
" " 60°	"	"	= 69.230	"
" " 80°	"	"	= 69.386	"
At the pole	"	"	= 69.407	"

It will be understood, of course, that the length of a degree at the pole is obtained by "extrapolation" from the measures made in lower latitudes.

The difference between the equatorial and polar degree of latitude is more than 3500 feet, while the probable error of measurement cannot exceed more than three or four feet to the degree.

Difficulty of deducing consistent results from the observations.

134. The deduction of the exact form of the earth from such measurements is an abstruse problem. Owing to errors of observation, and to local deviations in the direction of gravity due to unevenness of surface and variation of density in the rocks near the station, the different arcs do not give strictly accordant results, and the best that can be done is to find the result which most nearly satisfies all the observations.

According to the determination of Colonel Clarke, for a long time at the head of the English Ordnance Survey, the dimensions of the "spheroid of 1866" (which is still accepted by our Coast and Geodetic Survey as the basis of all its calculations) are as follows:

The earth's dimensions according to Colonel Clarke.

Equatorial radius	(A)	6 378206.4	meters	= 3963.307	miles.
Polar radius	(B)	6 356583.8	"	= 3949.871	"
Difference		21622.6	"	= 13.436	"
Oblateness		$\frac{13.436}{3963.307}$		= $\frac{1}{295.0}$	"

These numbers are likely to be in error as much, perhaps, as 100 meters, and possibly somewhat more; they can hardly be 300 meters wrong.[1]

The oblateness $\frac{A-B}{A}$ comes out $\frac{1}{295}$; but a comparatively small change in either the equatorial or polar radius would change the 295 by some units.

Uncertainty of present data.

At present the distance from a point on the earth's surface (say the observatory at Washington) to any point in the opposite hemisphere (say the observatory at the Cape of Good Hope) is uncertain by fully 1000 feet.

The deviation of the earth's form from a true sphere is due simply to its rotation, and might have been cited as proving it. As already shown, the centrifugal force caused by the rotation modifies the direction of gravity everywhere except at the equator and the poles, and the surface therefore necessarily takes the spheroidal form.

Availability of arcs of longitude, or of any extensive geodetic and astronomical surveys.

135. Arcs of longitude are also available for determining the earth's form and size. On an oblate or orange-shaped spheroid (since the surface lies wholly within the sphere which has the same equator) the degrees of longitude are evidently everywhere shorter than on the sphere, the difference being greatest at a latitude of 45°, and from this difference when actually determined the oblateness can be computed.

In fact, *arcs in any direction between stations of which both the latitude and longitude are known* can be utilized for the purpose; and thus all the extensive geodetic surveys[2] that have been made by different countries contribute to our knowledge of the earth's dimensions.

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[1] For Clarke's spheroid of 1878, see Appendix.

[2] It is extremely improbable that the actual *geoid* (the regular geometrical surface which *most nearly* fits the surface of the earth) is a perfect spheroid, or even a perfect *ellipsoid* of three axes. The local and continental irregularities are so great that it seems likely that it will be found best to adopt some one of the already computed spheroids as a final "surface of reference," and hereafter to investigate and tabulate the local deviations from this as a base, rather than to compute a new set of spheroid elements for every accession of new geodetic data.

Determination of form by pendulum observations.

136. Determination of the Earth's Form by Pendulum Experiments and purely Astronomical Observations. — Since t, the time of vibration of a pendulum (*Physics*, p. 79), equals $\pi\sqrt{\frac{l}{g}}$, we have $g = \pi^2 \frac{l}{t^2}$, and can therefore measure the variations of g, the force of gravity, at different parts of the earth by using a pendulum of invariable length and determining its time of vibration at each station. Extensive surveys of this sort have been made and are still in progress; and it is found from them that the force of gravity at the pole exceeds that at the equator by about $\frac{1}{190}$. In other words, a man who weighs 190 pounds at the equator (weighed by a spring-balance) would weigh 191 at the pole.

Loss of weight at equator is $\frac{1}{190}$ as compared with weight at the pole.

The centrifugal force of the earth's rotation accounts for about one pound in 289 of this difference; the remainder (about one pound in 555) has to be accounted for by the difference between the distances from the poles and from the equator to the center of the earth. At the pole a body is more than 13 miles nearer the center of the earth than at the equator, and as a consequence the earth's attraction upon it is greater. The difference of gravity between pole and equator depends, however, not only on the difference of *distance* from the center of the earth, but partly *on the distribution of density* within the globe.

Assuming what is probable, that the *strata of equal density are practically concentric*, Clairaut has proved that for any planet of small oblateness,

Clairaut's equation.

$$\Omega = 2\tfrac{1}{2}\, C - W,$$

in which C equals the centrifugal force at the planet's equator and W the diminution of gravity between pole and equator; *i.e.*, for the earth,

$$\Omega = 2\tfrac{1}{2} \times \frac{1}{289} - \frac{1}{190}, \text{ which gives } \Omega = \frac{1}{295.2}.$$

The *purely astronomical* methods for determining the form of the earth indicate a slightly smaller oblateness of about $\frac{1}{300}$. They depend upon precession and nutation (Secs. 168 and 170) and upon certain irregularities in the motion of the moon; but their discussion lies quite beyond our scope.

Result of purely astronomical methods.

From a combination of all the available data of every kind, Harkness (1891) gives as his final result for the oblateness,

$$\Omega = \frac{1}{300.2 \pm 3.0}.$$

137. Station Errors. — If the latitudes of all the stations in a triangulation as determined by astronomical observations are compared with their differences of latitude as deduced from the geodetic operations, we find discrepancies by no means insensible. They are far beyond all possible errors of observations and are due to *irregularities in the direction of gravity*, which depend upon the variations in density and form of the crust of the earth in the neighborhood of the station. Such irregularities in the direction of gravity *displace the astronomical zenith* of the station. They are called *station errors* and can be determined only by a comparison of astronomical positions by means of geodetic operations. According to the Coast Survey, station errors average about 1″.5 in the eastern part of the United States, affecting both the longitudes and latitudes of the stations and the astronomical azimuths of the lines that join them. Station errors of from 4″ to 6″ are not very uncommon, and in mountainous countries these deviations occasionally rise to 30″ or 40″.

Station errors due to irregular distribution of matter in crust of the earth.

138. Astronomical, Geographical, and Geocentric Latitudes.— (1) The *astronomical* latitude of the station is that actually determined by astronomical observations, — simply the observed altitude of the pole.

(2) The *geographical* latitude is the astronomical latitude *corrected for station error*. It may be defined as the angle

Astronomical, geographical, and geocentric latitudes defined and distinguished.

formed with the plane of the equator by a line drawn from the place *perpendicular to the surface of the standard spheroid* at that station. Its determination involves the adjustment and evening off of the discrepancies between the geodetic and astronomical results over extensive regions. The geographical latitudes (sometimes called *topographical*) are those used in constructing an accurate map.

For most purposes, however, the distinction between astronomical and geographical latitudes may be neglected, since on the scale of an ordinary map the station errors, amounting at most to a few hundred feet, would be entirely insensible.

Geocentric latitude and the angle of the vertical.

(3) *Geocentric Latitude.* While the astronomical latitude is the angle between the *plane* of the equator and the *direction of gravity* at any point, the *geocentric* latitude, as the name implies, is the angle, *at the center of the earth*, between the plane of the equator *and a line drawn from the observer to that center;* which line evidently does not coincide with the direction of gravity, since the earth is not spherical.

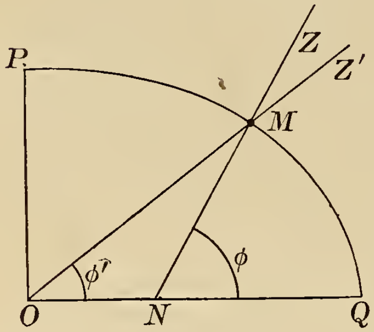

Fig. 49. — Astronomical and Geocentric Latitude

In Fig. 49 the angle MNQ is the *astronomical* latitude of the point M (it is also the *geographical* latitude, provided the station error at that point is insensible), and MOQ is the *geocentric* latitude.

The angle ZMZ', the difference of the two latitudes, is called *the angle of the vertical* and is about 11′ in latitude 45°.

Inverse relation of geocentric degrees to astronomical.

Geocentric degrees are longest near the equator, in precise contradiction to the astronomical degrees; and it is worth noticing that if we form a table like that of Sec. 133, giving the length of each degree of geographical latitude from the equator to the pole, the same table, *read backwards*, gives the length of the *geocentric degrees* without sensible inaccuracy; *i.e.*, at any distance *from the pole* a degree of geocentric latitude has within a few inches just the same length as the astronomical degree at the same distance from the equator.

Geocentric latitude is employed in certain astronomical calculations, especially such as relate to the moon and eclipses, in which it becomes necessary to "reduce observations to the center of the earth."

139. Surface and Volume of the Earth. — The earth is so nearly spherical that we can compute its surface and volume (or "bulk") with sufficient accuracy by the formulæ for a perfect sphere, provided we put the earth's *mean* semidiameter for radius in the formulæ.

Mean semidiameter of an oblate spheroid.

This mean semidiameter of an oblate spheroid is not $\frac{a+b}{2}$, but $\frac{2a+b}{3}$, because if we draw through the earth's center three axes of symmetry at right angles to each other, one only will be the axis of rotation, and both the others will be equatorial diameters.

The *mean* radius r of the earth thus computed is 3958.83 miles; its surface ($4\pi r^2$) is 196 944000 square miles, and its volume ($\frac{4}{3}\pi r^3$) 260000 million cubic miles, in round numbers.

III. THE EARTH'S MASS AND DENSITY

Definition of mass.

140. Definition of Mass. — The *Mass* of a body is *the quantity of matter* in it, *i.e.*, *the number of tons, pounds, or kilograms of material which it contains, — the unit of mass being a certain arbitrary body which has been selected as a standard.* A "kilogram," for instance, is the amount of matter contained in the block of platinum which is preserved as the standard at Paris. The *mass* of a body in the last analysis is measured by its *inertia*, i.e., *by the force required to give the body a certain velocity in a given time.*

Its relation to force.

Mass must not be confounded with "volume" or "bulk," which is simply the amount of space (cubic units) occupied by

Distinction between mass, on one hand, and volume and weight, on the other.

the body. A bushel of coal has the same volume as a bushel of feathers, but its mass is immensely greater.

Nor must mass be confounded with "weight," which is simply the *force* (push or pull) which urges the body towards the earth. It is true that under ordinary terrestrial conditions the mass of a body and its weight are *proportional* to each other and numerically equal. A *mass* of ten pounds *weighs* (very nearly) ten pounds anywhere on the earth's surface, but the word "pound" in the two parts of the sentence means two entirely different things; the pound of "mass" and the pound of "force" (stress) are as distinct as a "beam" of timber from a "beam" of sunlight.

Ambiguities arising from the identity of the ordinary names for the units of mass and force.

141. Mass and Force (Stress) distinguished. — This identity of names for the units of mass and force is on many accounts unfortunate, causing much ambiguity and much misunderstanding; but its reason is obvious, because we usually measure masses by *weighing*, and most often, not by weighing with a spring-balance, but by *balancing* the body against some standard mass, which standard is itself affected by variations of gravity in the same proportion as the body weighed, so that the ratio of their masses is correctly given notwithstanding such variations.

The *mass* of a given body — the number of "mass units" in it — remains invariable, wherever it may be; its *weight*, on the other hand — the number of "force units" which measures its tendency to fall, as judged by the effort required to lift it, or determined by a *spring-balance* — depends partly on where it is. At the equator it is nearly one half of one per cent less than at the pole, and on the surface of the moon it would be only about one sixth as great as on the earth.

Professor Newcomb's illustration.

To use an illustration suggested by Professor Newcomb: Suppose a base-ball team could somehow get to the moon. They would find their bats and balls very light to lift and hold; they themselves would be light on their feet and could jump six times as high and as far as on the earth, gravity and *weight* being so much less than here. But, since *masses* remain unchanged, the pitcher could not with a given exertion send the ball any more swiftly than here, nor could the batter hit it any harder or give it greater speed (though it would fly much farther before it fell), and the catcher in capturing the ball would receive just the same blow upon his hands as here. And if they had a steak for dinner that "weighed" only two pounds on their spring-balance, it would give them twelve pounds of meat; and, we may add, would also "weigh" twelve pounds *on a platform scale, or steelyard.*

The student must always be on his guard whenever he comes to the word "pound" or "kilogram," or any of their congeners, and must consider whether he is dealing with units of mass or of force.

142. The Scientific Unit of Force or Stress, — the Dyne, Megadyne, and Poundal. — Many high authorities now urge the entire abandonment of the old force units which bear the same names as the mass units, and the substitution, in all scientific work at least, of the *dyne* (*Physics*, p. 18) and its derivative, the *megadyne.* The change would certainly conduce to clearness, but for a time at least would be inconvenient, as making former mechanical literature almost unintelligible to those familiar with the new units only.

The scientific unit of force: the dyne.

The *Dyne* is the *force* (pull or push) *which acting constantly for one second upon a mass of one gram would give it a velocity of one centimeter a second.* It equals the "weight" (at Paris) of a mass of $\frac{10000}{9805}$[1], or 1.0199, milligrams. The *Megadyne* (a million dynes) is the weight (at Paris) of a mass of 1.0199 kilograms, or almost exactly 1.02 kilograms *in the latitude of Boston.*

The English poundal.

Many English authorities, however, insist on a unit of force based on the British units of mass and length and employ the *Poundal,* — the force which in one second would give a velocity of *one foot per second to a mass of one pound.* Since

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[1] 9.805 meters per second is the value of *g* at Paris.

g equals (nearly) $32\frac{1}{6}$ feet per second, the poundal is about $\frac{1}{32\frac{1}{6}}$ of the "weight" of a mass pound at London, or very nearly *half an ounce* of "pull." More accurately, the poundal equals 13.865 dynes.

143. Gravitation. The Cause of Weight. — Science cannot yet explain why bodies tend to fall towards the earth and push or pull towards it when held from moving; but Newton discovered and proved that the phenomenon is only a special case of the much more general fact which he inferred from the motions of the heavenly bodies and formulated as *the Law of Gravitation*, under the statement that *any two particles of matter attract each other with a force proportional to their masses and inversely proportional to the square of the distance between them.*

The law of gravitation expressed in words.

If instead of particles we have *bodies* composed of many particles, then the total force between the bodies is the sum of the attractions of all the different particles, each particle attracting every particle in the other body and being attracted by it.

The mystery of attraction not yet solved.

We must not imagine the word *attract* to mean too much. It merely states as a fact that there is a tendency for bodies to move towards each other, without including or implying any explanation of the fact. Thus far none has appeared which is less difficult to comprehend than the thing itself. It remains at present simply a fundamental fact, though it is not impossible (nor improbable) that ultimately it may be shown to be a necessary consequence of the relation between particles of ordinary matter and the all-pervading "ether" to which we refer the phenomena of light, radiant heat, electricity, and magnetism (*Physics*, p. 315).

Spheres attract each other as if their masses were all collected at their centers.

144. The Attraction of Spheres. — If the two attracting bodies are *spheres*, either homogeneous or made up of concentric shells which are of equal density and thickness throughout, then, as Newton demonstrated, the action on bodies outside the sphere is precisely the same as if *all the matter of each sphere were collected at its center.* If the distance between the bodies is *very* *great compared with their size*, then, whatever their form, the same thing is nearly, though not exactly, true.

He also showed that within a homogeneous *hollow* sphere of equal density and thickness throughout the attraction is everywhere *zero; i.e.*, a body anywhere within the hollow shell would not tend to fall in any direction.

Attraction zero within a hollow sphere.

If bodies which attract each other *are prevented from moving*, the effect of the attraction will be a *stress* (a push or pull), to be measured in *dynes* or force units (not in mass units), and this stress is given by the equation which embodies the law of gravitation, viz.,

Law of gravitation expressed as an equation giving the attraction as a pull in dynes.

$$F\,(\text{dynes}) = G \times \frac{M_1 \times M_2}{d^2},$$

where M_1 and M_2 are the masses of the two bodies (expressed in *grams*), d is the distance between their centers (in *centimeters*), and G is a factor known as *the Newtonian Constant* or the *Constant of Gravitation.*

145. The Constant of Gravitation. — The constant of gravitation is believed to be, like the velocity of light, an *absolute constant of nature*, — the same in all the universe of matter, — among the stars and planets as well as upon the earth. This is not yet absolutely proved, but there is no known phenomenon that contradicts it, and there is much probable evidence in its favor.

The constant of gravitation the same everywhere in the universe.

The numerical value of G depends on the units of mass, distance, and time; and in the C.G.S. system (centimeter-gram-second) it is, according to the elaborate determination of Boys in 1894, 0.00000006657 (6657×10^{-11}) or, quite within the limits of experimental error, *one fifteen-millionth.* If, for instance, the mass M_1 is 1000 grams, M_2 2000 grams, and the distance 10 centimeters, the force in dynes will be

Its numerical value *one fifteen-millionth* in the C.G.S. system of units.

$$\frac{1}{15\,000000} \times \frac{1000 \times 2000}{100},\ i.e.,\ \frac{1}{15\,000000} \times 20000,$$

or $\frac{1}{750}$ of a dyne.

It may be added that it is not yet proved that the equation $F = G \times M_1 \times M_2 \div d^2$ is the *complete* law. It is conceivable (though highly improbable) that the right-hand member may be only the principal term of a series which contains other terms (involving temperature, for instance) that may become sensible under conditions widely different from any yet observed. And "matter" may exist which does not "gravitate" though possessing "inertia," — the ether, for instance.

146. Acceleration by Gravitation. — If M_1 and M_2 *are set free* while under each other's attraction, they will at once begin to approach each other. At the end of the first second M_1 will have acquired a velocity $V_1 = G \times \frac{M_2}{d^2}$, which, the student will observe, depends entirely upon the mass of M_2 and not at all upon that of M_1 itself. (A grain of sand and a heavy rock will fall at the same rate in free space under the attraction of a given body when at the same distance from it.)

Formula giving the accelerating force due to gravitation in centimeters per second.

Similarly, M_2 will have acquired a velocity $V_2 = G \times \frac{M_1}{d^2}$. The speed with which the bodies *approach each other* will be the sum of these velocities, and if we denote this *relative acceleration* (or the *velocity of approach* acquired in one second) by f, we shall have

$$f = G\left(\frac{M_1 + M_2}{d^2}\right).$$

This is the form of the law of gravitation which is used in dealing with the *motions* of the heavenly bodies, caused by their attractions.

Notice that while the expression for F (the stress in *dynes*) has the *product* of the masses in its numerator, that for f (the *relative acceleration*) has their *sum*, and, like g, is expressed in centimeters per second.

147. We are now prepared to discuss the methods of measuring the earth's mass. It is only necessary to compare the *attraction* which the earth exerts on a body, m, *on its surface* (at a distance, therefore, of 3959 miles from its center) *with the* *attraction exerted upon* m *by some other body of a known mass at a known distance.* The practical difficulty is that the attraction of any manageable body is so extremely small, compared with the attraction of the earth, that the experiments are exceedingly delicate. Unless the mass employed for comparison with the earth is one of several tons, its attraction will be only a fraction of a dyne, — hard to detect even, and worse to measure.

Mass of earth measured by comparing its attraction upon a body at its surface with that of a known mass attracting the same body at a known distance.

The various experimental methods which have been actually used thus far for determining the earth's mass are enumerated and discussed in the author's *General Astronomy*, to which the student is referred. We present here only a single one, which is not difficult to understand and is probably the best, though not quite the earliest.

148. The Earth's Mass and Density determined by the Torsion Balance. — This is an apparatus invented by Michell and first employed by Lord Cavendish in 1798. A light rod carrying two small balls at its extremities is suspended at its center by a fine wire, so that the rod will hang horizontally. If it be allowed to come to rest, and then a very slight deflecting force be applied, the rod will be drawn out of position by an amount depending on the *stiffness* and *length* of the wire as well as the intensity of the force. When the deflecting force is removed the rod will vibrate back and forth until brought to rest by the resistance of the air.

The torsion balance.

The *Coefficient of Torsion*, as it is called, — *i.e.*, the stress which will produce a twist of one revolution, — can be accurately determined by observing the time of free vibration when the dimensions and masses of the rod and balls are known (see Anthony and Brackett, *Physics*, p. 117), and this coefficient will enable us to determine what force in dynes is necessary to produce a twist or deflection of any number of degrees. If the wire is stiff the coefficient will be large, and correspondingly small if very slender. It is therefore desirable that it should

Determination of the coefficient of torsion by observations of vibration.

be as slender as possible, while yet sufficiently strong to sustain the rod and balls.

Observation of deflection caused by attraction.

149. The Observations.— If, now, after the torsional coefficient has been carefully determined by observing the free vibrations of the rod, two large balls, *A* and *B* (Fig. 50), are brought near the smaller ones, a deflection will be produced by their attraction, and the small balls will move from *a* and *b* (their position of rest) to *a′* and *b′*. By shifting the large balls to the other side at *A′* and *B′* (which can be done by turning the frame upon which they are supported) we shall get a similar deflection in the opposite direction, — *i.e.*, from *a′* and *b′* to *a″* and *b″*, — and the difference between the two positions assumed by the two small balls — *i.e.*, the distance *a′a″* and *b′b″* — will be *twice* the deflection which is due to the *attraction of the two large balls* for the two small balls.

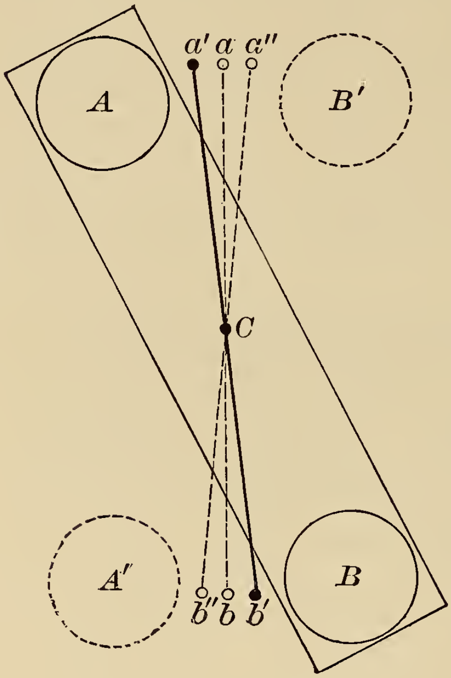


Fig. 50. — Plan of the Torsion Balance

The observations of vibration and deflection are best made by watching with a telescope from a distance the reflection of a fixed scale in a little mirror attached to the horizontal beam at *C*.

Exclusion of disturbing causes.

In observing the deflections it is not necessary, nor even best, to wait for the balls to come to rest. While still vibrating we note the extremities of their swing. The *middle point* of the swing (with a slight correction) gives the place of equilibrium. We must also carefully determine the distances *Aa′*, *A′b″*, *Bb′*, and *B′a″* between the center of each of the large balls and the center of the small ball when deflected. The utmost care must be taken to exclude air currents and electrical or magnetic disturbances, since these would seriously vitiate the results.

150. Calculation of the Earth's Mass from the Experiment. — The earth's attraction on each of the small balls, m, is evidently measured by the ball's *weight*, w, corrected for the centrifugal force of the earth's rotation at the observer's station *and reduced to dynes.*

Calculation of earth's mass.

The attractive force of the large ball on the small one near it is found from the observed deflection. If, for instance, this deflection is one degree, and the coefficient of torsion is such that it takes a force of ten dynes at the end of the rod to twist the wire one whole turn, then the deflecting force, which we will call $2f$, will be $\frac{1}{36}$ of a dyne. One half of this deflecting force, f, will be due to A's attraction of a, and half to B's attraction of b. Call the *mass* of the large ball B, determined by its weight, and that of the small ball m, and let d be the measured distance Bb' between their centers. We shall then have, from Sec. 144, the equation

$$f = G\frac{B \times m}{d^2}, \text{ or } B = \frac{f \cdot d^2}{G \cdot m}. \qquad (a)$$

Similarly, calling E the mass of the earth and R its radius, w being the corrected weight in dynes of the small ball, we shall have

$$w = G\left(\frac{E \times m}{R^2}\right), \text{ or } E = \frac{w \cdot R^2}{G \cdot m}. \qquad (b)$$

Dividing equation (a) by (b), G and m cancel out, and we have

$$\frac{E}{B} = \left(\frac{w}{f}\right)\left(\frac{R^2}{d^2}\right), \text{ or } E = B\left(\frac{w}{f}\right)\left(\frac{R^2}{d^2}\right),$$

which gives the mass of the earth in terms of B.

Determination of constant of gravitation.

By the same observations the value of the *constant of gravitation* is determined, since, from equation (a), $G = \dfrac{f \cdot d^2}{B \cdot m}$, B and m being both measured in *grams*, and d in *centimeters*.

151. Density of the Earth. — Having the mass of the earth it is easy to find its *density*. The volume, or bulk, in cubic miles has already been given (Sec. 139) and can be reduced to cubic feet by simply multiplying that number by the cube of 5280. Since a cubic foot of water contains $62\frac{1}{2}$ mass pounds (nearly), the mass which the earth would have, if composed of water, follows. Comparing this with its mass obtained from the observations, we get its *density*.

A combination of the results of all the different methods hitherto employed, taking into account their relative accuracy, gives about 5.53 as the most probable value of the earth's density compared with water, but the second decimal is not secure.

Direct determination of earth's density without previous calculation of its mass.

152. Density determined directly. — We can deduce the earth's density directly from the observations without any preliminary calculation of its mass.

Letting r and s represent the radius and density of the ball B, its mass is $\frac{4}{3}\pi r^3 s$. Similarly, E, the mass of the earth, is $\frac{4}{3}\pi R^3 s'$, s' being the earth's mean density.

Substituting these values of B and E in equations (a) and (b) of the preceding section, we have

$$\tfrac{4}{3}\pi r^3 s = \frac{fd^2}{Gm}, \qquad (c)$$

and

$$\tfrac{4}{3}\pi R^3 s' = \frac{wR^2}{Gm}. \qquad (d)$$

Dividing (d) by (c), we get $\dfrac{R^3 s'}{r^3 s} = \dfrac{wR^2}{fd^2}$,

whence, finally,

$$s' = s \times \frac{w}{f} \times \frac{r}{R} \times \frac{r^2}{d^2},$$

giving the density of the earth in terms of the density of the ball B, and other known quantities.

Early experiments.

153. In the earlier experiments, by this torsion-balance method, the small balls were of lead 1 or 2 inches in diameter, at the extremities of a light wooden rod 5 or 6 feet long inclosed in a case with glass ends, and their positions and vibrations were observed by a telescope looking directly at them from a distance of several feet. The attracting masses, A and B, were balls (also of lead) about a foot in diameter. Great difficulties were encountered from currents of air within the inclosure.

Experiments of Professor Boys.

Boys in 1894 used a most refined apparatus in which the small balls (of gold), one quarter of an inch in diameter, were hung at the end of a beam only a centimeter long, which was suspended by a delicate fiber of *amorphous quartz*, an ingenious invention of the experimenter. The deflections due to the attraction of two sets of lead balls, respectively 2¼ and 4½ inches in diameter, were measured by observing with a telescope the reflection of a scale in a little mirror attached to the beam. The whole apparatus was placed in an air-tight case no larger than an ordinary water pail, from which the air was exhausted and a little hydrogen admitted to take its place. His result for the earth's density was 5.527. It was from these experiments that the value of the constant of gravitation already given was deduced.

Different values for the earth's density, obtained by experiments during the last fifty years, range all the way from 5.8 to 4.9, omitting one or two which are very discordant from circumstances easily understood.

154. Constitution of the Earth's Interior. — Since the average density of the earth's crust does not exceed three times that of water, while the mean density of the whole earth is about 5.5, it is obvious that at the earth's center the density must be very much greater than at the surface, — very likely as high as eight or ten times the density of water, and perhaps higher, — equal to that of the heavier metals. There is nothing surprising in this. If the earth were ever fluid, it is natural to suppose that before solidification took place the densest materials would settle towards the center.

Increase of density towards earth's center.

Whether the interior of the earth is solid or fluid, it is difficult to say with certainty. Certain tidal phenomena, to be mentioned hereafter, have led Lord Kelvin to express the opinion that "the earth as a whole is solid throughout, and more rigid than glass," volcanic centers being mere "pustules," so to speak, in the general mass. To this most geologists demur, maintaining that at a depth of not many hundred miles the materials of the earth must be fluid or at least semi-fluid. This is inferred from the phenomena of volcanoes and from the

Question as to the rigidity or plasticity of the central portion.

fact that the temperature continually increases with the depth so far as we have yet been able to penetrate. But thus far the deepest penetration is but little more than a single mile, — a mere scratch, — not $\frac{1}{3500}$ part of the distance to the center of the earth.

EXERCISES

1. Does the transportation of sediment by the Mississippi tend to lengthen or to shorten the day?

2. If the diameter of the earth were doubled, keeping its mass unchanged, how would its density and the weight of bodies at its surface be affected?

3. If its diameter were trebled, keeping its density unchanged, how much would its mass and the weight of bodies at its surface be increased?

4. Supposing the earth to be homogeneous, how great (approximately) would be the force of gravity 1000 miles below its surface?

Solution. Inside of a hollow sphere the attraction is zero (Sec. 144). At the depth of 1000 miles, therefore, the effective attraction would be that of a sphere of only 3000 miles radius, all the shell of matter outside of this being without influence. We should therefore have gravity at the surface: gravity 1000 miles below the surface $= \frac{4}{3}\frac{\pi R^3}{R^2} : \frac{4}{3}\frac{\pi (R-1000)^3}{(R-1000)^2} = R : (R-1000)$; *i.e.*, as 4000 : 3000. In words, the attraction at this depth would be $\frac{3}{4}$ that at the surface of the earth, if it were of equal density throughout, which it is not.

5. Assuming g at the earth's surface to be 9.805 meters per second, what would it be in a balloon at an elevation of 2 miles? The radius of the earth may be taken as 4000 miles and centrifugal force neglected.

Ans. 9.7952 m per second.

Would the value of g be the same at the top of a mountain 2 miles high, and if not, why?

6. Given two spheres one of which has a mass m times greater than the other; on what point on the line joining their centers are their attractions equal?

Solution. Let d be the distance between their centers and x the distance of the point of equilibrium from the smaller body; then the attraction of the larger body at that point is $G\frac{m}{(d-x)^2}$, that of the smaller being $G\frac{1}{x^2}$. Canceling the G's and taking the square roots, we have $\frac{\sqrt{m}}{(d-x)} = \frac{1}{x}$, from which we have

Ans. $x = \frac{d}{1+\sqrt{m}}$.

7. Assuming the moon's mass as $\frac{1}{81}$ of the earth's, where is the equilibrium point on the line of centers?

Ans. At a point $\frac{1}{10}$ of the distance from the moon to the earth.

8. What is the attraction in dynes between two spheres, each having a mass of 1000 kilograms, at a distance of 1 meter between centers?

Ans. $6\frac{2}{3}$ dynes, or the weight of 6.8 mgm.

9. If these spheres were free to move under their mutual attraction, required their relative velocity at the end of one second.

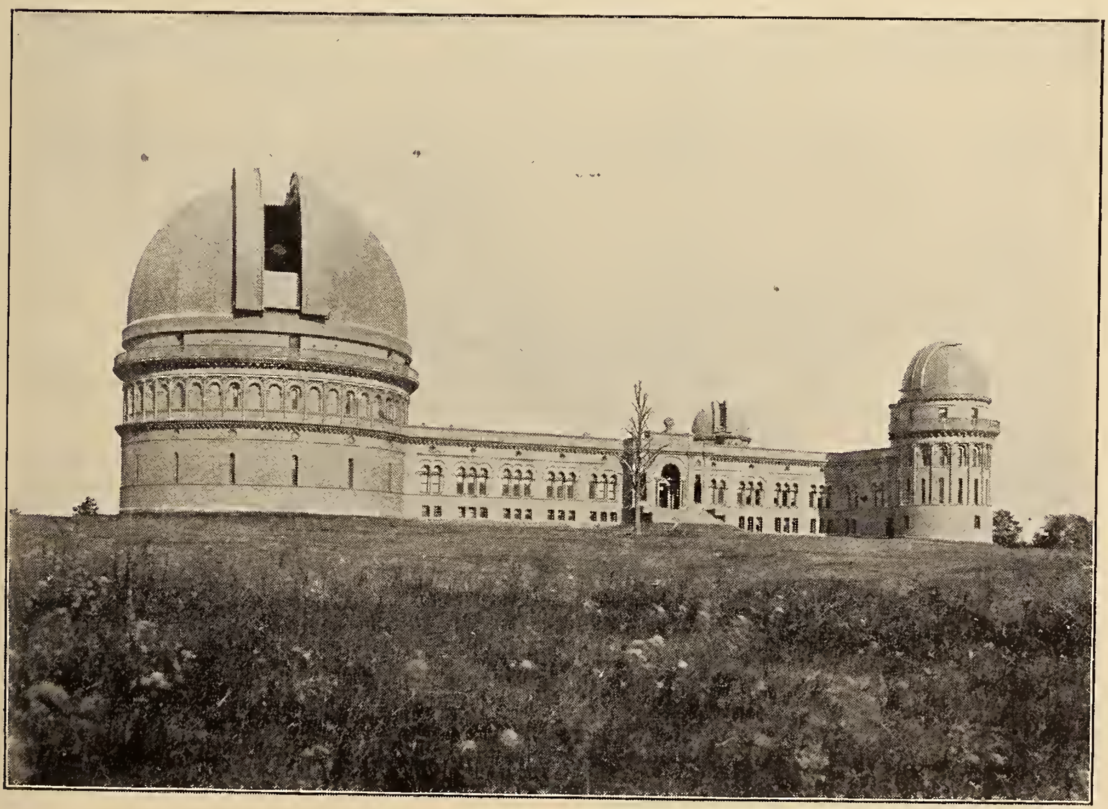
Ans. $\frac{1}{7500}$ mm per second.

10. If at a distance of half a meter from such a ball is suspended a small ball weighing 1 gram, what is the attraction between them?

Ans. $\frac{1}{37500}$ of a dyne.

11. If in this case the small ball were suspended by a fine thread 10 meters long, how many millimeters would it be drawn from a vertical position, and what angle would the thread make with the vertical?

Ans. Deviation, 0.000272 mm.
Deflection, $0''.00561$.



Yerkes Observatory

CHAPTER VI

THE ORBITAL MOTION OF THE EARTH

The Apparent Motion of the Sun, and the Orbital Motion of the Earth — Precession and Nutation — Aberration — The Equation of Time — The Seasons and the Calendar

155. The Sun's Apparent Annual Motion among the Stars. — This must have been among the earliest recognized of astronomical phenomena, as it is obviously one of the most important.

Sun's apparent motion in declination.

As seen by us in the northern hemisphere, the sun, starting in the spring at the vernal equinox, mounts higher in the sky each day at noon for three months, until the summer solstice, and then descends towards the south, reaching in the autumn the same noonday elevation which it had in the spring. It keeps on its southward course to the winter solstice in December and then returns to its original height at the end of a year, marking and causing the seasons by its course.

Its motion in right ascension.

Nor is this all. The sun's motion is not merely north and south, but it also advances continually *eastward* among the stars. In the spring the stars which at sunset are rising on the eastern horizon are different from those which are found there in summer or winter.

In March the most conspicuous of the eastern constellations at sunset are Leo and Boötes. A little later Virgo appears, in the summer Ophiuchus and Libra; still later Scorpio, while in midwinter Orion and Taurus are ascending as the sun goes down.

So far as the obvious appearances are concerned, it is quite indifferent whether we suppose the earth to revolve around the sun or *vice versa*. That the earth is the body which really moves, however, is absolutely demonstrated by two phenomena too delicate for observation without the telescope, but accessible to modern methods. The more conspicuous of them is the ***aberration of light;*** the other is the *annual parallax of the fixed stars.* These phenomena can be explained only by the actual motion of the earth and will be discussed later.

Facts which demonstrate that the apparent motion of the sun is due to the real motion of the earth.

156. The Ecliptic; its Related Points and Circles. — By observing daily with the meridian-circle both the sun's declination and also the difference between its right ascension and that of some chosen star, we obtain a series of positions of the sun's center which can be plotted on a globe, and we can thus mark out its path among the stars. It is a great circle, called the *Ecliptic* (Sec. 23), which cuts the equator at two opposite points (equinoxes), at an angle of approximately 23½° (23° 27′ 8″.0 in 1900). It gets its name from the fact, early discovered, that eclipses happen only when the moon is crossing it. It is *the great circle in which the plane of the earth's orbit cuts the celestial sphere.*

The ecliptic defined.

The angle which the ecliptic makes with the equator at the equinoctial points is called the *Obliquity of the Ecliptic* and is evidently equal to the *sun's maximum declination*, reached in June and December.

Obliquity of the ecliptic.

The two points in the ecliptic midway between the equinoxes are called the *Solstices*, because there the sun apparently "stands," *i.e.*, stops and reverses its motion in declination. The circles drawn through these solstices parallel to the equator are called the *Tropics.*

Solstices and tropics.

The *Poles of the Ecliptic* are the two points in the heavens 90° distant from every point in that circle. The northern one is the constellation Draco, about midway between the stars δ and ζ, and on the solstitial colure (right ascension, 18 hours), its distance from the pole of rotation being equal to the obliquity of the ecliptic (see Sec. 27).

The poles of the ecliptic.

It will be remembered that *celestial latitude and longitude are measured with reference to the ecliptic and not to the equator.*

The zodiac.

157. The Zodiac and its Signs. — A belt 16° wide (8° on each side of the ecliptic) is called the *zodiac*, or "zone of animals" (German, *Thierkreis*), the constellations in it, excepting Libra, being all figures of living creatures. It is taken of that particular width simply because the moon and the principal planets always keep within it. It is divided into the so-called "signs," each 30° in length, having the following names and symbols:

Signs of the zodiac.

Spring	Aries	♈	Autumn	Libra	♎
	Taurus	♉		Scorpio	♏
	Gemini	♊		Sagittarius	♐
Summer	Cancer	♋	Winter	Capricornus	♑
	Leo	♌		Aquarius	♒
	Virgo	♍		Pisces	♓

The symbols are for the most part conventional pictures of the objects. The symbol for Aquarius is the Egyptian character for water. The origin of the signs for Leo, Capricornus, and Virgo is not quite clear.

The zodiac is of extreme antiquity. In the zodiacs of the earliest history the Lion, Bull, Ram, and Scorpion appear precisely as now.

Distinction between the ecliptic and the orbit of the earth.

158. The earth's orbit is the path in space pursued by the earth in its revolution around the sun. The *ecliptic is not the orbit* and must not be confounded with it; it is simply a great circle of *the infinite celestial sphere*, while the orbit itself is (nearly) a circle, of finite diameter, *in space.* The fact that the ecliptic is a great circle gives us no information about the orbit, except that it lies *wholly in one plane, which passes through the sun;* it tells us nothing as to the orbit's real *form* or *size.*

Observations for determining the form of the orbit.

By reducing the daily observations of the sun's right ascension and declination made with a meridian-circle to celestial longitude and latitude (the latitude would always be exactly zero, except for some slight perturbations of the earth) and combining these data with observations of the *sun's apparent diameter*, we can, however, ascertain the *form* of the earth's orbit and the *law of its motion* in this orbit. The *size* of the orbit cannot be fixed until we find some means of determining the scale of miles.

159. To find the Form of the Orbit.—Take a point, *S* (Fig. 51), for the sun, and draw from it a line, *SO*, directed towards the vernal equinox, from which longitudes are measured. Lay off from *S* lines indefinite in length, making angles with *SO* equal to the earth's *longitude* as seen from the sun[1] on each of the days when observations were made. We shall thus get a sort of "spider," showing the *direction* of the earth as seen from the sun on each of those days.

Direction of earth from sun on days of observation.

Next, as to the *distances.* While the apparent diameter of the sun does not determine its *absolute* distance from the earth unless we know the diameter in miles, yet the *changes in the apparent diameter* do inform us as to the relative distance at different times,—the distance being *inversely proportional* to the sun's apparent diameter (Sec. 10). If, then, on this "spider" we lay off distances equal to the quotient obtained by dividing some constant, say 10000, by the sun's apparent diameter in seconds as observed at each date, these distances will be *proportional* to the true distance of the sun from the earth, and the curve joining the points thus obtained *will be a true map of the earth's orbit*, though without any scale of miles.

Relative distance from sun on these days.

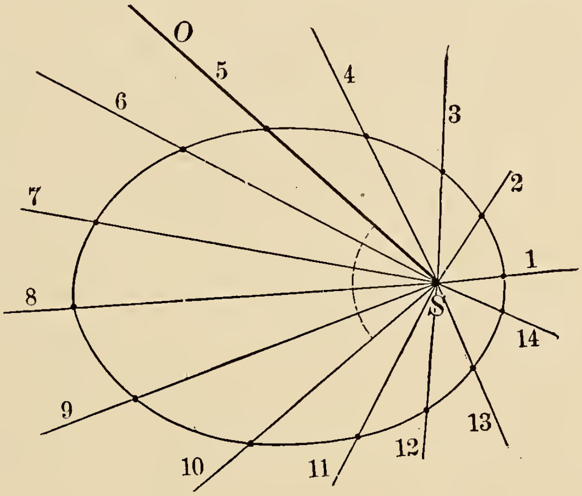


FIG. 51.—Determination of the Form of the Earth's Orbit

When the operation is performed, we find that the orbit is an ellipse of small eccentricity (about $\frac{1}{60}$), with the sun not in the center, but *at one of the two foci.*

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[1] This is 180° ± the sun's longitude *as seen from the earth.*

Definition of the ellipse, its axes, and eccentricity.

160. Definitions relating to the Orbital Ellipse. — *The Ellipse is a curve such that the sum of the two distances from any point on its circumference to two points within, called the foci, is always constant and equal to the major axis of the ellipse.*

In Fig. 52, wherever P is taken on the periphery of the ellipse, $SP + PF$ always equals AA', which is the major axis. AC is the *semi-major Axis* and is usually denoted by A or a. BC is the *semi-minor Axis*, denoted by B or b; the *eccentricity* of the ellipse is the fraction, or ratio, $\frac{SC}{AC}$, and is usually expressed as a decimal. It equals $\frac{\sqrt{a^2 - b^2}}{a}$.

If a cone is cut across obliquely by a plane, the section is an ellipse, which is therefore called one of the conic sections. (See Sec. 314.)

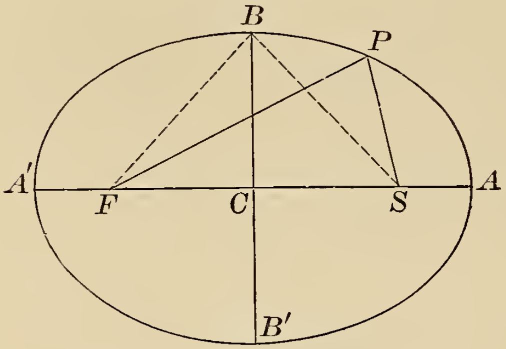

FIG. 52. — The Ellipse

Definition of perihelion, aphelion, radius vector, and anomaly.

Perihelion and *Aphelion* are respectively the points where the earth is nearest to and remotest from the sun, the line joining them being the major axis of the orbit. The *Line of Apsides* is the major axis indefinitely produced in both directions. A line drawn from the sun to the earth or any other planet at any point in its orbit, as SP in the figure, is called the planet's *Radius Vector*, and the angle ASP, reckoned from the perihelion point, in the direction of the planet's motion towards B, is called its *Anomaly*.

Discovery of the eccentricity of the earth's orbit by Hipparchus.

161. Discovery of the Eccentricity of the Earth's Orbit by Hipparchus. — The variations in the sun's diameter are too small to be detected without a telescope, so that the ancients failed to perceive them. Hipparchus, however, about 120 B.C., discovered that the earth is not in the center of the circular orbit,[1] which he supposed the sun to describe around it with uniform velocity.

Obviously, the sun's *apparent* motion is not uniform, because it takes 186 days for the sun to pass from the vernal equinox to the autumnal, and only 179 days to return. Hipparchus explained this difference by the hypothesis that the earth is out of the center of the circle.

In fact, the earth's orbit is so nearly circular that the difference between the radius vector of the ellipse and that of the eccentric circle of Hipparchus is everywhere so small that the method indicated in the preceding article would not *practically* suffice to discriminate between them. Other planetary orbits are, however, unmistakable ellipses, and the investigations of Newton show that the earth's orbit also is *necessarily* elliptical.

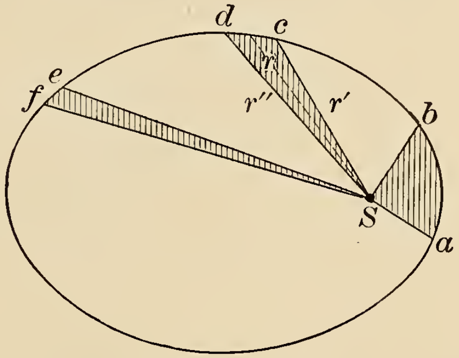

FIG. 53. — Equable Description of Areas

162. The Law of the Earth's Motion. — By comparing the measured apparent diameter of the sun with the differences of longitude from day to day, we can deduce not only the form of the orbit, but the *law of the earth's motion in it.* On arranging the daily motions and apparent diameters in a table, we find that the daily motions vary directly as the squares of the *diameters*, or *inversely* as the squares of the *distances* of the earth from the sun. In other words, the *product of the square of the distance by the daily motion is constant.* Now the area of any small elliptical sector cSd (Fig. 53) which is sensibly a triangle $= \frac{1}{2}\ Sc \cdot Sd\ \sin\ cSd$, or $\frac{1}{2}\ r'r''\ \sin\ cSd$. When the

The law of the earth's orbital motion — the law of equal areas in equal times.

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[1] Until the time of Kepler, it was universally assumed on metaphysical grounds that the orbits of the celestial bodies must necessarily be circular and described with a uniform motion, "because," as was reasoned, "the circle is the only *perfect* curve, and uniform motion is the only perfect motion proper to *heavenly* bodies."

Demonstration of the law of areas from observation.

angle is small $r' \times r'' =$ (sensibly) r^2, r being the "radius vector" drawn to the middle of cd; and for *sin* cSd we may put cSd itself. Hence, *area* $cSd = \frac{1}{2} r^2 \times cSd$, — a constant, as observation shows; or, in other words, *its radius vector describes areas proportional to the times*, a law which Kepler first discovered in 1609.

If in Fig. 53 *ab*, *cd*, and *ef* be portions of the orbit described by the earth in different weeks, the areas of the elliptical sectors *aSb*, *cSd*, and *eSf* are all equal. A planet near perihelion moves faster than at aphelion in just such proportion as to preserve this relation.

Proved by Newton to be a necessary consequence of mechanical principles.

163. As Kepler left the matter this is a mere fact of observation. He could give no reason for it. Newton afterwards proved that it is a necessary consequence of the fact that the earth moves under the action of a *force always directed towards the sun* (Secs. 303 and 304). The law holds good in every case of orbital motion under a central attraction and enables us to find the position of the earth or any planet, at any time, when we once know the time of its orbital revolution, or "period," and a date when it was at perihelion. Thus, the angle *ASP* (Fig. 54), or the *anomaly* of the planet, must be such that the shaded area of the elliptical sector *ASP* will be that portion of the whole ellipse which is represented by the fraction $\frac{t}{T}$, T being the number of days in the period and t the number of days since the planet last passed perihelion. The solution of this problem, known as Kepler's Problem, leads to a "transcendental" equation, and can be found in all books on physical astronomy, or in the Appendix to the author's *General Astronomy*.

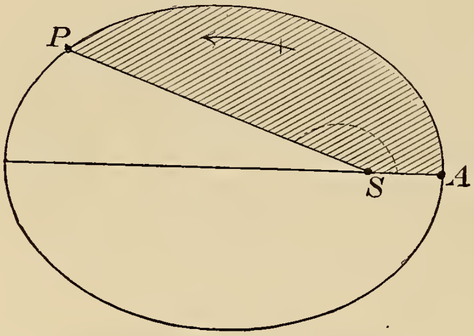

Fig. 54. — Kepler's Problem

Kepler's Problem.

164. Changes in the Orbit of the Earth. — Were it not for the attraction of the planets upon the earth and sun, the earth would maintain her orbit strictly unaltered from age to age, except that possibly in the course of millions of years the effect of falling meteors and the attraction of some of the nearer stars might become barely sensible. In consequence, however, of the attractions of the other planets, it is found that, while the *Major Axis* of the orbit and the *Length of the Year remain in the long run unchanged*, other elements undergo slow variations known as "secular perturbations."

Effect of planetary attractions upon the earth's motion.

Major axis and period unaffected in the long run.

(1) *Revolution of the Line of Apsides.* This line, which now stretches in both directions towards the opposite constellations of Gemini and Sagittarius, moves continually eastward (*i.e.*, in the same direction as the planetary motions) at a rate which would carry it completely around the circle in about one hundred and eight thousand years, if the rate continued always the same as at present, — which, however, it will not, since it is affected by changes in the eccentricity and by other circumstances.

Eastward revolution of the line of apsides.

(2) *Change of Eccentricity.* At present the eccentricity of the earth's orbit, now 0.016, is slowly diminishing, and according to Leverrier will continue to do so for about twenty-four thousand years, when it will be only 0.003; *i.e.*, the orbit will become almost circular. Then it will increase again for some forty thousand years and will continue to oscillate, *always keeping between zero and 0.07*. But the successive oscillations are not equal either in amount or time, — not at all like the "swing of a mighty pendulum," which has been rather a favorite figure of speech with some writers.

Change of eccentricity — at present slowly diminishing.

(3) *Change in the Obliquity of the Ecliptic.* The plane of the earth's orbit is also slowly changing its position, and as a consequence the ecliptic shifts its place among the stars, thus slowly altering their *latitudes* and the *angle* between the equator and the ecliptic. The *obliquity* is now about 24′ less than it

Obliquity of the ecliptic slowly diminishing.

was two thousand years ago,[1] and at present is decreasing about $0''.5$ yearly. After about fifteen thousand years, when the obliquity will be only 22½°, it will begin to increase again and will "oscillate" like the eccentricity. But the whole change can never exceed about 1½° on each side of the mean.

Slight periodic disturbances of earth in its orbit.

(4) *Periodic Disturbances of the Earth in its Orbit.* Besides these "secular perturbations" of the earth's *orbit*, the earth itself is all the time slightly disturbed in its orbit. On account of its connection with the moon, its center oscillates each month a few hundred miles above and below the true plane of the ecliptic; and by the action of the other planets it is sometimes set forward or backward or sideways to the extent of several thousand miles. Of course, every such displacement of the earth produces a corresponding slight change in the apparent position of the sun, and indeed of all bodies observed from the earth, except the moon, which accompanies the earth, and the stars, which are too far away to be sensibly affected.

Precession defined. Its discovery by Hipparchus.

165. Precession of the Equinoxes. — This is a slow *westward* motion of the equinoxes along the ecliptic first discovered by Hipparchus about 125 B.C. He found that the "year of the seasons," from solstice to solstice, *as determined by the gnomon*, was shorter than that determined by the *heliacal rising and setting of the stars* (*i.e.*, the times when certain constellations rise and set at sunset), just as if the equinox "preceded," *i.e.*, "stepped forward," a little to meet the sun. The difference between the year of the seasons and the sidereal year is about twenty minutes, the *precession* being $50''.2$ yearly, so that the equinox makes the complete circuit of the ecliptic in twenty-five thousand eight hundred years.

Amount of precession $50''.2$ yearly. Period 25800 years.

It is a motion of the *equator* and not of the *ecliptic* which causes the precession, as is proved by the fact that the *latitudes* of the stars have changed but slightly in the last two thousand years, showing that the *ecliptic* maintains its position among them nearly unaltered. The right ascension and declination of the stars, on the other hand, are both found to be constantly changing, and this makes it certain that it is the celestial equator which shifts its place. On account of this change in the place of the equinox the *longitudes* of the stars increase continually, — at a sensibly constant rate of $50''.2$ a year, — nearly 30° in the last two thousand years.

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[1] The ancients determined the "obliquity" with fair accuracy by observations of the length of the shadow of the gnomon at the two solstices. The angle *CBD*, or *SBS'* (Sec. 93, Fig. 36), is *twice* the obliquity. The Chinese records contain an observation which purports to be four thousand years old and is apparently genuine.

Due mainly to motion of the equator as proved by the constancy of the latitudes of stars.

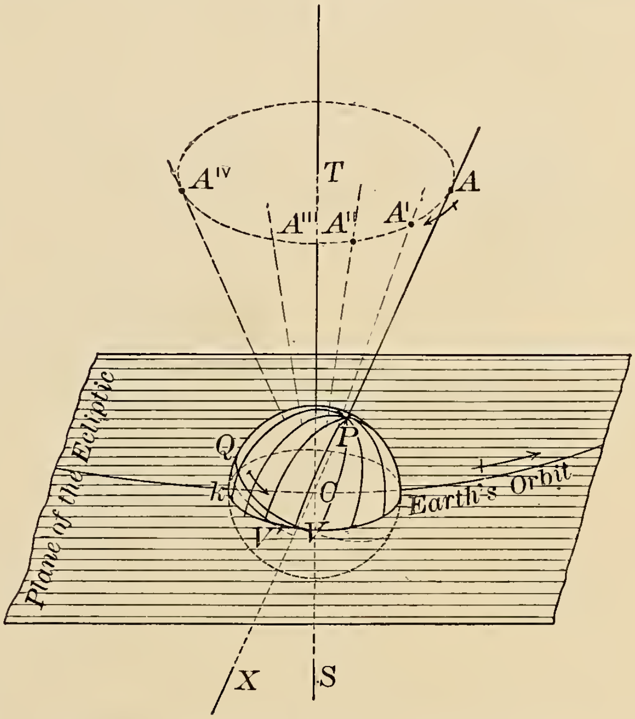

FIG. 55. — Conical Motion of Earth's Axis

166. Motion of the Pole of the Heavens around the Pole of the Ecliptic. — The *obliquity of the ecliptic*, which equals the angular distance of the pole of the heavens from the pole of the ecliptic, is not affected by precession. That is to say, as the earth travels around its orbit in the plane of the ecliptic (just as if that plane were the level surface of a sheet of water in which the earth swims half immersed), its axis, ACX (Fig. 55), always preserves very nearly the same constant angle of $23\frac{1}{2}°$ with the perpendicular, SCT, which points to the pole of the ecliptic. But, in consequence of precession, the axis, while keeping its inclination unchanged, shifts conically around the line SCT (like the axis of a spinning top), taking up successively the positions $A^{\mathrm{I}}C$, $A^{\mathrm{II}}C$, etc., thus carrying the equinox from V to V', and onwards.

Revolution of the pole of the heavens around the pole of the ecliptic.

In consequence of this shift of the axis the pole of the heavens, *i.e.*, that point in the sky to which the line *CA* happens to be directed at any time, describes a circle around the pole of the ecliptic in a period of about twenty-five thousand eight hundred years (360° ÷ 50″.2). While the pole of the ecliptic has remained almost fixed among the stars, the pole of the equator has traveled many degrees since the earliest observations. Fig. 56 shows *approximately* its path among the northern constellations; not *exactly*, of course, on account of the continual slight shifting of the plane of the earth's orbit, which makes the *pole of the ecliptic* move about a little, so that the center of the "precessional circle" is not an absolutely fixed point.

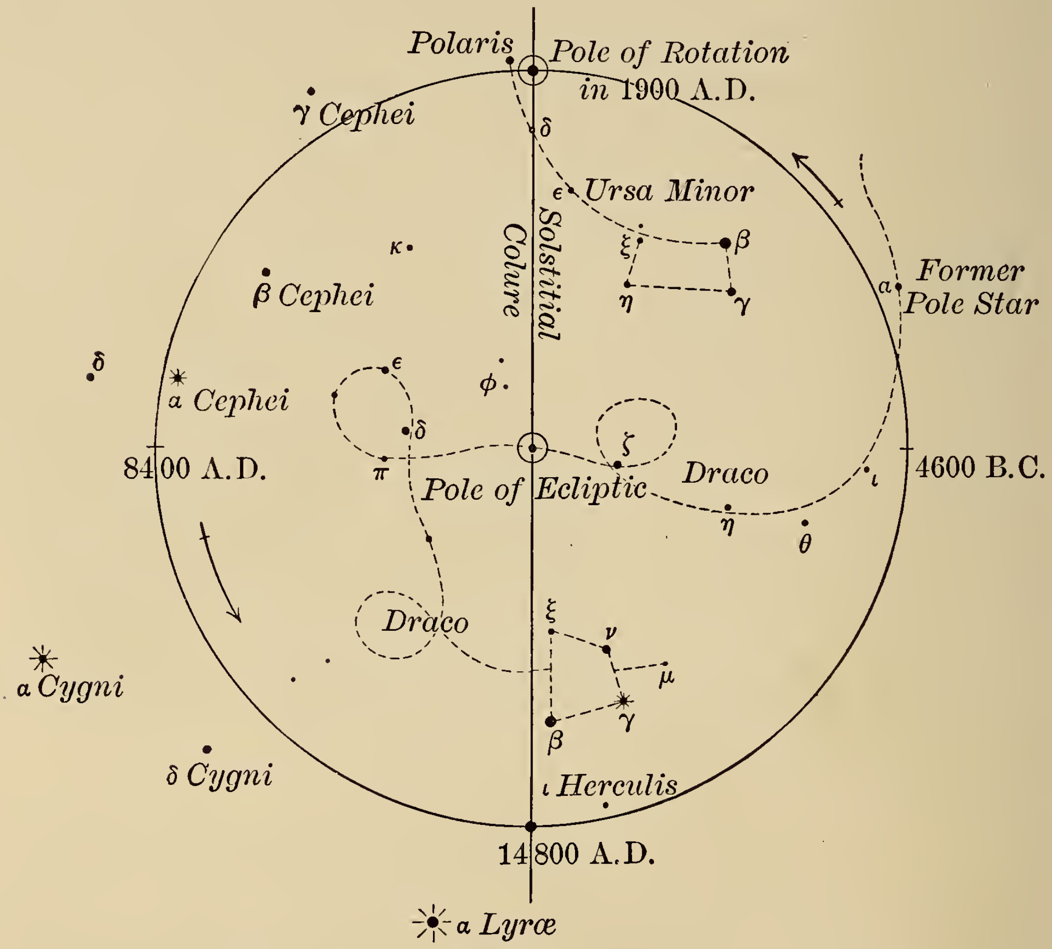
Polaris
Pole of Rotation in 1900 A.D.
γ Cephei
δ
Ursa Minor
ε
ξ
β
η
γ
κ
β Cephei
Solstitial Colure
Former Pole Star
α
δ
α Cephei
ε
φ
δ
ζ
8400 A.D.
π
Pole of Ecliptic
Draco
4600 B.C.
ι
η
θ
Draco
ξ
ν
μ
α Cygni
γ
β
δ Cygni
ι Herculis
14800 A.D.
α Lyræ

FIG. 56. — Precessional Path of the Celestial Pole

Path of the pole among the stars.

Reckoning back about four thousand six hundred years, we see from the figure that α Draconis was then the pole-star, and about five thousand six hundred years hence α Cephei will take the office. The circle passes not very far from Vega on the opposite side from the present pole-star, so that about twelve thousand years from now *Vega* (α Lyræ) will be the pole-star, — a splendid one but rather inconveniently far from the pole.

Former and future pole-stars.

N.B. — This precessional motion of the *celestial* pole must not be confounded with the motions of the *terrestrial* pole which cause the variations of latitude.

167. Displacement of the Signs of the Zodiac. — Another effect of precession is that the *signs* of the zodiac (Sec. 157) do not now agree with the *constellations* of which they bear the name. The *sign* of Aries is now in the *constellation* of Pisces, and so on. In the last two thousand years each sign has backed bodily, so to speak, into the constellation west of it.

Effect of precession on the signs of the zodiac.

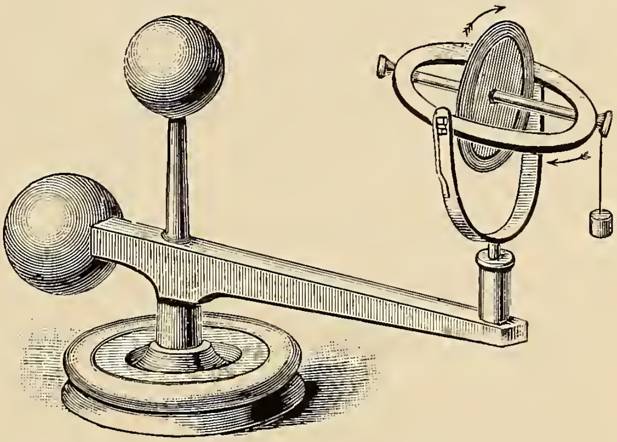

FIG. 57. — Precession illustrated by the Gyroscope

Great changes have taken place also in the apparent position of other constellations in the sky. Six thousand years ago the Southern Cross was visible in England and Germany, and Cetus never rose above the horizon.

168. Physical Cause of Precession. — The physical cause of this slow conical motion of the earth's axis was first explained by Newton, and lies in the two facts that the earth is not exactly spherical, but has, so to speak, a protuberant ring of matter around its equator, — the *equatorial bulge*, — and that the sun and moon act upon this ring, *tending* (but not able) to draw the plane of the equator into coincidence with the plane of the ecliptic, as a magnet tends to draw the plane of an iron ring into line with its pole.

Mechanical explanation of precession.

Combination of rotations illustrated by the gyroscope.

If it were not for the earth's rotation, this action of the sun and moon would actually bring the two planes of the equator and ecliptic into coincidence; but since the earth is spinning on its axis, we get the same result that we do with the whirling wheel of a gyroscope by hanging a weight at one end of its axis (Fig. 57). We then have a *combination of two rotations at right angles to each other*, — one the whirl of the wheel, the other the "tip" which the weight tends to give the axis. The resultant effect — very surprising when the experiment is seen for the first time — is that the axis of the wheel, instead of tipping, maintains its inclination unchanged, but *moves around conically* like the axis of the earth, as shown in Fig. 55. Any force tending to change the direction of the axis of a whirling body produces a motion *exactly at right angles* to its own direction.

Why precession is so slow.

Compared with the mass of the earth and its energy of rotation, this disturbing force is very slight, and consequently the rate of precession is extremely slow. If the earth were spherical, there would be no precession. If it revolved on its axis more slowly, precession would be more rapid, as it would be also if the sun and moon were larger or nearer, or if the obliquity of the ecliptic were greater, not exceeding 45°.

The moon, being nearer than the sun, is much the more effective of the two in producing the precession.

Equation of the equinox due to variations in the force which produces precession.

169. Equation of the Equinox. — The force which tends to pull the equator towards the ecliptic continually varies. When the sun and moon are crossing the celestial equator the action becomes zero — twice a year for the sun, twice a month for the moon. Moreover, as we shall see (Sec. 192), the maximum declination attained by the moon during the month changes all the way from 18° 07′ to 28° 47′, and its effect in producing precession varies correspondingly. As a consequence there is, superposed upon the *mean* precession of the equinoxes, a small periodic variation in its rate, producing the *equation of the* *equinox*, a slight advance or recession of the equinox from its mean place never amounting to more than a few seconds of arc.

170. Nutation. — This is a slight motion of the pole of the equator alternately towards and from the pole of the ecliptic, — a "nodding," so to speak, of the pole. In most positions of the sun or moon with respect to the equator, there is, in addition to the "tipping" force, a slight *thwartwise* action, *tending to accelerate or retard* the precession, just as if one should gently draw horizontally the weight W at the end of the axis (Fig. 57). The actual effect in this case is not to change the rate of precession in the least, but to *alter the inclination of the axis.* This causes a *nutation* amounting to about 9″.2 as a maximum and running through its principal changes in nineteen years, — the period in which the nodes of the moon's orbit complete their circuit (Sec. 192).

Nutation a slight periodical motion of the pole towards or from the pole of the ecliptic.

171. Aberration of Light. — *Aberration*[1] *is the apparent displacement of a heavenly body, due to the combination of the orbital velocity of the earth with the velocity of light.*

Aberration of light defined.

The fact that light is not transmitted instantaneously, but with a finite velocity, causes the displacement of an object viewed from any moving station, unless the motion is directly towards or from that object. If the motion of the observer is slow as compared with the speed of light, this displacement is insensible; but the earth moves swiftly enough (about 18½ miles per second) to make it easily observable with modern instruments. The direction in which we point our telescope to observe a star is usually not the same as if we were at rest, and the angle between the two directions is the star's *aberration* at the moment.

We may illustrate this by considering what would happen in the case of falling raindrops observed by a person in motion.

Illustration from the behavior of falling raindrops.

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[1] It was first discovered and explained in 1726 by Bradley, then the English Astronomer Royal.

Suppose the observer standing with a tube in his hand while the drops are falling vertically. If he wishes to have the drops descend through the tube without touching the side, he must obviously keep it vertical so long as he stands still; but if he advances in any direction, the drops will strike his face and he will have to draw back the bottom of the tube (Fig. 58) by an amount which equals the advance he makes during the time while a drop is falling through it; *i.e.*, he must incline the tube forward at an angle, a, which depends both upon the velocity of the raindrop and the velocity of his own motion, so that when the drop which entered the tube at B reaches A' the bottom of the tube will be there also. This angle is given by the equation

Equation giving the aberration in terms of the velocities.

$$\tan a = \frac{u}{V},$$

in which V is the velocity of the drop, and u the velocity of the observer at right angles to V.

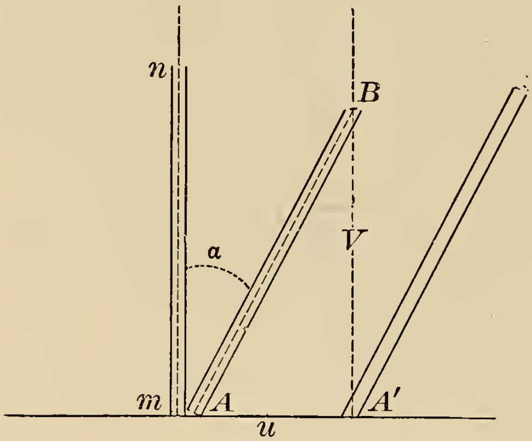

Fig. 58. — Aberration of a Raindrop

It is true that this illustration is not a demonstration, because light does not consist of *particles* coming towards us, but of *waves* transmitted through the ether of space. But it has been shown (though the proof is by no means elementary) that, within very narrow limits, if not exactly, the apparent direction of the motion of a *wave* is affected in precisely the same way as that of a moving projectile.

The constant of aberration.

172. The Constant of Aberration. — By the discussion of thousands of observations upon stars during the past fifty years, it is found that the *maximum aberration* of a star — the same for all stars — is about $20''.5$,[1] which is called the *Constant of Aberration.* This maximum displacement occurs, of course, whenever the sun's motion is at right angles to the line drawn from the earth to the star, always twice a year.

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[1] This value is uncertain by at least 0.02 or 0.03 of a second. The Astronomical Congress at Paris in 1896 adopted the value *20″.47.*

Aberrational orbit described annually by each star.

A star at the pole of the ecliptic is, however, permanently in a direction perpendicular to the earth's motion, and will therefore always be displaced by the same amount of 20″.5, but in a *direction continually changing.* It therefore appears to describe during the year as its "aberrational orbit" a *little circle 41″ in diameter.*

A star *on the ecliptic* (*latitude 0°*) appears simply to oscillate back and forth in *a straight line 41″ long.*

Between the ecliptic and its pole the aberrational orbit is an *ellipse* having its major axis parallel to the ecliptic and *always 41″ long*, while its minor axis depends upon the star's latitude, β, and always equals *41″ sin* β.

There is also a very slight *diurnal aberration* due to the rotation of the earth, its amount depending on the observer's *latitude* and ranging from 0″.31 at the equator to zero at the pole.

Distance of the sun determined by means of aberration.

173. Determination of the Earth's Orbital Velocity and the Mean Distance of the Sun by Means of Aberration. — From Sec. 171, $\tan a = \frac{u}{V}$, which gives $u = V \tan a$, u in this case being the velocity of the earth in its orbit and V the velocity of light, while a is the constant of aberration. The experiments of Michelson and Newcomb (*Physics*, p. 604) (confirmed in 1900 by Perrotin's experiments at Paris by a different method) make V equal 186330 miles a second, with a probable error of about 25 miles. We have, therefore, u, the velocity of the earth in its orbit, equals 186330 × tan 20″.47 = 18.5 miles.

The measured velocity of light.

Resulting value for the sun's distance.

The circumference of the orbit, regarded as circular (which in the case of the earth involves no sensible error), is found by multiplying this velocity, 18.5, by the number of mean solar seconds in the *sidereal* year (Sec. 182). Dividing this circumference by 2π, we find the radius of the orbit, or the mean distance of the sun, to be very nearly ***92 900000 miles.***

Amount of uncertainty.

The uncertainty of the constant of aberration affects the distance proportionally, by perhaps 100000 miles. Still the method is one of the very best of all that we possess for determining the value of "the astronomical unit."

Solar day about four minutes longer than the sidereal.

Apparent solar days are of unequal length.

174. Solar Time and the Equation of Time. — Since the sun makes the circuit of the heavens in a year, moving always towards the east, the solar day, as has been already explained in a preceding article, is about four minutes longer than the sidereal day, the difference amounting to exactly one day in a year; *i.e.*, while in a sidereal year there are 366¼ (nearly) *sidereal* days, there are only 365¼ *solar* days. Moreover, the sun's advance in right ascension between two successive noons varies materially, so that the apparent solar days are not all of the same length. December 23 is fifty-one seconds longer than September 16.

The fictitious sun.

Accordingly, as already explained (Sec. 98), *mean time* has been adopted, which is kept by a "fictitious," or "mean," sun moving uniformly in the equator at the same average rate as that of the real sun in the ecliptic. The hour angle of this *mean* sun is the local *mean time*, or clock time, while the hour angle of the *real* sun is the *apparent*, or *sun-dial*, time.

Definition of the equation of time.

The *Equation of time* is the difference between these two times reckoned as *plus* when the sun-dial is *slower* than the clock and *minus* when it is *faster; i.e.*, it is the correction which must be added (algebraically) to *apparent* time in order to get *mean* time, and this is simply equal to the difference between the right ascensions of the fictitious sun and the true sun; so that, calling the equation of time E, we may write $E = a_t - a_m$, in which a_t is the right ascension of the true sun and a_m the right ascension of the mean sun. When a_t is greater than a_m the real sun comes to the meridian *later* than the true sun, and the sun-dial is *slow* of mean time.

The principal causes of this difference are two:

(1) *The variable motion of the sun in the ecliptic, due to the eccentricity of the earth's orbit.*

(2) *The obliquity of the ecliptic.*

Causes of the equation of time.

175. Effect of the Eccentricity of the Earth's Orbit. — Near perihelion, which occurs about December 31, the sun's eastward motion on the ecliptic is most rapid. At this time, accordingly, the apparent solar days, for this reason, exceed the sidereal by more than the average amount, making the sun-dial days longer than the mean. The sun-dial will therefore *lose time* at this season and will, so far as this cause is concerned, continue to do so until the motion of the sun falls to its average value, as it will at the end of three months; at this time the difference will have amounted to about 7¾ minutes. Then the sun-dial will gain until aphelion, and at that time the clock and sun-dial will once more agree. During the remaining half of the year the action will be reversed. The equation of time, therefore, so far as due to this cause only, is about + 7¾ minutes in the spring, and − 7¾ in the autumn.

Effect of the unequal velocity of the earth in different parts of its orbit, producing an equation of ± 7¾ minutes.

176. Effect of the Obliquity of the Ecliptic. — Even if the sun's motion in *longitude* (*i.e.*, along the ecliptic) were uniform, its motion in *right ascension* would be variable. If the true and fictitious suns were together at the vernal equinox, one moving uniformly in the ecliptic and the other in the equator, they would indeed be together (*i.e.*, have the same right ascensions) at the two solstices and at the other equinox, because it is just 180° from equinox to equinox and the solstices are exactly half-way between them; but at any point between the solstices and equinoxes their right ascensions would differ.

Equation due to the obliquity of the ecliptic.

Uniform motion of sun on the ecliptic does not give a uniform motion in right ascension.

This is easily seen by taking a celestial globe and marking on the ecliptic the point m (Fig. 59), half-way between the vernal equinox E and the summer solstice C, and also marking a point n on the equator 45° from the equinox. It will be seen at once that the former point is west of n, the difference of right ascension being $m'n$, so that m in the apparent diurnal revolution of the sky will come first to the meridian.[1] In other words, about six weeks after each equinox, when the sun is halfway between the equinox and the solstice, the sun-dial, *so far as the obliquity of the ecliptic is concerned*, is *faster* than the clock, and this component of the equation of time is *minus*, amounting to nearly ten minutes. Of course, the same thing holds, with the necessary changes, for the other quadrants.

Equation of time due to this cause about $+10^{m}$ six weeks before each equinox, and -10^{m} six weeks after it.

If the ecliptic be divided into equal portions from E to C and hour-circles be drawn from P through the points of division, it is clear that near E the portions of the ecliptic are longer than the corresponding portions of the equator. On the other hand, near the solstice C the arc of the ecliptic is shorter than the corresponding arc of the equator, on account of the *divergence of the hour-circles* as they recede from the pole.

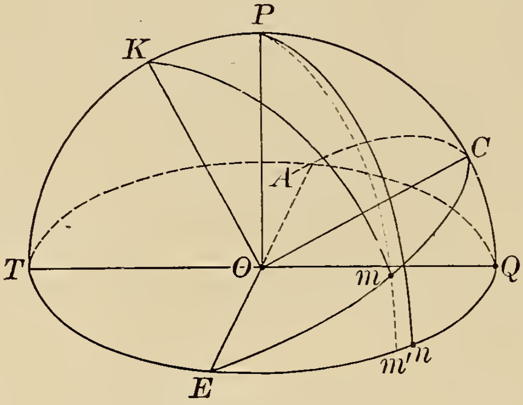

FIG. 59

177. Combination of the Effects of the Two Causes. — We can represent the two components of the equation of time and the result of their combination by a graphical construction (Fig. 60).

Graphical combination of the two components of the equation of time, showing the total result and effect.

The central horizontal line is a scale of *dates* one year long, the months being indicated at the top. The *dotted curve* shows that component of the equation of time which is due to the eccentricity of the earth's orbit. In the same way the *broken-line curve* denotes the effect of the obliquity of the ecliptic. The *heavy-line curve* represents the combined effect of the two

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[1] In the figure the observer is supposed to be looking at the globe *from the west*, E, the vernal equinox, being at the west point of the horizon. ECA is the ecliptic, its pole being K; while $EQAT$ is the celestial equator, its pole (of diurnal rotation) being P.

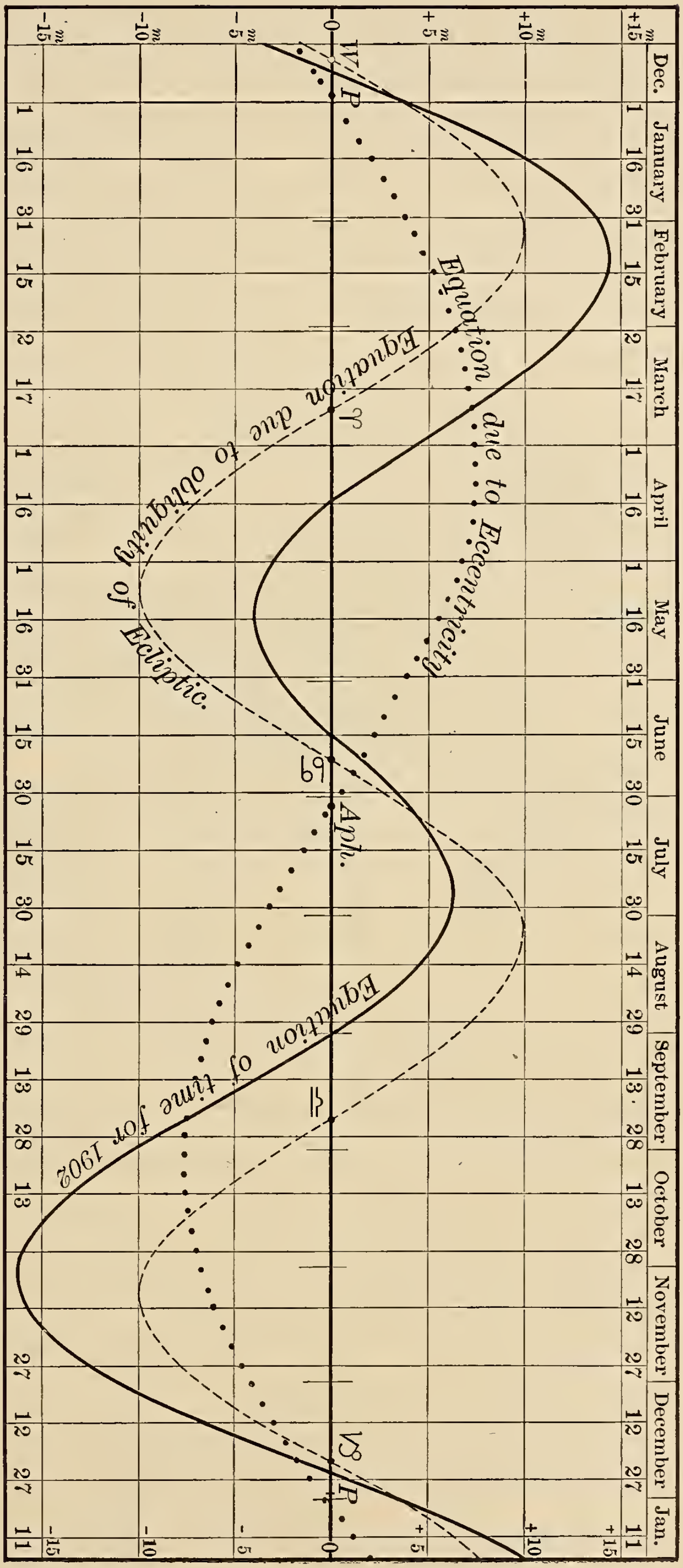

FIG. 60. — The Equation of Time

causes, its ordinate at each point being made equal to the algebraic sum of the ordinates of the other two curves. The heavy-line curve is carefully laid out from the Nautical Almanac for 1902 (a mean year in the "leap-year cycle") and will give the equation of time for any date during the next fifty years within less than half a minute; not exactly, because from year to year the equation of time for any day of the month varies a few seconds. The small rectangles reckoned horizontally represent *fifteen-day* intervals; vertically, intervals of *five minutes*.

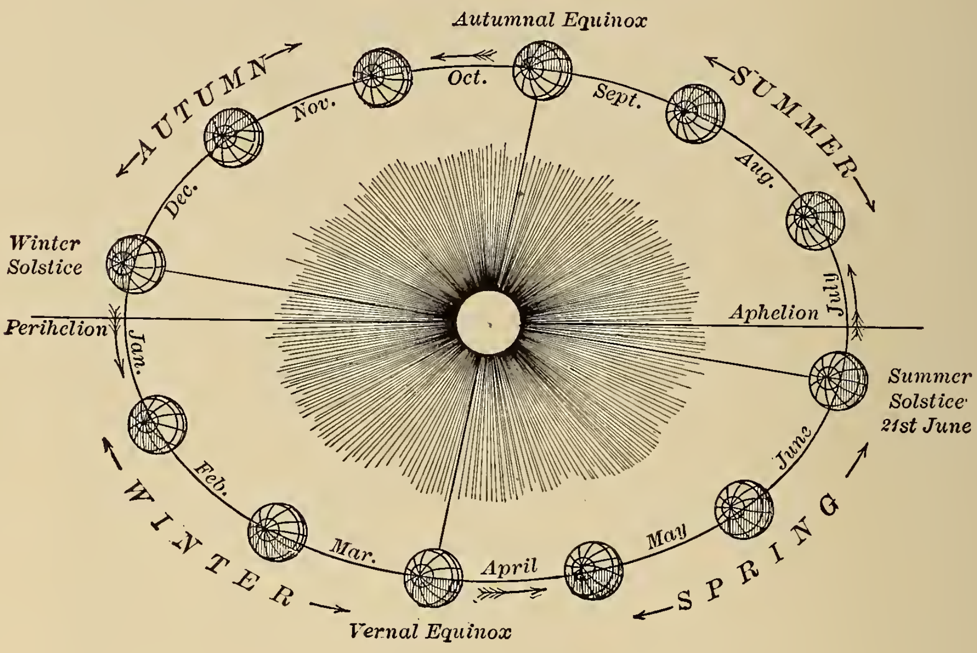
Autumnal Equinox
AUTUMN
Oct.
Nov.
Sept.
SUMMER
Dec.
Aug.
Winter Solstice
Perihelion
Jan.
Aphelion
July
Summer Solstice 21st June
June
Feb.
WINTER
Mar.
April
May
SPRING
Vernal Equinox

FIG. 61. — The Seasons

Other causes contribute slightly to the equation of time.

The two causes discussed above are only the principal ones. Every perturbation suffered by the earth slightly modifies the result, but all other causes combined never affect the equation of time by as much as ten seconds.

Dates when equation of time becomes zero.

The equation of time becomes zero four times yearly, as will be seen from the figure, — about April 15, June 14, September 1, and December 24; but the dates vary a little from year to year.

178. The Seasons. — The earth in its orbital motion keeps its axis nearly parallel to itself for the same mechanical reason that a spinning globe maintains the direction of its axis unless disturbed by some outside force, — very prettily illustrated by the gyroscope. Since this axis is not perpendicular to the plane of its orbit, the poles of the earth vary in their presentation to the sun, as shown in Fig. 61. At the two equinoxes, March 21 and September 22, the plane of the earth's equator passes through the sun, so that the circle which divides day from night upon the earth passes through the pole, as shown in Fig. 62, *B*, and day and night are then everywhere equal. On June 21 the earth is so situated that its *north* pole is inclined towards the sun by about 23½°, as shown in Fig. 62, *A*. The south pole is then in the unilluminated half of the globe, while the north pole receives sunlight all day long; and in all portions of the northern hemisphere the day is longer than the night, and *vice versa* in the southern hemisphere. At the time of the winter solstice these conditions are reversed and the south pole has perpetual sunshine. On the equator day and night are equal at all times of the year, and there are no seasons in the proper sense of the word.

Alternate presentation of north and south poles of the earth to the sun.

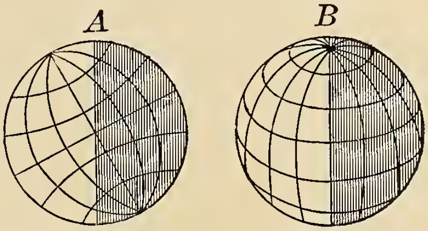

Fig. 62. — Position of Pole at Solstice and Equinox

The midnight sun and other phenomena in the neighborhood of the pole have already been discussed (Sec. 36).

179. Effects on Temperature. — The changes in the duration of *insolation* (exposure to sunshine) at any place involve changes of temperature and of other climatic conditions which produce the seasons. Taking as a standard the average amount of heat received from the sun in twenty-four hours on the day of the equinox, it is clear that the surface of the soil at any place in the northern hemisphere will receive each twenty-four hours

Station in northern hemisphere receives more than the average amount of heat in a day when sun is north of equator.

more than the average of heat whenever the sun is north of the celestial equator, and for two reasons:

Two reasons: day is then longer than the night, and during the day the sun's mean altitude is greater.

(1) Sunshine lasts more than half the day.

(2) The *mean altitude* of the sun while above the horizon is greater than at the time of the equinox.

Heating effect of sun's rays varies with the sine of sun's altitude, modified by atmospheric absorption.

Now the more obliquely the rays strike the less heat they bring to each square inch of the surface, as is obvious from Fig. 63. A beam of sunshine having a cross-section, $ABCD$, is spread over a larger area when it strikes obliquely than when vertically, its heating efficiency being in inverse ratio to the surface over which the heat is distributed. If Q is the amount of heat per square meter of area brought by the rays when falling perpendicularly, as on the surface AC, then on Ac, on which it strikes at the angle h (equal to the sun's altitude), the amount per square meter will be only $Q \times \sin h$. Moreover, this difference in favor of the more nearly vertical rays is exaggerated by the absorption of heat in the atmosphere, since rays that are nearly horizontal have to traverse a much greater thickness of air before reaching the ground.

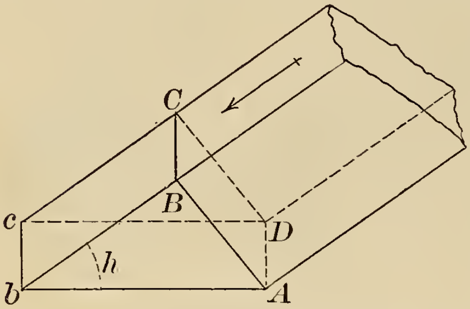

Fig. 63. — Effect of Sun's Elevation on Amount of Heat imparted to the Soil

For these two reasons, therefore, at a place in the northern hemisphere the mean temperature of the day rises rapidly as the sun comes north of the equator, thus causing summer.

Maximum temperature attained when amount of heat lost in a day equals that received.

180. Time of Highest Temperature. — We receive the most heat in twenty-four hours at the time of the summer solstice; but this is not the hottest time of the season for the obvious reason that the weather is still getting hotter, and the maximum will not be reached *until the increase ceases; i.e.*, not until the amount of heat *lost* in twenty-four hours equals that *received*, which occurs in our latitude about August 1. For similar reasons the minimum temperature of winter occurs about February 1.

Since the weather is not entirely "made on the spot where it is used," but is much influenced by winds and currents that come from great distances, the actual date of the maximum temperature at any particular place cannot be determined beforehand by mere astronomical considerations, but varies considerably from year to year. The great differences between the seasons of different years are as yet mostly without explanation.

Effect of the eccentricity of the earth's orbit in producing a difference between the seasons in the northern and southern hemispheres.

181. Difference between Seasons in Northern and Southern Hemispheres. — Since in December the distance of the earth from the sun is about three per cent less than it is in June, the earth as a whole receives hourly about six per cent more heat in December than in June, the heat received varying inversely as the *square* of the distance. For this reason the southern summer, which occurs in December and January, is hotter than the northern. It is, however, seven days shorter, because the earth moves more rapidly in that part of its orbit. The total amount of heat per acre received during the whole summer is therefore sensibly the same in each hemisphere, the shortness of the southern summer making up for its increased warmth.

Question whether the glacial period can be explained by this effect.

The southern *winter*, however, is both longer and colder than the northern, and it has been vigorously maintained by certain geologists that, on the whole, the mean annual temperature of the hemisphere which has its winter at the time when the earth is in aphelion is lower than the opposite one. It has been attempted to account for the glacial epochs in this way, but the explanation is very doubtful.

On account of the motion of the apsides of the earth's orbit (Sec. 164) the present state of things will be reversed in about ten thousand years; the perihelion will then be reached in *June*, and the northern summer will then be the shorter and the hotter one.

The three kinds of year.

182. The Three Kinds of Year. — Three different kinds of "year" are now recognized, — the *sidereal*, the *tropical* or *equinoctial*, and the *anomalistic*.

The sidereal year, — 365.25636 days.

The *Sidereal Year*, as its name implies, is the time occupied by the sun in apparently completing the circuit of the heavens *from a given star to the same star again.* Its length is $365^d 6^h 9^m 9^s$ of mean solar time ($365^d.25636$).

From the mechanical point of view this is the *true* year; *i.e.*, it is the time occupied by the earth in making one complete revolution around the sun from a given direction in space to the same direction again.

The tropical year, — 365.24219 days.

The *Tropical Year* is the time included between two successive passages of the vernal equinox by the sun. On account of precession (Sec. 165) the equinox moves yearly $50''.2$ towards the west, so that the tropical year is shorter than the sidereal, its length being $365^d 5^h 48^m 45^s.5$ ($365^d.24219$). Its length was determined by the ancients with considerable accuracy, as 365¼ days, by means of the gnomon; they noted the dates at which the noonday shadow was longest (or shortest), *i.e.*, the date of the solstice.

The tropical year the year of chronology.

Since the *seasons* depend on the sun's place with respect to the equinox, the tropical year is the year of chronology and civil reckoning. Whenever a period of so many years is spoken of we always understand tropical years, unless otherwise distinctly indicated.

The anomalistic year,— 365.25958 days.

The third kind of year is the *Anomalistic Year*, — the time between two successive passages of the perihelion. Since the line of apsides of the earth's orbit moves eastward about $11'$ a year (Sec. 164), this kind of year is nearly five minutes longer than the sidereal, its length being $365^d 6^h 13^m 48^s$ ($365^d.25958$).

It is little used, except in calculations relating to perturbations.

Natural time units.

183. The Calendar. — The natural units of time are the day, month, and year. The day is too short for convenience in dealing with considerable periods, — such as the life of a man, for instance; and the same is true even of the month, so that for all chronological purposes the *tropical year* — the year of the seasons — has always been employed. At the same time, so many religious ideas and observations have been connected with the changes of the moon that there was long a constant struggle to reconcile the *month* with the *year*. Since the two are incommensurable, no really satisfactory solution is possible, and the modern calendar of civilized nations entirely disregards the moon.

Lunar calendars.

In the ancient times the calendar was in the hands of the priesthood and was predominantly lunar, the seasons either being disregarded or kept roughly in place by the occasional intercalation or dropping of a month. The principal Mohammedan nations still use a purely lunar calendar having a year of twelve lunar months containing alternately 354 and 355 days. In their reckoning, therefore, the months and their religious festivals fall continually in different seasons, and their calendar gains on ours about one year in thirty-three.

The Julian calendar: every fourth year a leap-year.

184. The Julian Calendar. — When Julius Cæsar came into power he found the Roman calendar in a state of hopeless confusion. He therefore sought the advice of the Alexandrian astronomer Sosigenes, and in accordance with his suggestions established (45 B.C.) what is known as the *Julian calendar*, which still, either untouched or with a trifling modification, continues in use among all civilized nations. He discarded all consideration of the moon, and adopting $365\frac{1}{4}$ days as the true length of the year, he ordained that every fourth year should contain 366 days, the extra day being inserted by repeating the sixth day before the calends of March, whence such a year is called *bissextile*. He also transferred to January 1 the beginning of the year, which until then had been in March (as is indicated by the names of several of the months, as September, *i.e.*, the *seventh* month, etc.).

Why leap-year is called "bissextile."

Cæsar also took possession of the month Quintilis, naming it *July* after himself. His successor, Augustus, in a similar manner appropriated the next month, Sextilis, calling it *August*, and to vindicate his dignity and make his month as long as his predecessor's he added to it a day stolen from February.

The Julian calendar is still used unmodified in Russia and by the Greek Church generally.

Incorrectness of the Julian calendar.

185. The Gregorian Calendar. — The true length of the tropical year is not 365¼ days, but $365^d 5^h 48^m 45^s.5$, leaving a difference of $11^m 14^s.5$ by which the Julian year is too long. This amounts to a little more than three days in four hundred years. As a consequence, in the Julian calendar the date of the vernal equinox comes earlier and earlier as time goes on, and in 1582 it had fallen back to the 11th of March instead of occurring on the 21st, as it did at the time of the Council of Nice, A.D. 325.

The Gregorian calendar. Correction made, and error prevented from accumulating by new rule respecting leap-year.

Pope Gregory, therefore, under the advice of the distinguished astronomer Clavius, ordered that the calendar should be corrected by dropping *ten days*, so that the day following Oct. 4, 1582, should be called the 15th instead of the 5th; and further, to prevent any future displacement of the equinox, he decreed *that thereafter only such century years should be leap-years as are divisible by 400*. (Thus, 1700, 1800, 1900, 2100, and so on, are not leap-years, while 1600 and 2000 are.)

186. The change was immediately adopted by all Catholic countries, but the Greek Church and most Protestant nations refused to recognize the Pope's authority. It was, however, finally adopted in England by an act of Parliament, passed in 1751, providing that the year 1752 should begin on January 1 (instead of March 25, as had long been the rule in England) and that the day following Sept. 2, 1752, should be reckoned as the 14th instead of the 3d, thus dropping eleven days.

Adoption of the Gregorian calendar in England in 1752.

The change was bitterly opposed by many, and there were riots in consequence in various parts of the country, especially at Bristol, where several persons were killed. The cry of the people was, "Give us back our fortnight," for they supposed they had been robbed of eleven days, although the act of Parliament was carefully framed to prevent any injustice in the collection of interest, payment of rents, etc.

At present, since the years 1800 and 1900 were leap-years in the Julian calendar and not in the Gregorian, the difference between the two calendars is *thirteen days;* thus, in Russia the 22d of June is reckoned the 9th, but in that country both dates are ordinarily used for scientific purposes, so that the date would be written June $\frac{9}{22}$.

Present difference of the two calendars is thirteen days, and will remain so until the year 2100.

When Alaska was annexed to the United States the official date had to be changed by only eleven days, one day being provided for in the alteration from the Asiatic reckoning to the American (Sec. 111).

187. The Metonic Cycle and Golden Number. — In establishing a relation between the solar and lunar years, the discovery of the so-called *lunar* (or *Metonic*) *cycle* by Meton, about 433 B.C., considerably simplified matters. This cycle consists of 235 synodic months (from new moon to new again), which is very approximately equal to nineteen common years of 365¼ days. The calendar for the phases of the moon is, therefore (with very rare exceptions), the same for any two years nineteen years apart; *i.e.*, the calendar of the phases of the moon, and of all ecclesiastical holidays which depend upon them (Easter, etc.), is the same for 1901 as for 1872 and 1920. But the dates are liable to a shift of a single day, according to the number of leap-years which intervene. This cycle is still employed in the ecclesiastical calendar in determining the time of Easter.

The Metonic cycle — nineteen years — very nearly equal to 235 months.

The *golden number* of a year is its number in this Metonic cycle and is found by adding 1 to the date number of the year and dividing by 19. The remainder, unless zero, is the golden number. If it comes out zero, 19 is taken. Thus, the golden number for the year 1902 is 3.

The "golden number."

188. The Julian Period and Julian Epoch. — The Julian *Period* consists of 7980 Julian years (28 × 19 × 15), each containing exactly 365¼ days, and its starting-point, or *Epoch*, is Jan. 1, 4713 B.C., — the Julian date of Jan. 1, A.D. 1, being J.E. 4714.

The Julian period and epoch introduced by J. Scaliger.

Julian date and day numbers.

The system was proposed by J. Scaliger in 1582 as a universal harmonizer of the different systems of chronological reckoning then in use, and its adoption has brought order out of confusion. It is extensively employed in astronomical calculations, the date of any phenomenon being expressed beyond all ambiguity either by the (Julian) year and day, or still more simply by "day number" so and so of the Julian era. Thus, the date of the solar eclipse of Aug. 9, 1896, is J.E. 6609, 222d day, or simply Julian day 2 413781; and this is perfectly definite to every astronomer of whatever nation, — American, Russian, Arabian, or Chinese.

The number of days between any two events, even centuries apart, is at once found by merely taking the difference between their Julian day numbers.

The Almanac gives for each year its Julian number, and also the Julian day number for January 1 of that year.

1900 is Julian year 6613. Jan. 1, 1900, is Julian day 2 415021.
1902 " " " 6615. " 1, 1902, " " " 2 415751.
March 10, 1902, " " " 2 415820, etc.

For a fuller explanation of the considerations on which this system of reckoning is founded, the reader is referred to Herschel's *Outlines of Astronomy*, Art. 924.

EXERCISES

1. What is the meridian altitude of the sun at Princeton (Lat. 40° 21′) on the day of the summer solstice?

2. What is the sun's approximate right ascension at that time?

3. On what days during the year will the sun's right ascension be approximately an even hour (*i.e.*, 0 hours, 2 hours, 4 hours, etc.)?

4. On what days will it be an *odd* hour?

5. What is the (approximate) sidereal time at 10 P.M. on May 12? *Ans.* 13^h26^m.

6. At what time will Arcturus (R.A. = 14^h10^m) come to the meridian on August 1? *Ans.* About 5^h26^m P.M.

7. About what time of night is Mizar (R.A. = 13^h20^m) vertically under the pole on October 10? *Ans.* Midnight.

8. In what latitude has the sun a meridian altitude of 80° on June 21?
Ans. + 33° 27′.

9. What are the longitude and latitude (celestial) of the north celestial pole? *Ans.* Long. 90°, Lat. 66° 33′.

10. What are the right ascension and declination of the north pole of the ecliptic? *Ans.* R.A. 18^h, Dec. 66° 33′.

11. What are the greatest and least angles made by the ecliptic with the horizon at New York (Lat. 40° 43′)?

$$\textit{Ans.}\quad (90° - 40° 43') \pm 23° 27' = \begin{cases} \text{Max. } 72° 44'. \\ \text{Min. } 25° 50'. \end{cases}$$

12. Does the vernal equinox always occur on the same day of the month? If not, why not? How much can the date vary?

13. Will the ephemeris of the sun for one year be correct for every other year, and, if not, how much can it be in error?

Ans. A difference of $1\frac{3}{4}$ days' motion of the sun is possible; as, for instance, between 1897 and 1903, the leap-year being omitted in 1900.

14. When the sun is in the *sign* of Cancer in what *constellation* is he?

15. What obliquity of the ecliptic would reduce the width of the temperate zone to zero?

16. At a place west of Philadelphia an observer finds that his local *apparent* time on October 1, as determined from the sun by sextant, was 8^m30^s slow of eastern standard time. The equation of time on that date is -10^m8^s. What was his longitude from Greenwich? *Ans.* $5^h18^m38^s$.

17. At what standard time will the sun come to the meridian on March 21 at Boston (Long. 4^h44^m west of Greenwich), the equation of time being $+ 7^m28^s$? *Ans.* $11^h51^m28^s$.

18. When the equation of time is 16 minutes, as it is on November 1, how does the forenoon from sunrise till 12 o'clock compare in length with the afternoon from 12 o'clock till sunset?

19. Why do the afternoons begin to lengthen about December 8, a fortnight before the winter solstice?

20. There were five Sundays in February, 1880. The same thing has not occurred since, and will not until when? *Ans.* 1920.

21. What was the Russian date corresponding to Feb. 28, 1900, in our calendar? What corresponding to May 1 of the same year?
Ans. February 16; April 18.

CHAPTER VII

THE MOON

The Moon's Orbital Motion and the Month — Distance, Dimensions, Mass, Density, and Force of Gravity — Rotation and Librations — Phases — Light and Heat — Physical Condition — Telescopic Aspect and Peculiarities of the Lunar Surface

189. Next to the sun, the moon is the most conspicuous and *to us* the most important of the heavenly bodies, — in fact, the only one except the sun which exerts the slightest influence upon human life. If the stars and planets were all extinguished, our *eyes* would miss them, and that is all; but if the moon were annihilated, the interests of commerce would be seriously affected by the practical cessation of the tides. She owes her conspicuousness and importance, however, solely to her nearness, for she is really a very insignificant body as compared with the stars and the planets.

Importance of the moon in astronomy.

And yet, astronomically, she perhaps ranks highest among the heavenly bodies. The very beginnings of the science seem to have originated in the study of her motions and of the different phenomena which she causes, such as the eclipses and the tides; and in the development of modern theoretical astronomy the "lunar theory," with the problems it raises, has been perhaps the most fertile field of discovery and invention.

190. The Moon's Apparent Motion; Definition of Terms, etc. — One of the earliest observed of astronomical phenomena must have been the eastward motion of the moon with reference to the sun and stars and the accompanying changes of phase. If we note the moon to-night as very near some conspicuous star, we shall find her to-morrow night at a point about 13° farther east, and the next night as much farther still; she

Apparent motion of the moon among the stars.

makes a complete circuit of the heavens, from star to star again, in about $27\frac{1}{3}$ days. In other words, she "revolves around the earth" in that time, while she accompanies us in our annual journey around the sun.

Since the moon moves eastward among the stars so much faster than the sun, she overtakes and passes him at regular intervals; and as her phases depend upon her apparent position with respect to the sun, this interval from new moon to new moon is specially noticeable and is what we ordinarily understand as the *month*, — technically, the *synodic month.*

The moon's apparent motion with respect to the sun. The month.

The *Elongation* of the moon is her angular distance east or west of the sun at any time. At new moon it is zero, and the moon is said to be in *Conjunction.* At full moon the elongation is 180°, and she is said to be in *Opposition.* In both cases the moon is in *Syzygy*, *i.e.*, the sun, moon, and earth are ranged nearly along a straight line. When the elongation is 90° she is said to be in *Quadrature.*

Definitions of elongation, conjunction, etc.

191. Sidereal and Synodic Months. — The *Sidereal Month* is the time it takes the moon to make her revolution *from a given star to the same star again* as seen from the center of the earth. It averages $27^d 7^h 43^m 11^s.55$ ($27^d.32166$), but it varies some three hours on account of "perturbations." The mean daily motion is $360° \div 27.32166$, or $13° 11'$. Mechanically considered, the *sidereal month* is the true month.

The sidereal and synodic months.

The *synodic month* is the time between two successive conjunctions or oppositions, *i.e.*, between successive new or full moons. Its average value is $29^d 12^h 44^m 2^s.86$, but it varies nearly thirteen hours, mainly on account of the eccentricity of the lunar orbit. As has been said already, this synodic month is what we ordinarily mean when we speak of a "month."

If M be the length of the moon's sidereal period, E the length of the sidereal year, and S that of the synodic month, the three quantities are connected by a very simple relation. $\frac{1}{M}$ is the

Relation between the sidereal and synodic months.

fraction of a circumference moved over by the moon in a day. Similarly, $\frac{1}{E}$ is the apparent daily motion of the sun. The difference is the amount which the moon *gains* on the sun daily. Now it gains a whole revolution in one synodic month of S days, and therefore must daily gain $\frac{1}{S}$ of the circumference.

Equation of synodic motion.

Hence, we have the important equation $\frac{1}{M} - \frac{1}{E} = \frac{1}{S}$, "the equation of synodic motion," whence $S = \frac{E \times M}{E - M}$.

Number of sidereal months in a year exactly one more than the number of synodic months.

Another way of looking at the matter, leading, of course, to the same result, is this: In a sidereal year the number of sidereal months must be just one greater than the number of synodic months; the numbers are, respectively, 13.369+ and 12.369+.

192. The Moon's Path on the Celestial Sphere; the Nodes and their Motion.

The moon's path on the celestial sphere.

— By observing the moon's right ascension and declination daily with suitable instruments we can map out its apparent path, just as in the case of the sun (Sec. 156). It turns out to be (very nearly) a *great circle* inclined to the ecliptic at an angle of about 5° 8′, but varying 12′ each way, from 4° 56′ to 5° 20′.

The nodes.

The two points where the path cuts the ecliptic are called the *nodes*, the *ascending* node being the one where the moon passes from the south side to the north side of the ecliptic. The opposite node is called the *descending* node. (Ancient astronomers all lived in the *northern* hemisphere.)

The moon at the end of the month never comes back exactly to the point of beginning, on account of the so-called "perturbations," due to the attraction of the sun.

Regression of the nodes.

One of the most important of these perturbations is the *regression of the nodes*. These slide westward on the ecliptic in the same manner as the vernal equinox does, but much faster, completing their circuit in a little less than nineteen years instead of twenty-six thousand. The average time between two successive passages of the moon through the same node is called the *nodical* or *draconitic* month. It is 27.2122 days, — an important period in the theory of eclipses.

The nodical or draconitic month.

When the *ascending node* of the moon's orbit coincides with the vernal equinox the angle between the moon's path and the equator has its maximum value of 23° 27′ + 5° 8′, or 28° 35′; nine and one-half years later, when the descending node has come to the same point, the angle is only 23° 27′ − 5° 8′, or 18° 19′. In the first case the moon's meridian altitude will range during the month through about 57°. In the second case the range is reduced to 36° 38′.

Variation in the inclination of the moon's path to the celestial equator.

193. Interval between the Moon's Successive Transits; Daily Retardation of its Rising and Setting. — Owing to the eastward motion of the moon it comes to the meridian *later* each day. If we call the average interval between its successive transits a "moon day," we see at once that while in the synodic month there are 29.5306 *mean solar* days, there must be just one less of these "moon days," since the moon, in the synodic month, moves around eastward from the sun to the sun again, thus losing one complete relative rotation.

Interval between successive transits of the moon, — $24^h 50^m.5$.

It follows, therefore, that the length of the "moon day" must be $24^h \times \frac{29.5306}{28.5306}$, or $24^h 50^m.51$, the average "daily retardation" being $50\frac{1}{2}$ minutes. It ranges, however, all the way from 38 minutes to 66 minutes on account of the variations in the rate of the moon's motion in right ascension, — due partly to perturbation, but mainly to the oval form of its orbit and its inclination to the celestial equator, — variations precisely analogous to the inequalities of the sun's motion, which produce the equation of time (Sec. 174), but many times greater.

How the daily retardation is computed. Its range.

The average retardation of the moon's daily *rising and setting* is also the same 50.51 minutes, but the actual retardation is much more variable than that of the transits, depending largely

Daily retardation of moon's rising and setting ranges in our latitude from 23ᵐ to 1ʰ17ᵐ.

on the latitude of the observer. At New York the range is from 23 minutes to 77 minutes. In higher latitudes it is still greater. Indeed, in latitudes above 61° 20′, the moon, when it has its greatest possible declination of 28° 47′, will become *circumpolar* for a certain time each month and will remain visible without setting at all for a whole day or more, according to the latitude of the observer. As a consequence of this daily retardation it follows that there is always one day in the month on which the moon does not rise, and another on which it does not set.

One day in each month when the moon does not rise.

194. Harvest and Hunter's Moon. — The full moon that comes nearest the autumnal equinox is known as the *harvest-moon;* the one next following is the *hunter's moon.* At that time of the year the moon while nearly full rises for several consecutive nights at about the same hour, so that the moonlight evenings last for an unusual length of time. The phenomenon is much more striking in Northern Europe than in the United States.

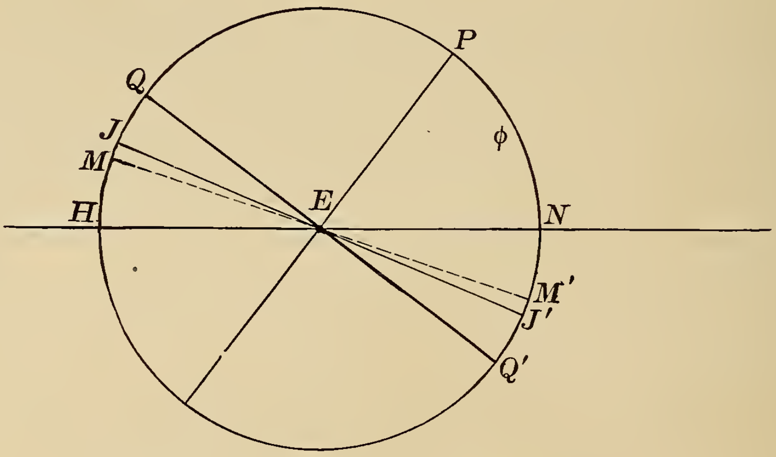

Fig. 64. — Explanation of the Harvest-Moon

Harvest and hunter's moon.

In the autumn the full moon is near the vernal equinox (since the sun is at the autumnal) and is in the portion of its path which is least inclined to the eastern horizon, where it rises. This is obvious from Fig. 64, which represents a celestial globe looked at from the east. *HN* is the horizon, *E* the east point, *P* the pole, and *EQ* the equator. If, now, the *first of Aries* is rising at *E*, the line *JEJ′* will be the ecliptic and will be inclined to the horizon at an angle less than *QEH* (the inclination of the equator) by $23\frac{1}{2}°$.

Explanation of the phenomenon.

If the *ascending node* of the moon's orbit happens to coincide with the first of Aries, then, when this node is rising, the moon's path will lie still more nearly horizontal than *JJ′*, as shown by the line *MM′*, and the phenomenon of the harvest-moon will be specially noticeable.

The moon's orbit an ellipse with a mean eccentricity of about $\frac{1}{18}$.

195. Form of the Moon's Orbit. — By observation of the moon's apparent diameter, combined with observations of her place in the sky, we can determine the *form of her orbit* around the earth in the same way that the form of the earth's orbit around the sun was worked out in Sec. 160. The moon's apparent diameter ranges from 33′ 33″, when as near as possible, to 29′ 24″, when most remote.

The orbit turns out to be an ellipse like that of the earth around the sun, but of much greater eccentricity, averaging about $\frac{1}{18}$. We say "averaging" because it varies from $\frac{1}{15}$ to $\frac{1}{21}$ on account of perturbations.

Definition of perigee, apogee, and apsides.

The point of the moon's orbit nearest the earth is called the *perigee* (πέρι γῆ), that most remote the *apogee* (ἀπό γῆ), and the indefinite line passing through these points and continuing to the heavens the *line of apsides*, the major axis being that portion of this line which lies between perigee and apogee. On account of perturbations the line of apsides is in continual motion like the line of nodes, but it moves *eastward* instead of westward, completing its revolution in about nine years.

Eastward motion of the line of apsides.

In her motion around the earth the moon also very nearly observes the same "law of areas" that the earth does in her orbit around the sun.

Law of moon's orbital motion.

196. Method of determining the Size of the Moon's Orbit, *i.e.*, her Distance and Parallax. — In the case of any heavenly

Determination of the moon's distance from the earth.

body one of the first and most fundamental inquiries relates to its distance; until this has been measured we can get no knowledge of the real dimensions of its orbit, nor of the size, mass, etc., of the body itself. The problem is usually solved by measuring the "parallactic displacement" (Sec. 78) due to a known change in the position of the observer. Many methods are applicable in the case of the moon. We limit ourselves to a single one, the simplest, though perhaps not the most accurate, of the different methods that are practically available.

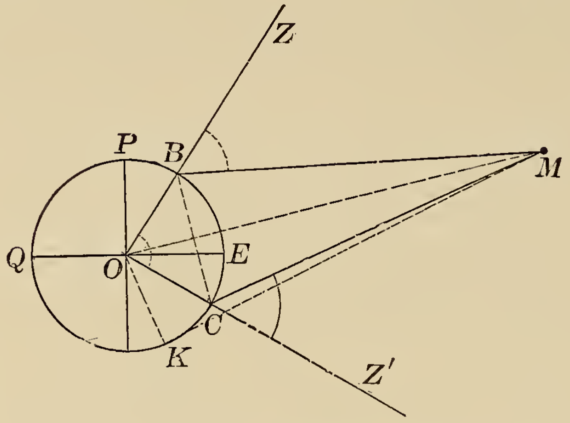

FIG. 65.—Determination of the Moon's Parallax

Simultaneous meridian-circle observations of the moon's zenith-distance from two stations on the same meridian but widely separated in latitude.

At each of two observatories, *B* and *C* (Fig. 65), on, or very nearly on, the same meridian and very far apart (Berlin and Cape of Good Hope, for instance), the moon's zenith-distance, *ZBM* and *Z′CM*, is observed simultaneously with the meridian-circle. This gives in the quadrilateral *BOCM* the two angles *OBM* and *OCM*. The angle *BOC*, at the center of the earth, is the difference of the *geocentric* latitudes of the two observatories (numerically their sum). Moreover, the sides *BO* and *CO* are known, being radii of the earth.

The quadrilateral can, therefore, be solved by a simple trigonometrical process. (1) In the triangle *BOC* we have given *BO*, *OC*, and the included angle *BOC*; hence, we can find the side *BC* and the two angles *OBC* and *OCB*. (2) In the triangle *BCM* we now have given *BC* and the two angles *MBC* and *MCB* (which are got by simply subtracting *OBC* from *OBM* and *OCB* from *OCM*); hence, we can find *BM* and *CM*. (3) In the triangle *OBM* or *OCM* we now know the two sides and the included angle at *B* or *C*, from which we can find *OM*, *the* *moon's distance* from the center of the earth. (4) When OM is determined we at once find the *horizontal parallax* from the equation

$$\sin \Pi = \frac{OB}{OM} = \frac{r}{R} \text{ (Sec. 79).}$$

197. Parallax, Distance, and Velocity of the Moon. — The moon's *equatorial horizontal parallax* is found to average 3422″.0 (57′ 2″.0), according to Neison, but it varies considerably from day to day on account of the eccentricity of the orbit. Her average distance from the earth is about 60.3 times the earth's equatorial radius, or *238840 miles*, with an uncertainty of 10 or 20 miles.

Mean parallax of moon 57′ 02″.

Distance 238840 ± 15 miles.

The maximum and minimum values of the moon's distance are given by Neison as 252972 and 221614 miles. It will be noted that the "average" distance is not the mean of the two extremes.

Range of distance about 31400 miles.

Knowing the size and form of the moon's orbit, the velocity of her motion is easily computed. It averages 2287 miles an hour, or about 3350 feet per second. Her mean angular velocity in the celestial sphere is about 33′ an hour, just a little greater than the apparent diameter of the moon itself.

The moon's orbital velocity.

198. Form of the Moon's Orbit with Reference to the Sun. — While the moon moves in a small oval orbit around the earth, it also moves around the sun in company with the earth. This common motion of the moon and earth, of course, does not affect their relative motion, but to an observer outside the system looking down upon moon and earth the moon's motion around the earth would be a very small component of the moon's whole motion as seen by him.

The distance of the moon from the earth is only about $\frac{1}{390}$ part of the distance of the sun. The speed of the earth in its orbit around the sun is also more than thirty times greater than that of the moon around the earth; for the moon, therefore, the resulting path *in space* is one which deviates very slightly

The moon's path relative to the sun always concave towards the sun.

from the orbit of the earth and is *always concave towards the sun*, as shown in Fig. 66. It is *not* as shown in Figs. 67 and 68, although often so represented.

If we represent the orbit of the earth by a circle with a radius of 100 inches (8 feet 4 inches), the moon would deviate from it by only one fourth of an inch on each side, crossing it twenty-four or twenty-five times in one revolution around the sun, *i.e.*, in a year.

199. Diameter, Area, and Volume, or Bulk, of the Moon. — The mean apparent diameter of the moon is 31′ 7″. Knowing its mean distance, we easily compute from this (Sec. 12) its real *diameter*, ***2163 miles.*** This is 0.273 of the earth's diameter, — somewhat more than one quarter.

Size of the moon: diameter, area, and volume.

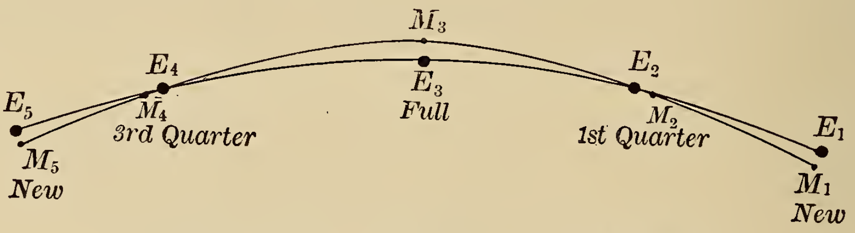

Fig. 66. — Moon's Path Relative to the Sun

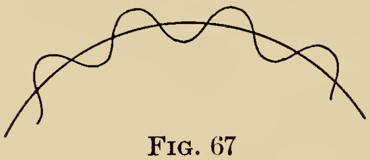

Fig. 67

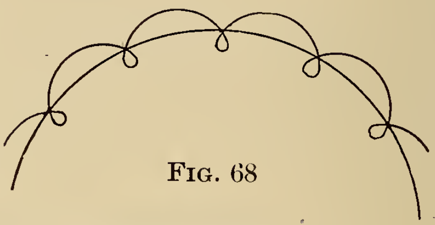

Fig. 68

Erroneous Representation of the Moon's Path

Since the surfaces of globes vary as the *squares* of their diameters, and their volumes as the *cubes*, this makes the *surface area* of the moon equal to 0.0747 (about $\frac{1}{14}$) of the earth's, and the *volume*, or bulk, 0.0204 (almost exactly $\frac{1}{49}$) of the earth's.

No other satellite is nearly as large as the moon in comparison with its primary planet. The earth and moon together, as seen from a distance, are really in many respects more like a *double planet* than a planet and satellite of ordinary proportions.

When Venus happens to be nearest us (at a distance of about twenty-five millions of miles) her inhabitants, if she has any, see the earth about twice as brilliant as Venus herself at her best appears to us, and the moon, about as bright as Sirius, oscillating backwards and forwards about half a degree on each side of the earth.

Earth and moon as seen from Venus.

200. Mass, Density, and Superficial Gravity of the Moon. —

The accurate determination of the moon's *mass* is practically a difficult problem. Though she is the nearest of all the heavenly bodies, it is far more difficult to *weigh* her than to determine the mass of Neptune, the remotest of the planets. There are many different methods of dealing with the problem.

Determination of the mass of the moon.

One, perhaps the best, consists in determining the position of the *center of gravity*, or *center of mass*, of earth and moon. It is this point and not the earth's center which describes around the sun what is called the "orbit of the earth." Now the earth and moon revolve together around this common center of gravity every month in orbits exactly alike in form, but differing greatly in size, the earth's orbit being as much smaller than the moon's as its mass is greater.

On account of this monthly motion of the earth's center, there results necessarily a "lunar equation," *i.e.*, a slight alternate eastward and westward displacement in the heavens of every object viewed from the earth as compared with the place the object would occupy if the earth had no such motion. In the case of the stars or the remoter planets the displacement is not sensible; but it *can* be measured by observing through the month the apparent motion of the sun, or better, of one of the nearer planets, as Mars or Venus, or the newly discovered Eros.

The "lunar equation" in the apparent motion of the sun and nearer planets.

From such observations it is found that the radius of the monthly orbit of the earth's center (*i.e.*, the distance from the center of the earth to the common center of gravity of the earth) is *2880 miles*. This is just about $\frac{1}{82.5}$ of the distance from the

The moon's mass is $\frac{1}{81.5}$ the mass of the earth.

earth to the moon, whence we conclude that the *mass* of the moon is $\frac{1}{81.5}$ that of the earth.

For other methods of determining the mass of the moon, the reader is referred to the *General Astronomy*, Art. 243.

Density of the moon about three fifths the density of the earth.

201. Since the density of a body is equal to its mass ÷ volume, the *density* of the moon compared with the earth is $\frac{1}{81.5}$ divided by $\frac{1}{49}$, which equals 0.601, or about 3.4 the density of water, the earth's density being 5.53. This is a little above the average density of the rocks which compose the crust of the earth. This low density of the moon is not at all surprising, nor at all inconsistent with the belief that it once formed a part of the earth, since, if such were the case, the moon was probably formed by the separation of the outer portions of that mass, which would be likely to be lighter than the rest.

Gravity on the moon's surface about one sixth of gravity on the earth; important in relation to the constitution of the moon.

The *superficial gravity*, or the attraction of the moon for bodies at its surface, is *mass* ÷ *radius*^{2}, *i.e.*, $\frac{1}{81.5}$ divided by 0.273^2, and comes out about *one sixth* of gravity at the surface of the earth. That is, a body weighing six pounds on the earth's surface would, at the surface of the moon, weigh only *one* pound (by a spring-balance). A man who can leap to a height of 5 feet here would reach 30 feet there, and so on.[1]

This is a point that must be borne in mind in connection with the enormous scale of the surface structure of the moon. Volcanic forces on the moon would throw ejected materials to a vastly greater distance than on the earth.

Rotation of the moon on its axis. The period of rotation equals the sidereal month.

202. Rotation of the Moon. — The moon rotates on its axis once a sidereal month, *in exactly the same time as that occupied by its revolution around the earth;* its day and night, therefore, the interval between sunrise and sunset, are each nearly a fortnight in length, and *in the long run it keeps the same side always towards the earth.* We see to-day precisely the same aspect of the moon as Galileo did in the days when he first turned his telescope upon it.

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[1] But see Sec. 141 for Professor Newcomb's illustration.

Many find difficulty in seeing why a motion of this sort should be called a "rotation" of the moon, since it is extremely like the motion of a ball carried on a revolving crank (Fig. 69). "Such a ball," they say, "revolves around the shaft, but does not rotate on its own axis." It does rotate, however; for if we mark one side of the ball, we shall find the marked side presented successively to every point of the compass as the crank turns, so that the ball turns on its own axis as really as if it were whirling upon a pin fastened to the table.

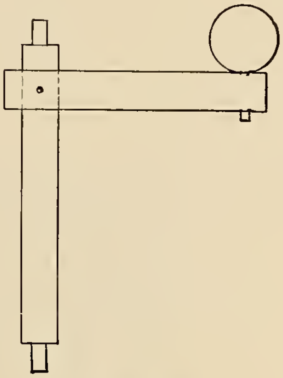

FIG. 69

Rotation of a ball carried by a crank arm.

By virtue of its connection with the crank, the ball has two distinct motions: (1) the *motion of translation*, which carries its center in a circle around the axis of the shaft; (2) an additional *motion of rotation*[1] around a line drawn through its center parallel to the shaft. The pin *A* (in the figure) and the hole in which it fits both rotate at the same rate, so that the ball, while it turns on its "axis" (an imaginary line), *does not turn on the pin, nor the pin in the hole.*

The rotation of the moon independent of its orbital revolution though having the same period.

203. Librations. — While in the "long run" the moon keeps the same face towards the earth, *it is not so in the short run;* there is no crank connection between the earth and moon, and the moon in different parts of a single month does not keep exactly the same face towards the earth, but rotates with perfect independence of her orbital motion. With reference to the center of the earth the moon's face is continually oscillating slightly, and these oscillations constitute what are called *librations*, of which we distinguish three, — viz., the libration in *latitude*, the libration in *longitude*, and the *diurnal* libration.

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[1] Rotation consists essentially in this: that a line connecting any two points, and not parallel to the axis of the rotating body, will sweep out a circle on the celestial sphere, if produced to it.

Inclination of the moon's equator to the plane of her orbit, causing libration in latitude.

(1) The *libration in latitude* is due to the fact that the moon's equator does not coincide with the plane of its orbit, but makes with it an angle of about $6\frac{1}{2}^\circ$. This inclination of the moon's equator causes its north pole at one time in the month to be tipped $6\frac{1}{2}^\circ$ towards the earth, while a fortnight later the south pole is similarly inclined to us; just as the north and south poles of the earth are alternately presented to the sun, causing the seasons.

Rotation uniform, while orbital motion is variable, causing libration in longitude.

(2) The *libration in longitude* depends on the fact that the moon's angular motion in its oval orbit is *variable*, while the motion of rotation is *uniform*, like that of any other undisturbed body; the two motions, therefore, do not keep pace exactly during the month, and we see alternately a few degrees around the eastern and western edge of the lunar globe. This libration amounts to about $7\frac{3}{4}^\circ$.

Apparent libration due to observer's displacement by the rotation of the earth.

(3) *The diurnal libration.* Again, when the moon is rising we look over its upper, which is then its *western*, edge, seeing a little more of that part of the moon than if we were observing it from the center of the earth; and *vice versa* when it is setting. This constitutes the so-called *diurnal libration*, and amounts to about one degree. Strictly speaking, this diurnal libration is not a libration of the moon, but of the observer. The telescopic effect is the same, however, as that of a true libration.

Minute physical libration.

In addition to this there is also a very slight *physical libration* of the moon. It is probable that the diameter of the moon directed towards the earth is a little longer than the diameter at right angles, the difference being perhaps a few hundred feet; and as the moon revolves around the earth this longest diameter oscillates slightly from side to side, changing its position apparently about $1\frac{1}{2}$ miles on the moon's disk.

Coincidence of periods of rotation and orbital revolution probably to be explained by tidal evolution.

The exact, long-run agreement between the moon's time of rotation and of her orbital revolution cannot be accidental. It has probably been caused by the action of the earth on some protuberance on the moon's surface, analogous to a tidal wave. If the moon were ever plastic, the earth's attraction must necessarily have been to produce a huge tidal bulge upon her surface, and the effect would have been ultimately to force an agreement between the lunar day and the sidereal month. The subject will be resumed later in connection with tidal evolution (Sec. 346).

Phases of the moon due to the fact that we see only a varying portion of her illuminated hemisphere.

204. The Phases of the Moon. — Since the moon is an opaque body shining merely by reflected light, we can see only that hemisphere of her surface which happens to be illuminated, and of this hemisphere only that portion which happens to be turned towards the earth. When the moon is between the

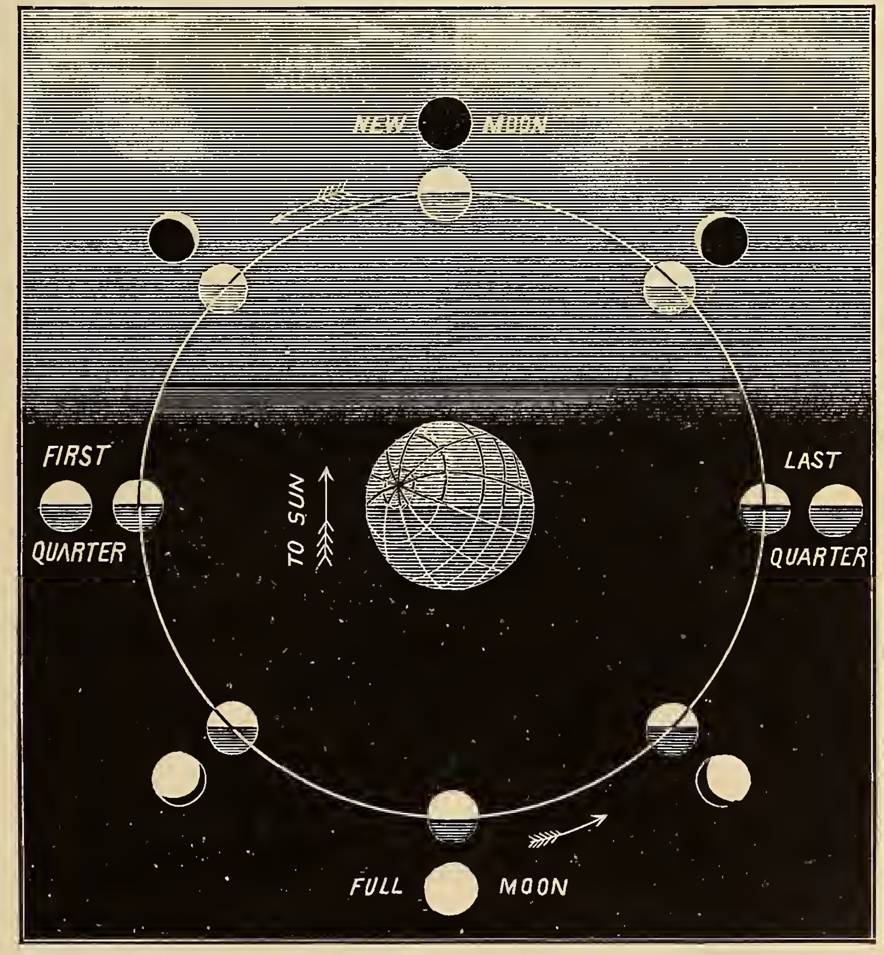


FIG. 70. — Explanation of the Moon's Phases

earth and the sun (at new moon) the dark side is then presented directly towards us, and the moon is entirely invisible. A week later, at the end of the first quarter, half of the illuminated hemisphere is visible, just as it is a week after the full. Between the new moon and the half-moon, during the first and last quarters of the lunation, we see *less* than half of the illuminated portion and then have the "crescent" phase. Between half-moon and the full moon, during the second and third quarters of the lunation, we see *more* than half of the moon's illuminated side and have then what is called the "gibbous" phase.

Fig. 70 (in which the light is supposed to come from a point far above the circle which represents the moon's orbit) shows how the phases are distributed through the month.

The terminator always a semi-ellipse.

205. The Terminator. — The line which separates the dark portion of the disk from the bright is called the *terminator* and is always a *semi-ellipse*, since it is a semicircle viewed obliquely. The illuminated portion of the moon's disk is, therefore, always a figure which is made up of a semicircle plus or minus a semi-ellipse, as shown in Fig. 71 *A*. At new or full moon, however, the semi-ellipse becomes a semicircle. It is sometimes incorrectly attempted to represent the crescent form by a construction like Fig. 71 *B*, in which a smaller circle has a portion cut out of it by an arc of a larger one.

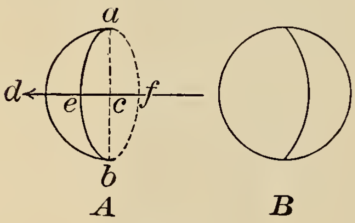


FIG. 71

Direction of the horns of the crescent always away from the sun.

It is to be noticed also that *ab*, the line which joins the "cusps," or points of the crescent, is always perpendicular to a line drawn from the moon to the sun, so that *the horns are always turned away from the sun.* The precise position, therefore, in which they will stand at any time is perfectly predictable and has nothing whatever to do with the weather. Artists are sometimes careless in representing a crescent moon with its horns pointed downwards, which is impossible.

206. Earth-Shine on the Moon. — Near the time of new moon the whole disk is easily visible, the portion on which sunlight does not fall being illuminated by a pale reddish light. This light is *earth-shine*, the earth as seen from the moon being then nearly "full."

Earth-shine on the moon.

Seen from the moon, the earth would show all the phases that the moon does, the earth's phase being in every case exactly supplementary to that of the moon as seen by us at the time. Taking everything into account, the earth-shine by which the moon is illuminated near new moon is probably from fifteen to twenty times as strong as the light of the full moon. The ruddy color is due to the fact that the light sent to the moon from the earth has passed twice through our atmosphere and so has acquired the sunset tinge.

PHYSICAL CHARACTERISTICS OF THE MOON

207. The Moon's Atmosphere. — The moon's atmosphere, if any exists, is extremely rare, probably not producing at the moon's surface a barometric pressure to exceed $\frac{1}{25}$ of an inch of mercury, or $\frac{1}{750}$ of the atmospheric pressure at the earth's surface. The evidence on this point is twofold.

No sensible atmosphere on the moon.

First, *the telescopic appearance*. The parts of the moon near the edge of the disk, or "limb," which, if there were any atmosphere, would be seen through its greatest possible depth, are visible without the least distortion. There is no haze, and all the shadows are perfectly black; there is no evidence of clouds or storms, or of anything like the ordinary phenomena of the terrestrial atmosphere.

No haze, all shadows perfectly black; no clouds or atmospheric phenomena.

Second, *the absence of refraction at the moon's limb, when the moon intervenes between us and any more distant object*. At an eclipse of the sun there is no distortion of the sun's limb where the moon cuts it. When the moon "occults" a star there is no distortion or discoloration of the star disk, but both the

No sensible refraction of rays of light which pass close to the moon's limb.

disappearance and reappearance are practically instantaneous. Moreover, an atmosphere of even slight density, quite insufficient to produce any sensible distortion of the image, would notably diminish the time during which the star would be concealed behind the moon, since the refraction would bend the rays from the star around the edge of the moon so as to render it visible, both after it had really passed behind the limb and before it emerged from it. There are some rather doubtful indications of a very slight effect of this kind, corresponding to what would be produced by an atmosphere about $\frac{1}{1000}$ as dense as our own.

No water on the moon.

208. Water on the Moon's Surface. — Of course, if there is no atmosphere there can be no *liquid* water, since if there were it would immediately evaporate and form an atmosphere of vapor. It is not impossible, however, nor perhaps improbable, that *solid* water, *i.e.*, ice and snow, may exist on parts of the moon's surface at a temperature too low to liberate vapor enough to make an atmosphere observable from the earth.

How came the moon to lose her atmosphere?

Partly, perhaps, by absorption, occlusion, and chemical combination in rocks.

Perhaps by flight.

209. What has become of the Moon's Air and Water? — If the moon ever formed a part of the same mass as the earth, she must once have had both air and water. There are a number of possible, and more or less probable, hypotheses to account for their disappearance: (1) The air and water may have *struck in*, — partly absorbed by porous rocks and partly disposed of in cavities left by volcanic action; partly also, perhaps, by chemical combination as water of crystallization, and by simple occlusion. (2) The atmosphere may have *flown away;* and this is perhaps the most probable hypothesis, though it is quite possible that this cause and the preceding may have coöperated. If the "kinetic" theory of gases is true, no body of small mass, not extremely cold, can permanently retain any considerable atmosphere. A particle leaving the moon with a speed exceeding the "critical velocity" of 1½ miles a second would never return (Sec. 319). If she was ever warm, the molecules of her atmosphere must have been continually acquiring velocities greater than this, and deserting her one by one. (See *Physics*, pp. 270, 271.)

However it came about, it is quite certain that at present no substances that are gaseous or vaporous at low temperatures exist in any considerable quantity on the moon's surface, — at least, not on our side of it.

210. The Moon's Light. — As to *quality*, it is simply sunlight, showing a spectrum identical in every detail with that of light coming directly from the sun itself; and this may be noted incidentally as an evidence of the absence of a lunar atmosphere, which, if it existed in any quantity, would produce markings of its own in the spectrum.

Light of the moon. In quality identical with sunlight.

The *brightness* of full moonlight as compared with sunlight is estimated as about $\frac{1}{600000}$. According to this, if the whole visible hemisphere were packed with full moons, we should receive from it about *one-eighth* part of the light of the sun.

Light of full moon about $\frac{1}{600000}$ of sunlight.

Moonlight is not easy to measure, and different experimenters have found results for the ratio between the light of the full moon and sunlight ranging all the way from $\frac{1}{300000}$ (Bouguer) to $\frac{1}{800000}$ (Wollaston). The value now generally accepted is that determined by Zöllner, viz., $\frac{1}{618000}$.

The half-moon does not give, even approximately, half as much light as the full moon. Near the full the brightness suddenly and greatly increases, probably because at any time except at the full moon the moon's visible surface is more or less darkened by *shadows*.

Sudden increase of brightness near full moon.

The average *albedo*, or reflecting power of the moon's surface, Zöllner states as 0.174; *i.e.*, the moon's surface reflects a little more than one-sixth part of the light that falls upon it.

This corresponds to the reflecting power of a rather light-colored sandstone. There are, however, great differences in the brightness of the different portions of the moon's surface. Some spots are nearly as white as snow or salt, and others as dark as slate.

Moon reflects about one sixth of the light which it receives.

Heat received from the full moon probably about $\frac{1}{150000}$ of that received from the sun.

211. Heat of the Moon. — For a long time it was impossible to detect the moon's heat by observation. Even when concentrated by a large lens, it is too feeble to be shown by the most delicate thermometer. The first sensible evidence of it was obtained by Melloni in 1846, with the newly invented *thermopile*, by a series of observations from the summit of Vesuvius.

With modern apparatus it is easy enough to *perceive* the heat of lunar radiation, but the *measurements* are extremely difficult.

Mainly obscure heat.

A considerable percentage of the lunar heat seems to be heat simply reflected like light, while the rest, perhaps three quarters of the whole, is "obscure heat," *i.e.*, heat which has first been absorbed by the moon's surface and then radiated, like the heat from a brick surface that has been warmed by sunshine. This is shown by the fact that a comparatively thin plate of glass cuts off some eighty-six per cent of the moon's heat.

The total amount of heat radiated by the full moon to the earth is estimated by Lord Rosse at about *one eighty-thousandth* part of that sent us by the sun; but this estimate is probably too high. Prof. C. C. Hutchins in 1888 found it $\frac{1}{185000}$.

Mean temperature of the moon extremely low, but range of temperature probably very great.

212. Temperature of the Moon's Surface. — As to the *temperature* of the moon's surface, it is difficult to affirm much with certainty. On the one hand, the lunar rocks are exposed to the sun's rays in a cloudless sky for fourteen days at a time, so that if they were protected by air like the rocks upon the earth they would certainly become intensely heated. During the long lunar night of fourteen days the temperature must inevitably fall appallingly low, perhaps 200° below zero.

Oscillations of opinion.

There have been great oscillations of opinion on this subject. Some years ago Lord Rosse inferred from his observations that the maximum temperature attained by the moon's surface was not much, if at all, below that of boiling water; but his own later investigations and those of Langley threw great doubt on this conclusion, rather indicating that the temperature never reaches that of melting ice. The latest observations, however — the elaborate work of Very in 1899 — corroborate Lord Rosse's earlier results and show almost conclusively that on the moon's equator at lunar noon the temperature rises very high, falling correspondingly low when night comes on.

Sudden disappearance of lunar heat when immersed in the earth's shadow.

Lord Rosse has also found that during a total eclipse of the moon her heat radiation practically vanishes and does not regain its normal value until some hours after she has left the earth's shadow. This seems to indicate that she loses heat nearly as fast as it is received.

213. Lunar Influences on the Earth. — The moon's *attraction* coöperates with that of the sun in producing the *tides*, to be considered later.

Influences of the moon on the earth: only tidal action and a very slight magnetic disturbance.

There are also certain distinctly ascertained disturbances of *terrestrial magnetism* connected with the approach and recession of the moon at perigee and apogee; and this ends the chapter of *ascertained* lunar influences.

Numerous superstitions for which no evidence can be found.

The multitude of current beliefs as to the controlling influence of the moon's phases and changes upon the weather and the various conditions of life are simply superstitions, mostly unfounded or at least unverified.

It is quite certain that if the moon has any influence at all of the sort imagined it is extremely slight, so slight that it has not yet been demonstrated, though numerous investigations have been made expressly for the purpose of detecting it. It is not certain, for instance, whether it is warmer or not, or less cloudy or not, at the time of full moon.

214. The Moon's Telescopic Appearance and Surface. — Even to the naked eye the moon is a beautiful object, diversified with markings which are associated with numerous popular myths.

The moon as a telescopic object.

In a powerful telescope these markings mostly vanish and are replaced by a multitude of smaller details which make the moon, on the whole, the finest of all telescopic objects, — especially so for instruments of a moderate size (say from 6 to

10 inches in diameter), which generally give a more pleasing view of our satellite than instruments either much larger or much smaller.

An instrument of this size, with magnifying powers between 250 and 500, brings the moon optically within a distance ranging from 1000 to 500 miles; and since an object half a mile in diameter on the moon subtends an angle of about $0''.43$, it would be distinctly visible. A long line, or streak, even less than a quarter of a mile across can probably be seen. With larger telescopes the power can now and then be carried very much higher, and correspondingly smaller details made out, *when the seeing is at its best*, not otherwise. The right-hand illustration opposite gives an excellent idea of the moon's appearance with a moderate magnifying power of about 100.

Best time to look at the moon.

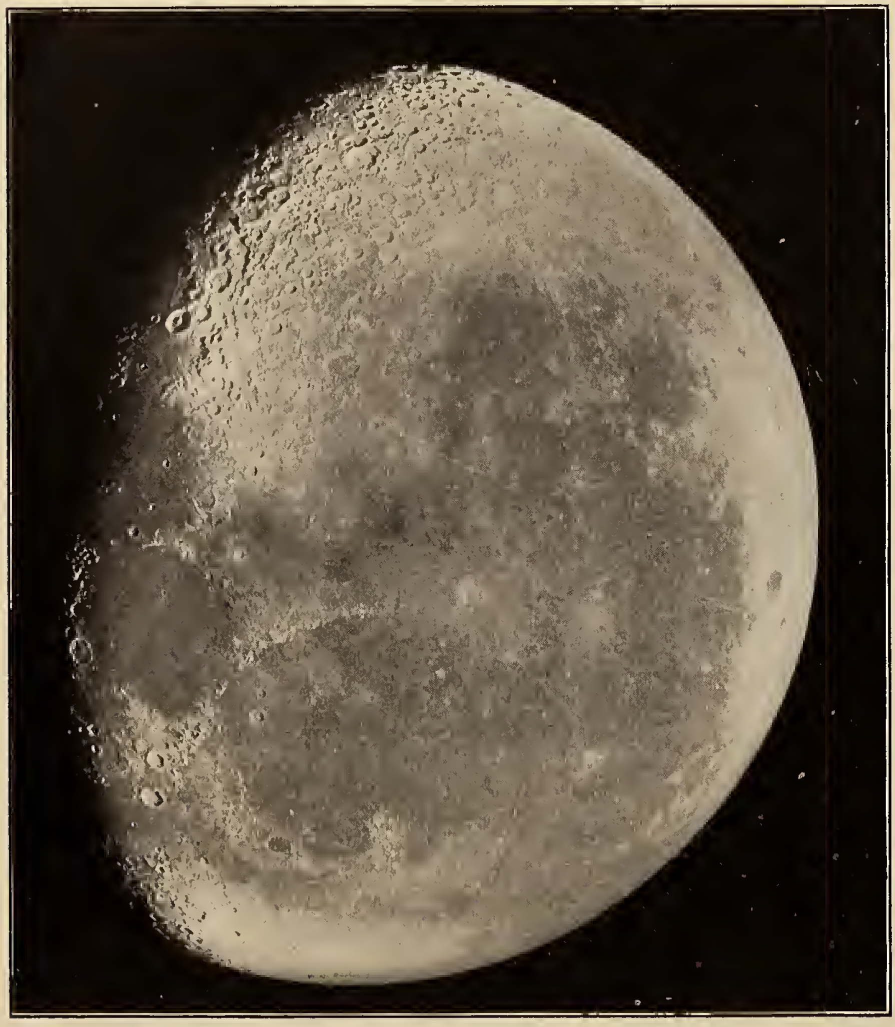
For most purposes the best time to look at the moon is when it is between six and ten days old. At the time of full moon few objects on the surface are well seen, as there are then no shadows to give relief.

It is evident that while with the telescope we should be able to see such objects as lakes, rivers, forests, and great cities, if they existed on the moon, it would be hopeless to expect to distinguish any of the minor indications of life, such as buildings or roads.

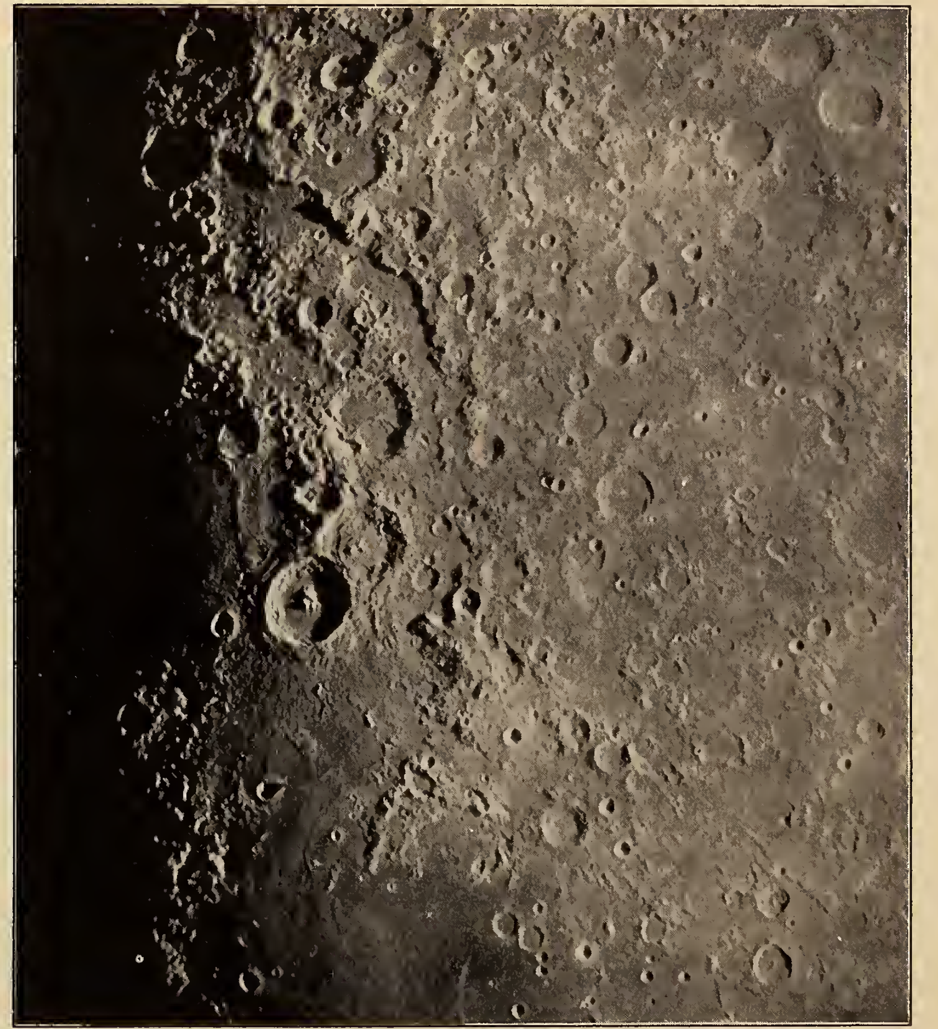
The moon's surface very much broken by a few mountain ranges and numerous craters.

215. The Moon's Surface Structure. — The moon's surface for the most part is extremely broken. With us the mountains are mostly in long ranges, like the Andes and Himalayas. On the moon the ranges are few in number; but, on the other hand, the surface is pitted all over with great *craters*, which resemble very closely the volcanic craters on the earth's surface, though on an immensely greater scale. The largest terrestrial craters do not exceed 6 or 7 miles in diameter; many of those on the moon are 50 or 60 miles across, and some have a diameter of more than 100 miles, while smaller ones from 5 to 20 miles in diameter are counted by the hundred.

Dimensions of craters.



Moon in Last Quarter. Photographed at Kenwood Observatory, 1892



Enlargement of Part of Photograph by Ritchey at Yerkes Observatory, 1900

The normal lunar crater.

The normal lunar crater (Fig. 72) is nearly circular, surrounded by a ring of mountains which rise anywhere from 1000 to 20000 feet above the surrounding country. The floor within the ring may be either above or below the outside level; some craters are deep, and some filled nearly to the brim. In a few cases the surrounding mountain ring is entirely absent, and the crater is a mere hole in the plain. Frequently in the center of the crater there rises a group of peaks, which attain about the same elevation as the encircling ring, and these central peaks sometimes show holes or craters in their summits. Fig. 73 is from a drawing by Nasmyth of the crater Gassendi, which is on the southeast quadrant of the moon's surface, and comes into view about three or four days before full moon. It is 58 miles in diameter and about 8000 feet deep. Fig. 74 is also from one of Nasmyth's drawings and is a fine

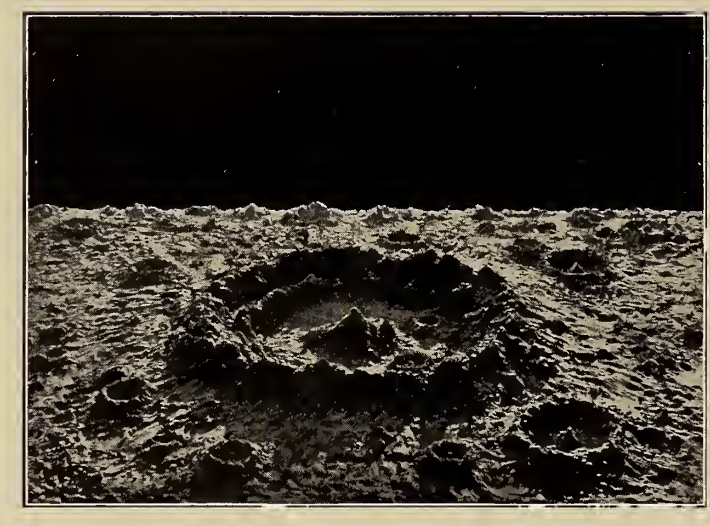


Fig. 72. — Normal Lunar Crater

Gassendi.

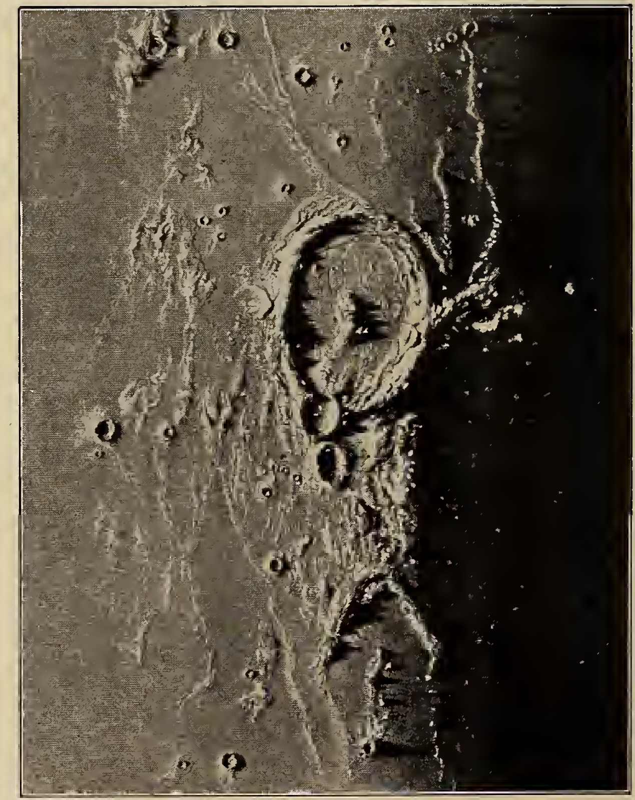


Fig. 73. — Gassendi

Copernicus.

representation of Copernicus, a crater not quite so large or deep as Gassendi, but very interesting on account of the number of surrounding ridges and the manner in which the neighboring region is thickly sown with craterlets and holes. It is on the terminator a day or two after the half-moon.

Theophilus.

In the enlarged photograph of a portion of the moon's surface on page 187 the great crater at the left is Theophilus, 64 miles in diameter and nearly 19000 feet deep.

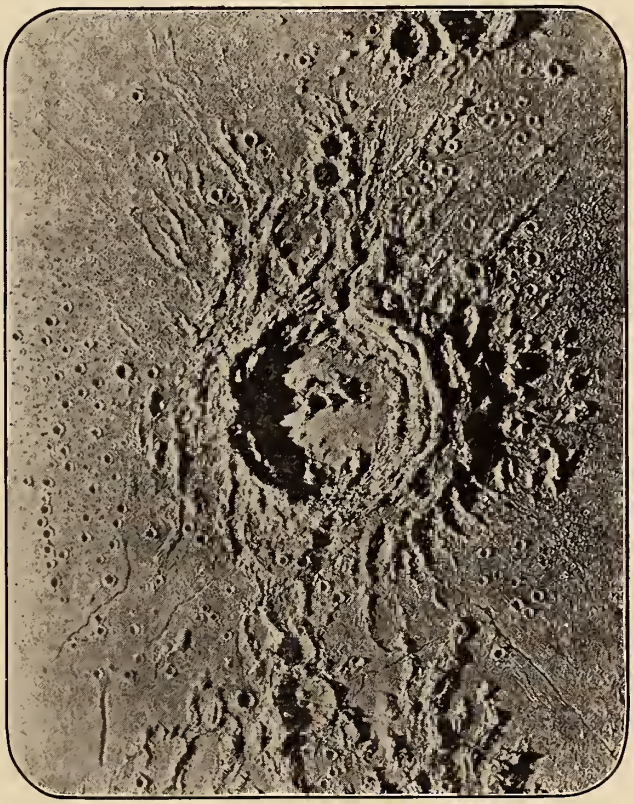


Fig. 74. — Copernicus

The youngest craters usually the deepest.

On certain portions of the moon these craters stand very thickly; older craters have been encroached upon, or more or less completely obliterated, by the newer, so that the whole surface is a chaos of which the counterpart is hardly to be found on the earth, even in the roughest portions of the Alps. This is especially the case near the moon's south pole. It is noticeable, also, that as on the earth the youngest mountains are generally the highest, so on the moon the newer craters are generally deeper and more precipitous than the older.

Measurement of elevations on the moon.

The *height of a lunar mountain* or depth of a crater can be measured with considerable accuracy by means of its shadow, or, in the case of a mountain, by the measured distance between its summit and the terminator at the time when the top first catches the light, looking like a star quite detached from the bright part of the moon, as seen in Fig. 73.

Lunar craters are probably of volcanic origin, but the explanation is not free from difficulty.

216. The striking resemblance of these formations to terrestrial volcanic structures, like those exemplified by Vesuvius and others, makes it natural to assume that they had a similar origin. This, however, is not absolutely certain, for there are considerable difficulties in the way, especially in the case of what are called the great "Bulwark Plains." These are so extensive that a person standing in the center could not see even the summit of the surrounding ring at any point; and yet there is no line of discrimination between them and the smaller craters — the series is continuous. Moreover, on the earth volcanoes necessarily require the action of air and water, which do not *at present* exist on the moon. It is obvious, therefore, that if these lunar craters are the result of volcanic eruptions, they must be, so to speak, "fossil" formations, for it is quite certain that there is *absolutely no evidence of present volcanic activity.*

No volcanic action at present evident on the moon.

217. Other Lunar Formations. — The craters and mountains are not the only interesting formations on the moon's surface. There are many deep, narrow, crooked valleys that go by the name of "rills," some of which may once have been water-courses. Fig. 74 shows several of them. Then there are numerous straight "clefts," half a mile or so wide and of unknown depth, running in some cases several hundred miles, straight through mountain and valley, without any apparent regard for the accidents of the surface: they seem to be deep cracks in the crust of our satellite. Most curious of all are the light-colored streaks, or "rays," which radiate from certain of the craters, extending in some cases a distance of many hundred miles. These are usually from 5 to 10 miles wide and neither elevated nor depressed to any considerable extent with reference to the general surface. Like the clefts, they pass across valley and mountain, and sometimes through craters, without any change in width or color. They have been doubtfully explained as a staining of the surface by vapors ascending from rifts too narrow to be visible.

Rills and clefts.

Streaks, or rays.

The most remarkable of these "ray systems" is the one connected with the great crater Tycho, not very far from the moon's south pole, well shown in the (nearly) full-moon photograph on page 187. The rays are not very conspicuous until within a few days of full moon, but at that time they and the crater from which they diverge constitute by far the most striking feature of the whole lunar surface.

Ray system of Tycho.

218. Lunar Maps. — A number of maps of the moon have been constructed by different observers. The most extensive is that by Schmidt of Athens, on a scale 7 feet in diameter, published by the Prussian government in 1878. Of the smaller maps available for ordinary lunar observation, perhaps the best is that given in Webb's *Celestial Objects for Common Telescopes.* Two new photographic, large-scale, lunar maps have lately been published from negatives made at the Lick and Paris observatories. Our maps of the visible part of the moon are on the whole as complete and accurate as our maps of the earth, taking into account the polar regions and the interior of the continents of Asia and Africa.

Lunar maps.

219. Lunar Nomenclature. — The great plains upon the moon's surface were called by Galileo "oceans" or "seas" (*maria*), for he supposed that these grayish surfaces, which are visible to the naked eye and conspicuous in a small telescope, though not with a large one, were covered with water.

Lunar nomenclature.

The ten *mountain ranges* on the moon are mostly named after terrestrial mountains, as Caucasus, Alps, Apennines, though two or three bear the names of astronomers, like Leibnitz, Doerfel, etc.

The conspicuous *craters* bear the names of eminent ancient and medieval astronomers and philosophers, as Plato, Archimedes, Tycho, Copernicus, Kepler, and Gassendi; while hundreds of smaller and less conspicuous formations bear the names of more modern astronomers.

This system of nomenclature seems to have originated with Riccioli, who in 1650 made the first map of the moon.

220. Changes on the Moon. — It is certain that there are no *conspicuous* changes: there are no such transformations as would be presented by the *earth* viewed telescopically, — no clouds, no storms, no snow of winter, and no spread of vegetation in the spring. At the same time it is confidently maintained by some observers that here and there alterations

Question of changes on the moon's surface. None that are obvious, but some probable.

do take place in the details of the lunar surface, while others as stoutly dispute it.

The difficulty in settling the question arises from the great changes in the appearance of a lunar object under varying illumination. To insure certainty in such delicate observations, comparisons must be made between the appearance of the object in question, as seen at *precisely the same phase of the moon*, with telescopes (and eyes too) of equal power, and under substantially the same conditions in other respects, such as the height of the moon above the horizon and the clearness and

Difficulty of the problem.

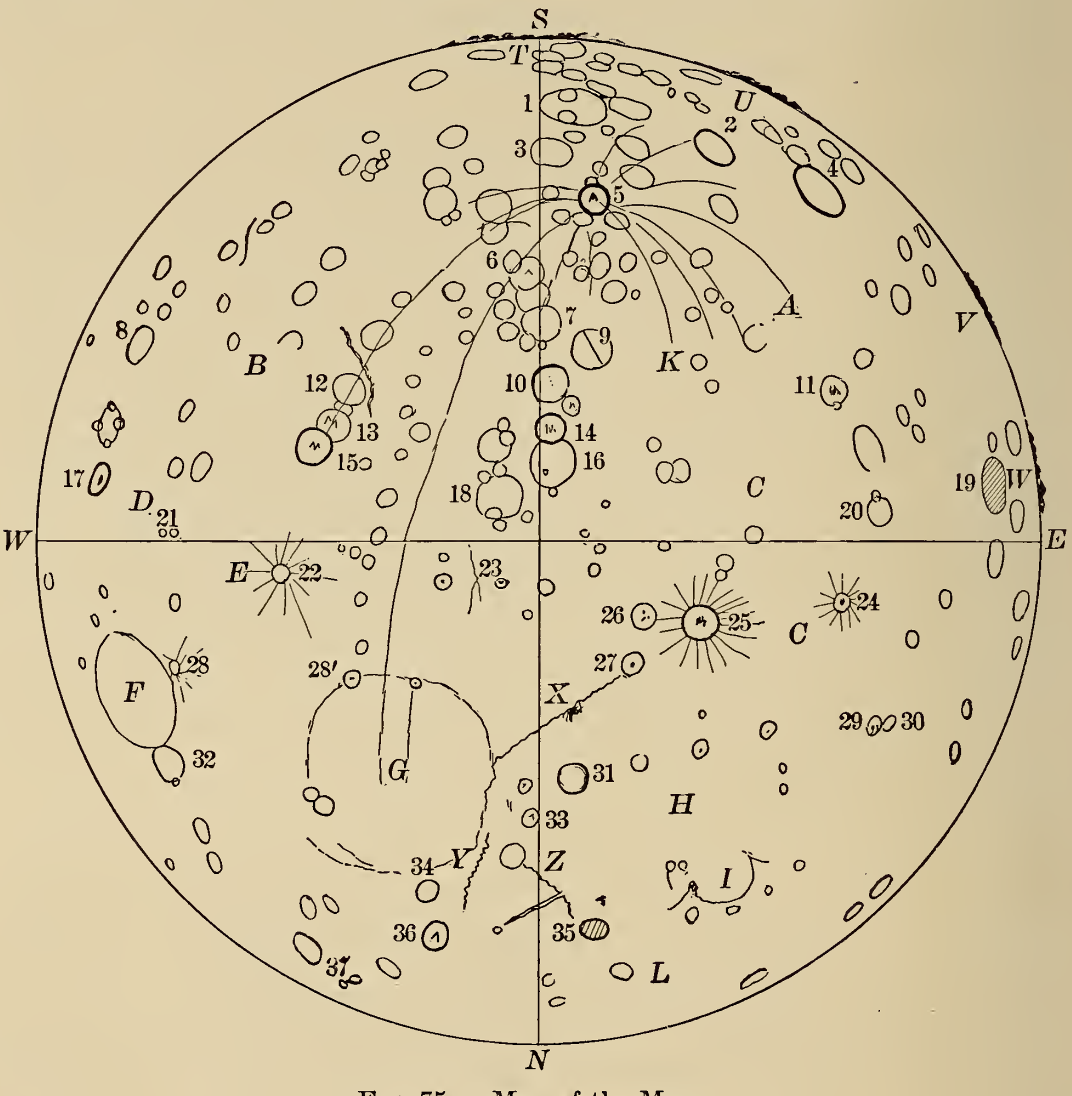

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Fig. 75. — Map of the Moon
Reduced from Neison

steadiness of the air. It is, of course, very difficult to secure such identity of conditions. (For an account of certain supposed changes, see Webb's *Celestial Objects for Common Telescopes.*)

Skeleton map of the moon.

221. Fig. 75 is reduced from a skeleton map of the moon by Neison and, though not large enough to exhibit much detail, will enable a student with a small telescope to identify the principal objects by the help of the key.

KEY TO THE PRINCIPAL OBJECTS INDICATED IN FIG. 75

A. Mare Humorum.
B. Mare Nectaris.
C. Oceanus Procellarum.
D. Mare Fecunditatis.
E. Mare Tranquilitatis.
F. Mare Crisium.
G. Mare Serenitatis.
H. Mare Imbrium.
I. Sinus Iridum.
K. Mare Nubium.
L. Mare Frigoris.
T. Leibnitz Mountains.
U. Doerfel Mountains.
V. Rook Mountains.
W. D'Alembert Mountains.
X. Apennines.
Y. Caucasus.
Z. Alps.

1. Clavius.
2. Schiller.
3. Maginus.
4. Schickard.
5. Tycho.
6. Walther.
7. Purbach.
8. Petavius.
9. "The Railway."
10. Arzachel.
11. Gassendi.
12. Catherina.
13. Cyrillus.
14. Alphonsus.
15. Theophilus.
16. Ptolemy.
17. Langrenus.
18. Hipparchus.
19. Grimaldi.
20. Flamsteed.
21. Messier.
22. Maskelyne.
23. Triesnecker.
24. Kepler.
25. Copernicus.
26. Stadius.
27. Eratosthenes.
28. Proclus.
28′. Pliny.
29. Aristarchus.
30. Herodotus.
31. Archimedes.
32. Cleomedes.
33. Aristillus.
34. Eudoxus.
35. Plato.
36. Aristotle.
37. Endymion.

Lunar photography.

222. Lunar Photography. — It is probable that the question of changes upon the moon's surface will in the end be authoritatively decided by means of photography. The earliest success in lunar photography was that of Bond of Cambridge, U.S., in

Originated in this country.

1850, using the old daguerreotype process. This was followed by the work of De la Rue in England, and by Dr. Henry Draper and Mr. Lewis M. Rutherfurd in this country. Until very recently Mr. Rutherfurd's pictures have remained absolutely unrivaled; but since 1890 there has been a great advance. At various places, especially at Cambridge and at the Lick and Yerkes observatories in this country, and at Paris, most admirable photographs have been made, which bear enlargement well and show *almost* as much detail as can be seen with the telescope, — not quite, however.

Latest successes.

The half-tone engraving of the entire moon on page 187 is slightly enlarged from a photograph made by Professor Hale at his Kenwood Observatory (Chicago) in 1892 with a 13½-inch *photographic* object-glass. The other covers a small portion of the moon's surface on a much larger scale, including the great crater Theophilus with its neighbors Cyrillus and Catherina. It is enlarged from a magnificent photograph made in 1900 by Ritchey of the Yerkes Observatory with the *non-photographic* object-glass of the great 40-inch telescope, a yellowish color screen being used in front of the sensitive plate to cut off the red, violet, and ultra-violet rays, according to the method introduced by Professor Hale. The original negative is certainly not surpassed by any thus far obtained with photographic lenses or reflectors.

CHAPTER VIII

THE SUN

Its Distance, Dimensions, Mass, and Density — Its Rotation and Equatorial Acceleration — Methods of studying its Surface — The Photosphere — Sun-Spots — Their Nature, Dimensions, Development, and Motions — Their Distribution and Periodicity — Sun-Spot Theories

THE sun is the nearest of the *stars*, — a hot self-luminous globe, enormous as compared with the earth and moon, though probably only of medium size compared with other stars; but to the earth and the other planets which circle around it it is the most magnificent and important of all the heavenly bodies. Its attraction controls their motions, and its rays supply the energy which maintains every form of activity upon their surfaces.

The sun's supremacy.

223. The Distance of the Sun; the Astronomical Unit. — The problem of finding accurately the *sun's distance* is one of the most important and difficult presented by astronomy, — important because this distance, *i.e.*, the radius of the earth's orbit, is the fundamental *Astronomical Unit* to which all measurements of celestial distance are referred; difficult because the measurements which determine it are so delicate that any minute error of observation is enormously magnified in the result.

Importance and difficulty of determining the distance of the sun, the fundamental astronomical unit.

Without a knowledge of the sun's distance we cannot form any idea of its real dimensions, mass, and density, and the tremendous scale of solar phenomena.

We have already given one method for finding this distance, depending upon the experimental determination of the *velocity of light*, combined with the observed *constant of aberration*, and we postpone until later the consideration of the methods by which we measure the sun's *parallax* (Sec. 79) and so determine

Already determined by aberration of light.

his distance in terms of the radius of the earth. From the combination of all the material now available the sun's mean distance comes out very closely ***92 900000*** miles (149 500000 kilometers), the *horizontal parallax* being 8″.80 ± 0″.02.

Its distance 92 900000 miles.

The distance is still *uncertain* by perhaps 100000 miles, and because of the eccentricity of the earth's orbit it is *variable* to the extent of about 3 000000 miles, being the least on January 1 and greatest early in July.

Earth's orbital velocity.

The *orbital velocity* of the earth, found by dividing the circumference of the orbit by the number of seconds in a year, is 18½ miles a second, as already determined by aberration (Sec. 173). (Compare this velocity with that of a cannon-shot — seldom exceeding 2500 feet per second.)

Illustrations.

Perhaps one of the simplest illustrations of the distance of the sun is that such a shot would require over six years to reach the sun, traveling without change of speed. A railroad train running at 60 miles an hour, without stop or slackening, would require 175 years, and the fare one way, at two cents a mile, would be $1 860000. A bicyclist traveling 100 miles a day would be nearly 2550 years in making the journey, and if he had started from the sun in the year A.D. 1, he would by this time have covered only about three quarters of the distance. *Light* makes the journey in 499 seconds.

224. Dimensions of the Sun. — The sun's *mean apparent diameter* is 32′ 4″ ± 2″. Since at the distance of the sun one second equals 450.36 miles (92 900000 ÷ 206264.8), its real diameter is 866500 miles, or 109½ times that of the earth. It is quite possible that this diameter is *variable* to the extent of a few hundred miles, since the sun is not solid.

The sun's diameter 109.5 times that of the earth.

If we suppose the sun to be hollowed out, and the earth placed at the center, the sun's surface would be 433000 miles away. Now, since the distance of the moon from the earth is about 239000 miles, she would be only a little more than halfway out from the earth to the inner surface of the hollow globe, which would thus form a very good sky background for the

Illustration: radius of the sun compared with the distance of the moon.

study of the lunar motions. Fig. 76 illustrates the size of the sun, and of such objects upon it as the sun-spots and "prominences," compared with the size of the earth and the moon's orbit.

If we represent the sun by a globe 2 feet in diameter, the earth on the same scale would be 0.22 of an inch in diameter, the size of a very small pea, at a distance from the sun of just about 220 feet; *and the nearest star, still on the same scale, would be 8000 miles away at the antipodes.*

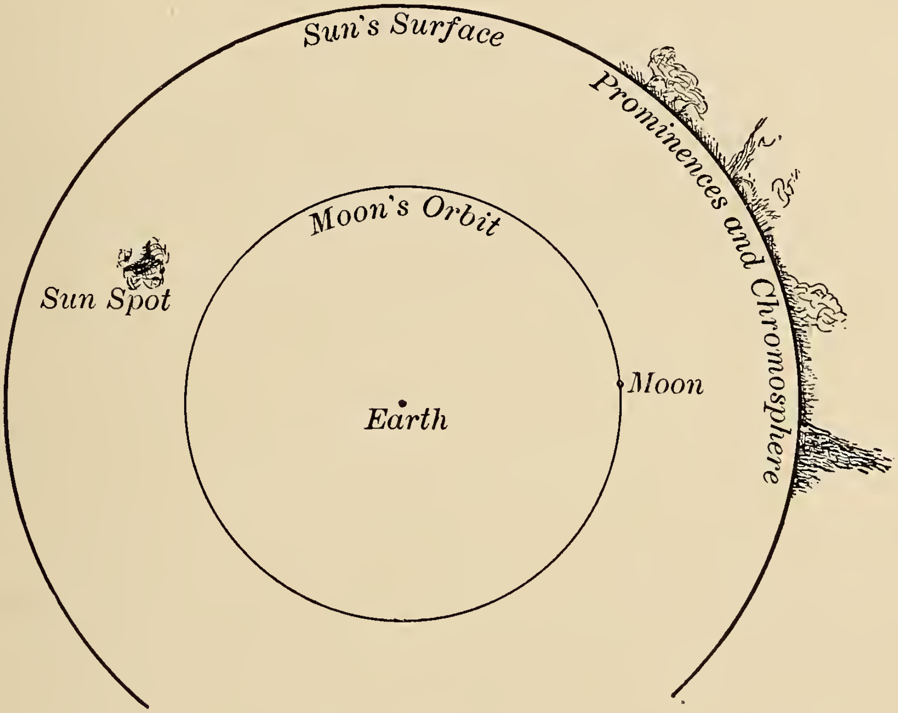

Fig. 76. — Dimensions of the Sun compared with the Moon's Orbit

Since the *surfaces* of globes are proportional to the *squares* of their radii, the surface of the sun exceeds that of the earth in the ratio of $109.5^2:1$; *i.e.*, the area of its surface is about 12000 times the surface of the earth.

Surface area of the sun 12000 times as great as that of the earth. Bulk 1 300000 times as great as that of the earth.

The *volumes* of spheres are proportional to the cubes of their radii. Hence, the sun's *volume* (or *bulk*) *is* 109.5^3, *or 1 300000, times that of the earth.*

225. The Sun's Mass. — *This is about* ***333000*** *times that of the earth.* For our purpose the most convenient way of reaching

Mass of the sun 333000 times that of the earth.

this result (for another method, see Sec. 380) is by comparing g^1, the earth's attraction for bodies at her surface, with the sun's attraction for the earth as measured by our orbital motion. Call this attraction f^1, and let R be the radius of the earth's orbit, r the radius of the earth, S the mass of the sun, and E that of the earth. Then, from the law of gravitation (Sec. 146), we have[1]

$$f : g :: \frac{S}{R^2} : \frac{E}{r^2},$$

How determined.

or

$$S = E\left(\frac{f}{g}\right) \times \left(\frac{R}{r}\right)^2.$$

But, from the law of central force, $f = \frac{V^2}{R}$, in which V is $18\frac{1}{2}$ miles a second and R 92 900000 miles. Reducing R and V to inches, and making the computation, we find $f = 0.2332$ inches, and since g (corrected for centrifugal force) is 386.8 inches, $\frac{f}{g} = \frac{1}{1658.7}$. Also, $\frac{R}{r} = \frac{92\,900000}{3958.8} = 23467$, the square of which is 550 686000. Finally, therefore, $S = E \times \frac{550\,686000}{1658.7} = 332400\ E$. But the last three figures are uncertain.

Earth's orbit departs from a straight line only a little more than one ninth of an inch in 18.5 miles.

226. The Curvature of the Earth's Orbit and Total Force of Sun's Attraction. — The distance which the earth would fall towards the sun in a second if its orbital motion were arrested is $\frac{1}{2} f$, or 0.116 inches, just as $\frac{1}{2} g$, $16\frac{1}{12}$ feet, is the distance a body falls towards the earth in the first second; and this 0.116 inches is the amount by which the earth deviates from a tangent to its orbit in a second. In other words, the earth in traveling 18.5 miles is deflected towards the sun but a little more than *one ninth of an inch.*

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[1] Since the attractions of the sun and earth are here measured by the *accelerations* f *and* g, the proportion would strictly be $f : g = \frac{S + E}{R^2} : \frac{E + m}{r^2}$, where m is the small body by the fall of which g is determined. But E and m are so small as compared with S and E, respectively, that they may be omitted without sensible error, as in the proportion given.

It would seem that a feeble force only would be needed to produce so slight a deviation from a straight line. But since the sun's attraction is $\frac{1}{1659}\,g$, a mass of 1659 pounds on the earth is attracted towards the sun with a force of about one pound.[1]

It follows, therefore, that the total attraction between the earth and sun amounts to the amazing pull of *3 600000 millions of millions of tons* ($\frac{1}{1659}$ of the earth's mass, which is 6×10^{21} tons). This would be the breaking strain of a steel rod more than 3000 miles in diameter, — a force inexplicably exerted through, or transmitted by, apparently empty space, in which the planets move without sensible resistance.

Total attraction between sun and earth equal to the breaking strain of a steel rod 3000 miles in diameter.

227. Superficial Gravity, or Gravity at the Surface of the Sun. — This is found by dividing the sun's *mass* by the *square of its radius* (both compared with the earth), *i.e.*, $\frac{332000}{(109.52)^2}$, which gives *27.6*. A mass of ten pounds would weigh 276 pounds on the sun, and a person who weighs 150 pounds here would weigh over *two tons* there. Locomotion would be impossible. A body would fall 444 feet in the first second, and a pendulum which here vibrates in a second would vibrate in less than one fifth of a second there. But (putting temperature out of consideration) *a watch* would go no faster there than here, since neither the *inertia* of the balance-wheel nor the *elasticity* of the spring would be affected by the increased gravity.

Gravity on sun's surface nearly 28 times as great as on the earth.

228. The Sun's Density. — Its mean *density* as compared with that of the earth may be found by simply dividing its *mass* by its *volume* (both as compared with the earth); *i.e.*, the sun's density equals $332000 \div 1\,300000 = 0.255$, — a little more than *one quarter* of the earth's density.

Mean density of the sun only about one fourth of the earth's density, or 1.4 times that of water.

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[1] This does not imply, and it is not true, that when the sun is overhead a 166-pound man weighs one tenth of a pound less (by a spring-balance) than when the sun is rising. (Why not?) The difference is really only about $\frac{1}{20\,000000}$ of his weight (Sec. 333).

To get its *specific gravity — its density compared with water* — we must multiply this by the earth's mean specific gravity, 5.53, which gives 1.41. That is, *the sun's mean density is less than one and one-half times that of water.*

Its significance as indicating the sun's tremendous heat.

This is a most remarkable and significant fact, considering the sun's tremendous force of gravity and that a considerable portion of its mass is composed of *metals*, as proved by the spectroscope. The obvious, and only possible, explanation is that the temperature of the sun is such that its materials are almost wholly in the condition of vapor, — not solid or even liquid.

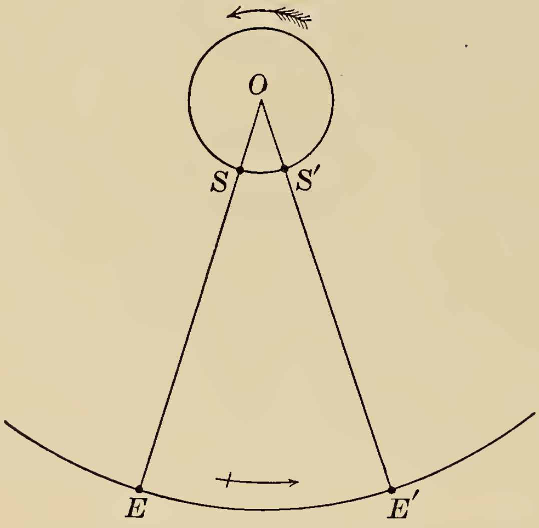

FIG. 77. — Synodic and Sidereal Revolution of the Sun

Axial rotation of the sun. Average apparent or synodic rotation period 27.25 days.

229. The Sun's Rotation. — This is made evident by the behavior of the dark sun-spots which cross the sun's disk from east to west. The times in which they make their circuits differ slightly, but the average, as seen from the earth, is 27.25 days. This, however, is not the *true or sidereal* time of the sun's rotation, but the *synodic*, as is evident from Fig. 77. Suppose that an observer on the earth at E sees a spot on the center of the sun's disk at S; while the sun rotates the earth will also move forward in its orbit, and when he next sees the spot on the center of the disk he will be at E', the spot having gone around the whole circumference plus the arc SS'.

Equation for determining the sidereal period from the synodic.

The equation by which the true period is deduced from the synodic is the same as in the case of the moon, viz.,

$$\frac{1}{T}-\frac{1}{E}=\frac{1}{S},$$

T being the true *sidereal* period of the sun's rotation, E the length of the year, and S the observed *synodic* rotation. This gives $T = 25.35$ days. In a year the number of sidereal revolutions exceeds that of the synodic by exactly one.

Sidereal period about 25.3 days.

Different observers, however, get slightly different results, because the spots are not fixed in their positions on the sun's surface. Carrington finds 25.38 days and Spoerer 25.23 days.

The paths of the spots across the sun's disk are usually more or less oval, showing that the sun's axis is not perpendicular to the ecliptic, but so inclined that the north pole is tipped a little more than 7° towards the position which the earth occupies on September 7. The inclination of the sun's equator to the plane of the terrestrial *equator* is about 26° 15′; but different investigators get slightly different values.

Inclination of sun's equator to the ecliptic about 7°.

The position of the point in the sky towards which the sun's axis is directed is in right ascension 19^{h}, declination + 63° 48′, very nearly half-way between the bright star α Lyræ and the pole-star. Twice a year, when the earth is in the plane of the sun's equator, the sun-spot paths become straight, — on June 3 and December 5. (See Fig. 78.)

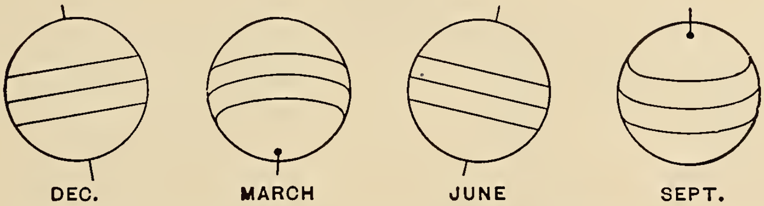

FIG. 78. — Path of Sun-Spots across the Sun's Disk

Position of the sun's pole in the heavens.

Dates when earth passes the plane of the sun's equator.

230. The Equatorial Acceleration. — It was noticed quite early that different spots give different results for the period of rotation, but the researches of Carrington about 1860 first brought out the fact that the differences are systematic, and that *at the solar equator the time of rotation is shorter than on either side of it.* For spots near the sun's equator it is about twenty-five days; in solar latitude 30°, twenty-six and one-half days; and in solar latitude 40°, twenty-seven days. In latitude 45° it is fully two days longer than at the equator; but we are unable to carry the observations to higher latitudes because spots almost never appear beyond the parallels of 45°.

The sun's equatorial acceleration.

Formulæ for sun's rate of rotation in different solar latitudes.

Various formulæ have been proposed to represent the motion; that of Faye, which agrees with the observations as well as any, is $X = 862' - 186' \sin^2 l$, X being the daily motion. Spoerer's formula, as modified by Wolfer, is $X = 8° 55' + 5°.80 \cos l$. This looks very different from Faye's, but gives very nearly the same results for the regions in which spots are observable. None of the formulæ proposed rest on any sound theoretical explanation.

Sun's surface not solid.

Clearly the sun's visible surface is not solid, but permits motions and currents like those of our air and oceans. It might be argued that the spots misrepresent the sun's real rotation, not being fixed upon its surface, floating like our clouds. The faculæ, however, give substantially the same result, and so do Dunér's spectroscopic observations of the shift of lines in the spectrum at the eastern and western limbs. (See Sec. 254.)

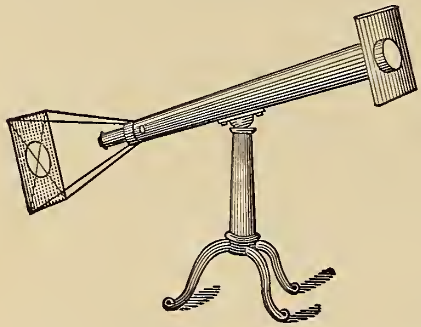

Fig. 79. — Telescope and Screen

Cause of equatorial acceleration not yet certainly ascertained. Probably a survival from past conditions.

Possibly this equatorial acceleration may be, in some way, an effect of the tremendous outpour of heat from the solar surface, as Emden of Munich attempts to show in a paper just published. Other recent investigators, however, have reached the conclusion that it cannot be explained by causes now acting, but is *a lingering survival from the sun's past history*, and destined ultimately to disappear.

Arrangements for study of sun's surface.

231. Arrangements for the Study of the Sun's Surface.—The heat and light of the sun are so intense that we cannot look directly at it with a telescope. A very convenient method of exhibiting the sun to a number of persons at once is simply to attach to a telescope a small frame carrying a screen of white paper at a distance of a foot or more from the eyepiece, as shown in Fig. 79. A screen should also be used at the object end, as shown in the figure, in order to shade the paper upon which the image is formed. When the focus is properly

Telescope and screen.

adjusted a distinct image appears, which shows the sun's principal features very fairly; indeed, with proper precautions, almost as well as the most elaborate apparatus. Still, it is generally more satisfactory to look at the sun directly with a suitable eyepiece.

Solar eyepieces and helioscopes.

With a small telescope, not more than $2\frac{1}{2}$ or 3 inches in diameter, a simple shade glass is often used between the eyepiece and the eye; but the dark glass soon becomes very hot and is apt to crack. With larger instruments it is necessary to use eyepieces specially designed for the purpose, and known as *solar eyepieces*, or *helioscopes*, which reject most of the light coming from the object-glass and permit only a small fraction of it to enter the eye.

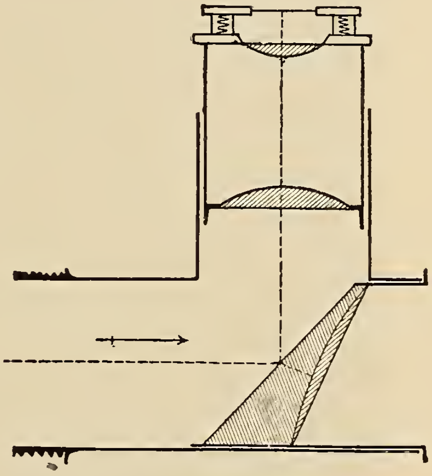

FIG. 80. — Herschel Eyepiece

The simplest of them, and a very good one, is known as Herschel's, in which the sun's rays are reflected at right angles by a plane of unsilvered glass. The reflector is made wedge-shaped, as shown in Fig. 80, in order that the reflection from the back surface may not interfere with the image. Most of the light passes through the glass and out through the open end of the eyepiece, but the reflected light is still too intense for the unprotected eye. Only a thin shade glass is required, however, which does not become very much heated. A more elaborate polarizing helioscope, figured in the *General Astronomy*, is still better.

It is not a good plan to cap the object-glass in order to reduce the light. To cut down the aperture is to sacrifice the definition of delicate details (Sec. 46).

Solar photography; the heliograph.

232. The Heliograph. — In the study of the sun's surface, photography is for some purposes very advantageous and much used. The instrument (called a *heliograph*) must, however, have lenses specially constructed for photography, since a visual object-glass would be nearly worthless for the purpose unless possibly the use of a color screen might make it available (Sec. 222). Arrangements must be made also to secure an extremely rapid exposure, and it is best to use special slow plates. The disk of the sun on the negatives is usually from 2 to 10 inches in diameter, but photographs of small portions of the solar surface are often made on a very much larger scale.

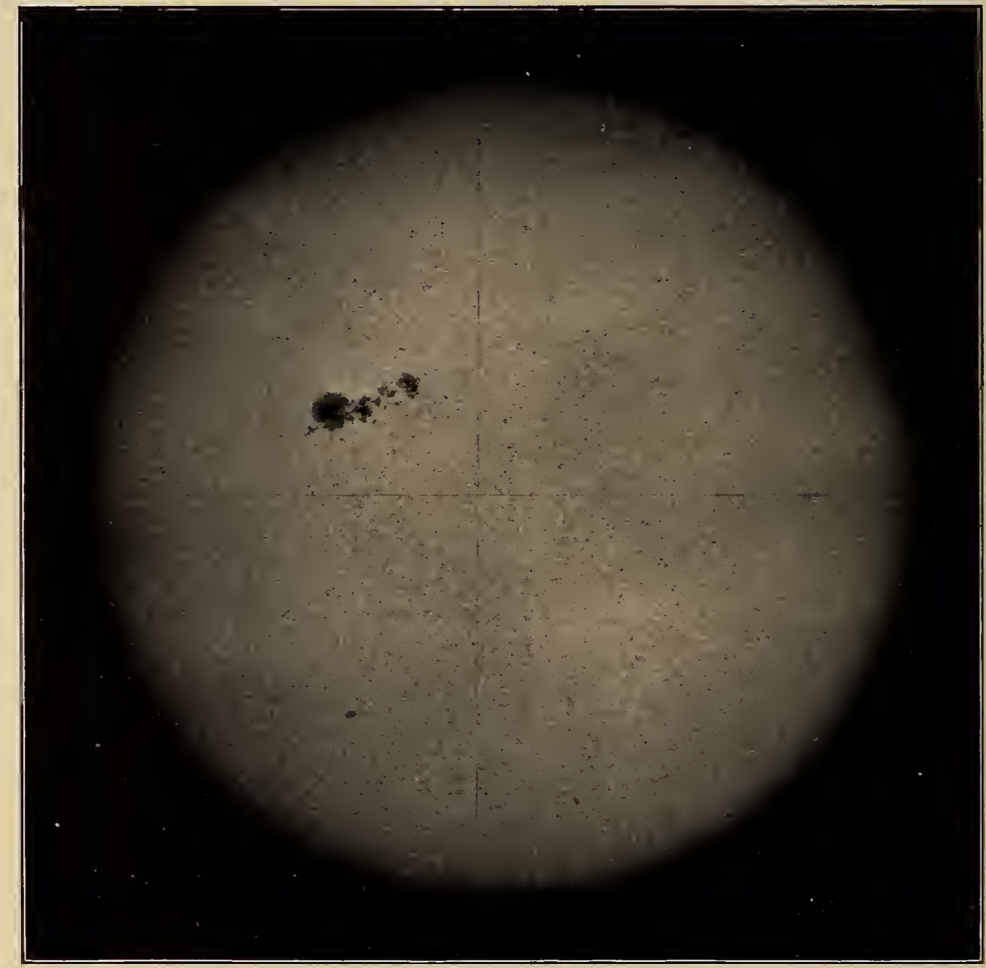

FIG. 81. — Greenwich Photograph of Sun-Spot, Sept. 10, 1898

Fig. 81 is reduced from a 9-inch photograph made with one of the heliographs at Greenwich, on Sept. 10, 1898.

Advantages and disadvantages of photography.

Photographs have the great advantage of freedom from prepossession on the part of the observer, and in an instant of time secure a picture of the whole surface of the sun such as would require hours of labor for a skilful draftsman. On the other hand, they take no advantage of the instants of fine seeing, but offer merely, as some one puts it, "a brutal copy" of whatever happened to appear when the plate was exposed, affected by all the momentary distortions due to atmospheric disturbance.

The photosphere: its appearance.

233. The Photosphere. — The sun's visible surface is called the *Photosphere*, *i.e.*, the "light sphere." When studied with a telescope under favorable conditions, and a rather low power, it appears not smoothly bright, but mottled, looking much like rough drawing-paper. It is considerably darker at the edge than in the center, the difference between the center and limb being especially conspicuous in photographs, as in Fig. 81. With a high power and the best atmospheric conditions, the surface is

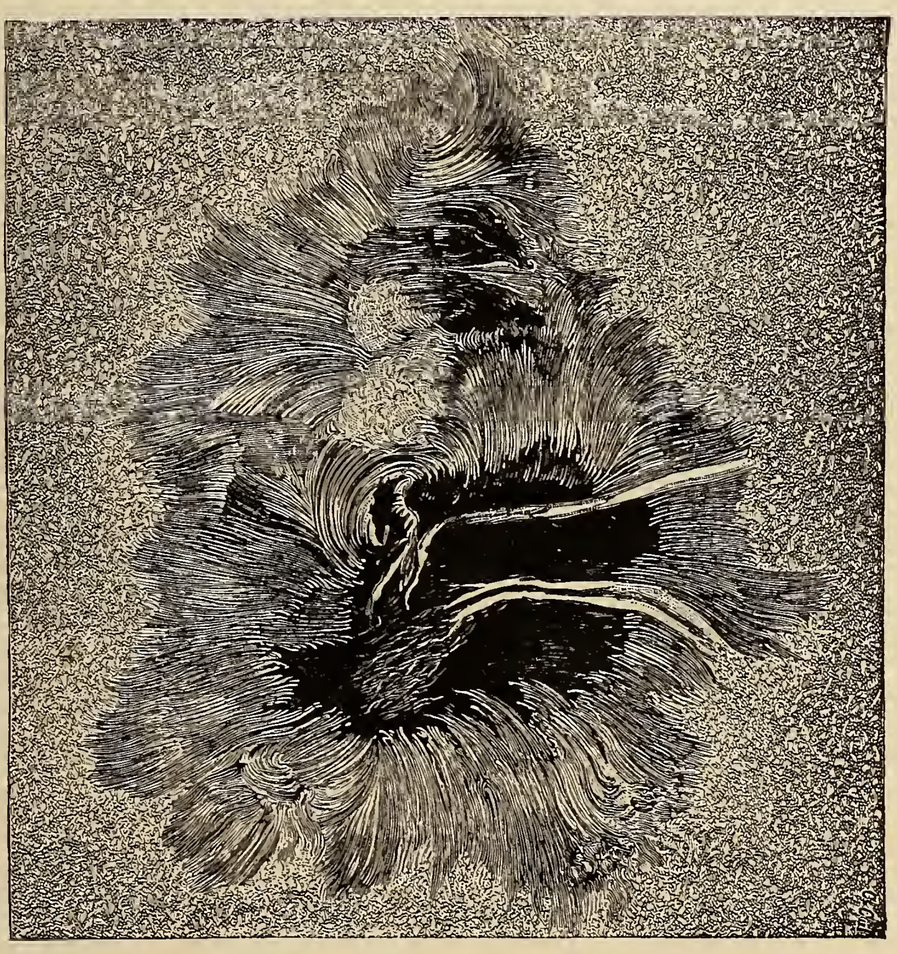

Fig. 82. — Nodules and Granules on the Sun's Surface
After Langley

shown to be made up, as seen in Fig. 82, of a comparatively darkish background sprinkled over with grains, or "nodules," as Herschel calls them, of something much more brilliant, — "like snowflakes on gray cloth," according to Langley. These nodules, or "rice grains," are from 400 to 600 miles across, and when the seeing is best they themselves break up into "granules" still more minute. Generally the nodules are about as broad as they are long, though irregular, but here and there, especially in the neighborhood of the spots, they are drawn out into long streaks, and then are called "filaments," "willow leaves," or "thatch straws."

Nodules, granules, rice grains, filaments, etc.

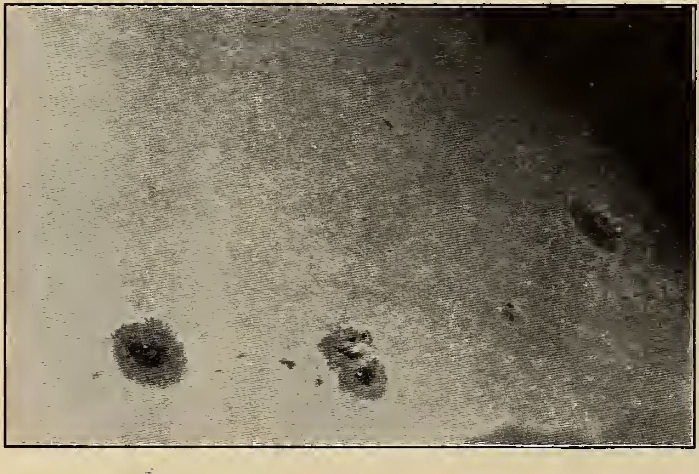

FIG. 83. — Spots and Faculæ
After De la Rue

Certain bright streaks and patches called *Faculæ* are also usually visible here and there upon the sun's surface, and though not very obvious near the center of the disk, they become conspicuous near the limb, especially in the neighborhood of spots, as shown in Fig. 83. Probably they are of the same nature as the rest of the photosphere, only elevated above the general level and intensified in brightness because less affected by the absorption of the overlying gases.

The photosphere probably a stratum of incandescent clouds, acting like a Welsbach mantle.

The photosphere is probably, according to the view now generally accepted, *a sheet of clouds floating in a less luminous atmosphere*, just as the clouds formed by the condensation of water vapor float in our air. It is intensely brilliant, for the same reason that the mantle of a Welsbach burner outshines the gas flame which heats it; *the radiating power* of the solid and liquid particles which compose the clouds is extremely high as compared with that of the gases in which they float. (See also Sec. 278.)

The normal spot.

234. Sun-Spots. — Sun-spots, whenever visible, are the most conspicuous and interesting objects upon the solar surface. The appearance of a normal sun-spot (Fig. 84), fully formed and not yet beginning to break up, is that of a dark central *umbra*, with a fringing *penumbra* composed of converging filaments. The umbra itself is not uniformly dark throughout, but is overlaid with filmy clouds, which usually require a good helioscope to make them visible, but sometimes, though rather infrequently, are conspicuous, as in the figure. Usually, also, within the umbra there are a number of round and very black spots, sometimes called "nucleoli," but often referred to as "Dawes' holes," after the name of their first discoverer. Even the darkest portions of the sun-spot, however, are dark only by contrast. Photometric observations show that the umbra gives about one per cent as much light as a corresponding area of the photosphere, so that the blackest portion of a sun-spot is really more brilliant than a calcium light (Sec. 265).

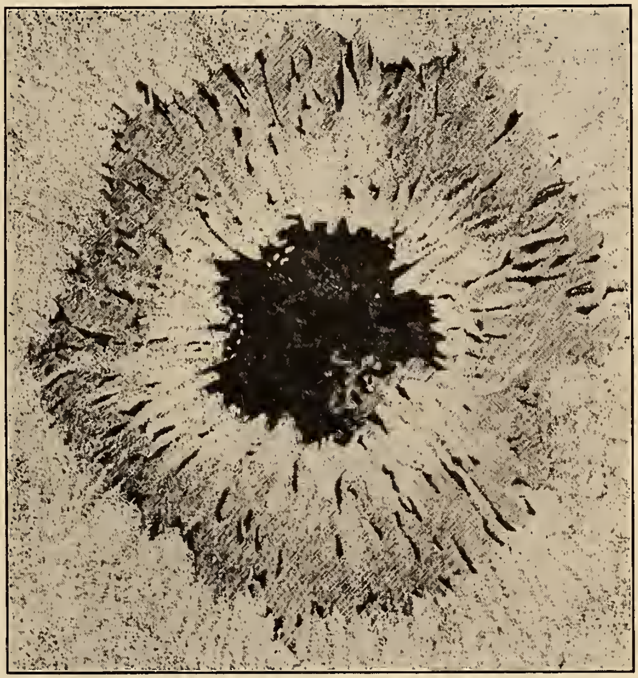

FIG. 84. — Normal Sun-Spot
After Secchi

Penumbra, umbra, nucleoli, etc.

Darkest part of sun-spot as bright as a calcium light.

Very few spots are strictly normal. They are often gathered in groups within a common penumbra, which is partly covered with brilliant "bridges" extending across from the outside

Irregularities in sun-spots.

photosphere. Frequently the umbra is not in the center of the penumbra, or has a penumbra on one side only; and the penumbral filaments, instead of converging regularly towards the nucleus, are often distorted in every conceivable way. Fig. 85 is enlarged from a Greenwich photograph of the spot of September, 1898.

Sun-spots believed to be usually depressions in the photosphere.

235. Nature of Sun-Spots. — Until very recently sun-spots have been believed to be *cavities* in the photosphere filled with gases and vapors, cooler, and therefore darker, than the surrounding region. This theory is founded on the fact that many spots as they cross the sun's disk behave as shown in Fig. 86. Near the limb they look just as they would if they were saucer-shaped hollows, with sloping sides colored gray and the bottom black.

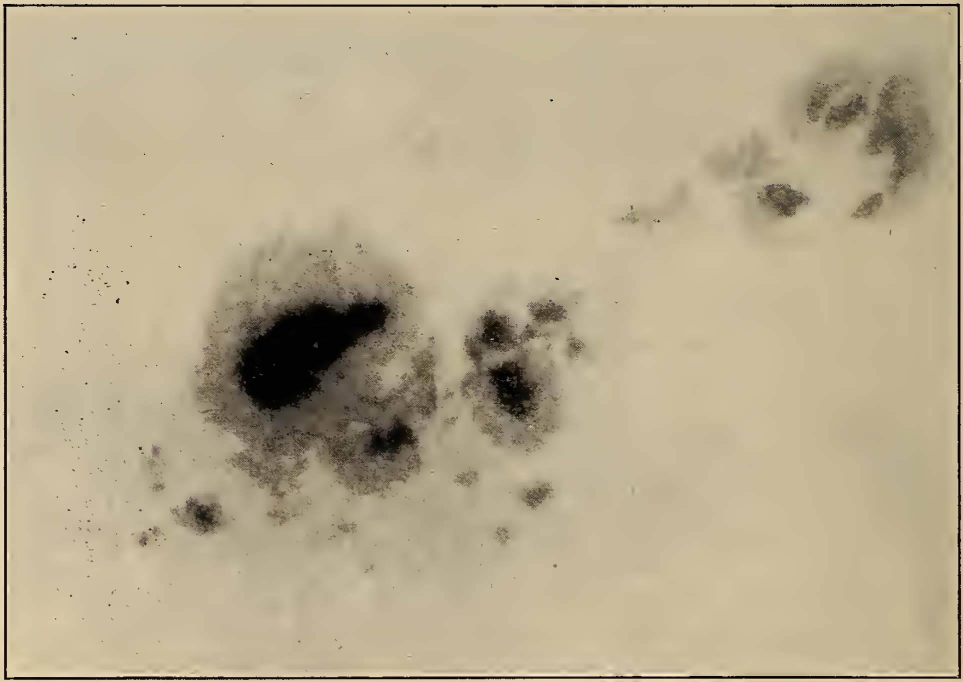

Fig. 85. — Group of Spots from a Greenwich Photograph, Sept. 11, 1898

This theory has, however, of late been seriously called in question; many spots, possibly a majority, as shown by photographs and drawings, fail to present the appearances described. But the principal objection lies in the behavior of spots in respect to their heat radiation. Near the center of the disk the thermopile shows that, as they are darker, so also they emit less heat than the photosphere around them; but near the "limb" (*i.e.*, the *edge* of the sun's disk) the difference becomes less and in some cases is even reversed, a fact most easily explained by supposing the spot to be high above the photosphere.

Evidence that in some cases they are elevated above it.

On the whole, it now seems most probable that different spots lie at very different levels, some low down, forming hollows in the photosphere, but others at a considerable elevation.

The penumbra.

The *penumbra* is usually composed of "thatch straws," or long-drawn-out filaments of photospheric cloud, and these, as has been said, converge in a general way towards the center of the spot, though not infrequently more or less spiral in their course.

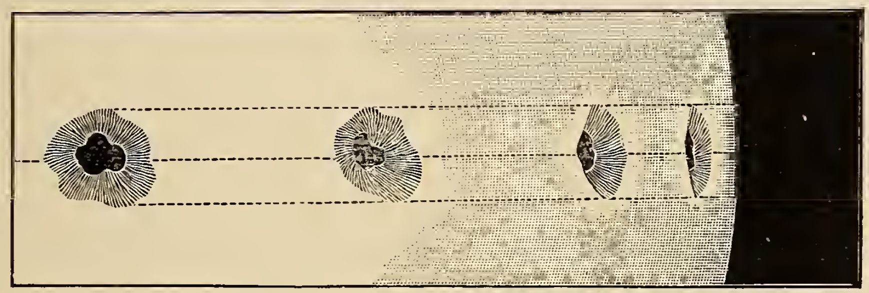

FIG. 86. — Sun-Spots as Cavities

Terminal form of penumbral filaments.

At its inner edge the penumbra, from the convergence of these filaments, is usually brighter than at the outer. The inner ends of the filaments are ordinarily club-formed; but sometimes they are drawn out into fine points, which seem to curve downward into the umbra, like the rushes over a pool of water. The outer edge of the penumbra is usually pretty sharply bounded, and there the penumbra is darkest. In the neighborhood of the spot the surrounding photosphere is usually much disturbed and elevated into faculæ, which ordinarily appear before the spot is formed and continue after it disappears.

Size of sun-spots.

Sometimes visible to the naked eye.

236. Dimensions of Sun-Spots. — The diameter of the *umbra* of a sun-spot varies all the way from 500 miles, in the case of a very small one, to 40000 or 50000 miles, in the case of the largest. The *penumbra* surrounding a group of spots is sometimes 150000 miles across, though that is exceptional. Not infrequently sun-spots are large enough to be visible with the naked eye and can actually be thus seen at sunset or through a fog or by the help of a colored glass.

The Chinese have many records of such objects, but their real discovery dates from 1610, as an immediate consequence of Galileo's invention of the telescope. Fabricius and Scheiner, however, share the honor with him as being independent observers.

Duration of sun-spots and faculæ.

237. Duration, Development, and Changes of Spots. — The duration of sun-spots is very variable; but they are always, astronomically speaking, short-lived phenomena, sometimes lasting for a few days only, though more usually, if of any size, for two or three months. In a single recorded instance (1840–41) a spot persisted for eighteen months.

Tendency of sun-spots to recur at points on sun's surface where spots have disappeared.

The faculæ in the surrounding region generally endure much longer than the spots, and not infrequently a new group of spots breaks out in the same region where one has disappeared some time before, — as if the *local disturbance which caused the spots and faculæ still continued deep below the surface.*

Initial stage in life of a sun-spot.

The *development* of a spot or spot group usually begins, according to Secchi, with the formation of faculæ interspersed with small dark points, or "pores." These pores grow rapidly larger, coalesce, and the neighboring "granules" of the photosphere are transformed into the filaments of the penumbra, converging towards the umbra. Ordinarily this process takes several days, but sometimes only a few hours.

Cortie's account of the later stages.

According to Cortie, the irregular group of scattered incipient spots soon passes into a second stage, stretching out east and west with two predominant spots, one a leader, the other a rear-guard of the flock. The preceding one (in the direction of the sun's rotation) is usually more compact and regular, though the other is sometimes the larger. The leader apparently pushes forward upon the photosphere and so increases the length of the train of "spotlets" between the two principals. Then a third stage follows, as well shown in Fig. 85, Sec. 234. After a time these small spots generally disappear, usually followed pretty soon by the larger spot in the rear, leaving the leader to settle down into a well-formed "normal" spot, which may endure without much change for weeks or months; not infrequently, however, the leader disappears with the rest. Frequently a large spot divides into several, separated by brilliant bridges, and the "segments" fly apart with a speed of sometimes a thousand miles an hour. An active spot is an extremely interesting telescopic object; not infrequently a single day works a complete transformation.

Segmentation of spots

When a large spot vanishes it is most usually by the rapid encroachment of the surrounding atmosphere, which seems, as Secchi expresses it, to "tumble pell-mell into the cavity," if it be one, forming a *facula* to replace the spot.

238. Proper Motions of the Spots. — Spots within 15° of the equator usually drift slightly *towards it*, while those in higher latitudes drift from it; but the drift in latitude is seldom rapid, and exceptions to the rule are numerous.

Drift of sun-spots in latitude.

Active spots as a rule drift pretty steadily forward in the direction of the sun's rotation. The quiet ones move slowly, if at all. Within and close around the spot the motion with reference to the spot is usually inward and downward, so far as it can be observed. Occasionally fragments of the penumbral filaments break off, move towards the center of the spot, and disappear as if swallowed up by a vortex (but there are other possible explanations of their vanishing, such as dissolving into invisible vapor). Sometimes, but rarely, the downward motion in the umbra of a spot is swift enough to be detected by the

Eastward drift of active spots.

Vertical motion: downward in umbra, upward in region just outside the penumbra.

displacement of lines in the spectrum (Sec. 254). On the other hand, around the outer edges of the penumbra there is often a vigorous boiling up from below, evidenced by the eruption of prominences (Sec. 260) and by spectroscopic phenomena within the spot itself. Cyclonic action is often observed; sometimes there are two or more "whirlpools" within the same spot, not infrequently rotating in opposite directions.

Occasional cyclonic action.

239. Distribution of the Spots. — For the most part the spots are confined to two belts between 5° and 40° of north and south latitude (Fig. 87). A few appear near the equator at the time of the sun-spot maximum, and practically none beyond the forty-fifth degree, though in somewhat higher latitudes what Trouvelot calls "veiled spots" sometimes appear, looking like dark masses floating a little below the surface of the photosphere and only dimly seen through the overlying cloud.

Distribution of spots in belts on each side of sun's equator.

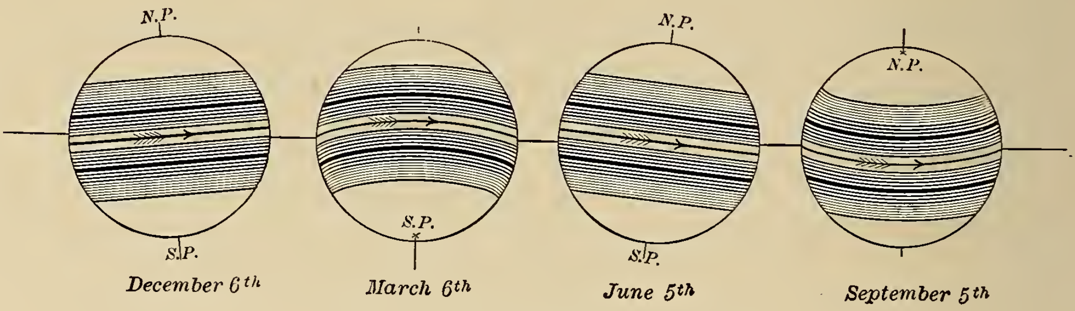

FIG. 87. — Spot Belts and Paths

Generally the numbers are about equal in the two hemispheres, but sometimes there is a marked difference for years. From 1672 to 1704 not a single spot was observed on the northern hemisphere, and the breaking out of a few in 1705 occasioned great surprise and was reported to the French Academy as an anomaly. No reason for such a one-sided inactivity has thus far been discovered.

Periodicity of sun-spots: the eleven-year cycle.

240. Sun-Spot Periodicity. — The number of spots varies greatly in different years and shows an approximately regular *periodicity* of about eleven years. The fact was first discovered by Schwabe of Dessau, in 1843, as the result of his systematic watching of sun-spots for nearly twenty years, and has since been abundantly confirmed.

Wolf of Zurich, who died in 1893, has collected all the observations available and summarized them in the diagram of which Fig. 88 is the reproduction. Fig. 89 continues it to date. The last maximum occurred in 1894; the last minimum near the beginning of 1901.

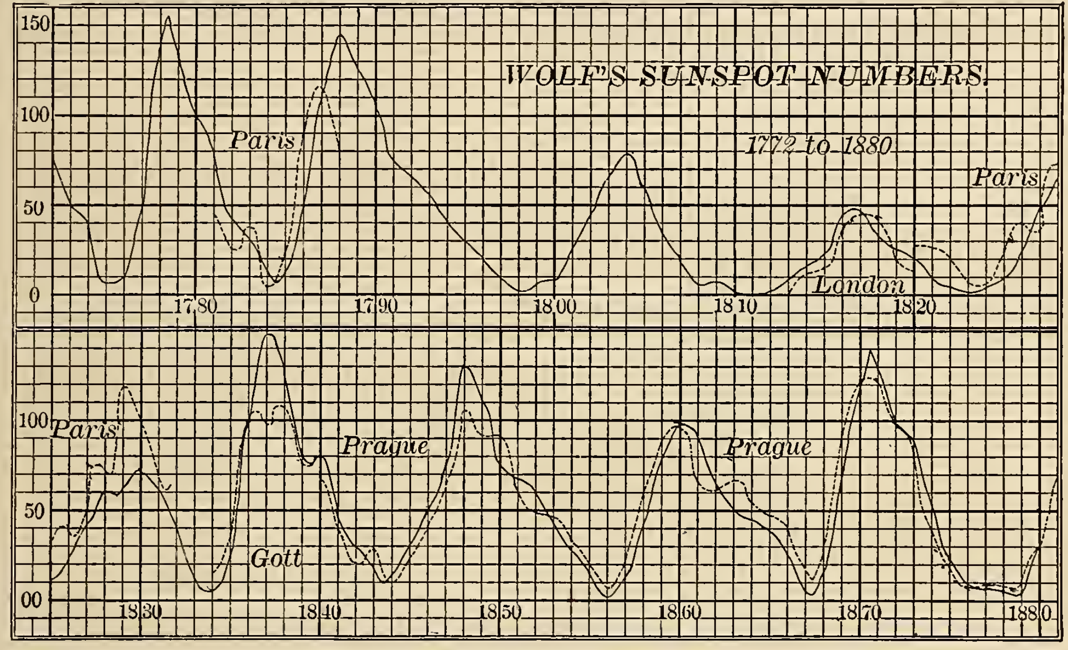

Fig. 88. — Wolf's Sun-Spot Numbers

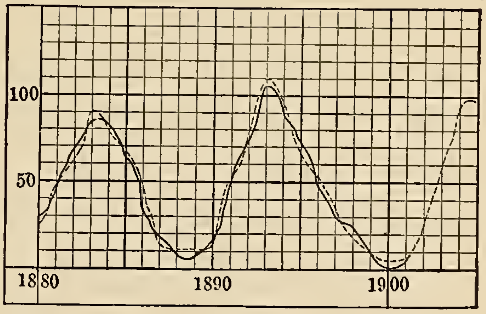

Fig. 89. — Continuation of Fig. 88

During the maximum, the surface of the sun is never free from spots; sometimes a hundred are visible at once. During the minimum, weeks, and months even, pass without a single one. The rise from minimum to maximum is much more rapid than the fall that follows, and evidently from the diagram the maxima are not of equal intensity, nor are their intervals equal. Dr. J. S. Lockyer (son of Sir Norman), from a recent investigation of all data (including magnetic as well as strictly solar) between 1833 and 1901, finds that this variation appears to be itself periodical, with a period of about thirty-five years. But the time covered by the material is hardly sufficient to warrant a sure conclusion.

Cause not yet explained.

Many attempts have been made to connect these phenomena in some way with planetary action, but so far without success, and the general impression has lately been that it is probably due to causes within the sun itself or its atmosphere — a sort of geyser-like action — rather than to anything external.

But a recent thorough investigation by Professor Newcomb puts rather a different phase upon the matter. He finds a regular period of 11.13 years (4.62 + 6.51) as a *uniform cycle underlying the periodic variations of sun-spot activity;* just as the regular period of 365¼ days underlies the more or less variable seasons. He adds, "Whether the cause of this cycle is to be sought in something external to the sun, or within it, . . . we have at present no way of deciding."

Spoerer's law of sun-spot latitudes.

241. Spoerer's Law of Sun-Spot Latitudes. — Speaking broadly, the disturbance which produces the spots of a given period first appears in two belts, about 30° north and south of the sun's equator. These seats of disturbance then move gradually towards the equator, and the spot maximum occurs when their latitude is about 16°, while the disturbance dies out at a latitude of from 5° to 10°, about thirteen or fourteen years after its first outbreak. Two or three years before this disappearance, however, two new zones of disturbance show themselves in latitude 30° to 35°. Thus, at the spot minimum there are usually four well-marked spot belts, one on each side of the equator, due to the expiring disturbance, and two in high latitudes, due to the one just beginning.

Sun-spot theories: none fully established.

242. Cause of Sun-Spots. — Absolute knowledge is wanting here. Numerous theories have been proposed; many of them are now refuted, and of those that remain no one can be said to be fully established. On the whole, perhaps, at present the most probable view is that they are the result of *eruptions*,— not, however, that they are *craters* through which the eruptions break out, as was at one time thought. It is more likely that when an eruption takes place a hollow or "sink" results in the neighboring surface of the photosphere, in which hollow the cooler gases and vapors collect.

The theory of eruptions.

Another theory, first proposed by Sir John Herschel and now favored by Lockyer and others, is that the spots are formed, not by any action from within, but by cool matter descending from above, and probably of meteoric origin; but it is not easy to reconcile this with the peculiar distribution of the spots upon the sun's surface, though it falls in well with their periodicity.

Theory that they are due to meteors.

Faye considers them to be *cyclones* in the solar atmosphere somewhat analogous to terrestrial storms.

In 1894 E. Oppolzer of Vienna proposed the newest theory, which attributes them to bodies of gas and vapor which, ascending from the polar regions, drift towards the equator and descend in the spot zones, becoming *warmed* and *dried* by the descent,—just as is the case with descending currents in the earth's atmosphere. If he is right, the spots are actually *hotter* than the underlying photosphere, but less luminous because, being purely gaseous, they radiate less powerfully.

Meteorological theories.

243. Terrestrial Influence of Sun-Spots.—One correlation between sun-spots and the earth is perfectly proved. When the spots are numerous *magnetic disturbances* ("magnetic storms") are most numerous and intense, and, as Ellis has also showed, the *regular daily variations* of terrestrial magnetism are also greatest; in many instances also (but by no means always) notable disturbances upon the sun have been accompanied by violent magnetic storms and electric earth-currents, with brilliant exhibitions of the aurora borealis, as in 1859 and 1883. The fact of the connection between terrestrial magnetism and solar disturbances is beyond doubt, though the nature and mechanism of this connection is as yet unknown,—we do not know whether

The terrestrial influence of sun-spots.

Correlation of terrestrial magnetic disturbances with sun-spots.

the solar disturbance *causes* the terrestrial, or whether both disturbances are due to some *external* influence.

The dotted lines in Figs. 88 and 89 represent the magnetic "storminess" at the indicated dates, and its correspondence with the sun-spot curve makes it impossible to doubt their connection.

Question of effect upon the meteorology of the earth.

It has also been attempted to show that solar disturbances are accompanied by effects upon the earth's *meteorology*,—upon its temperature, barometric pressure, storminess, and amount of rainfall. It can only be said that the matter is still under debate. While some particular investigations appear to show a correspondence for a time, others contradict them. If, as is not antecedently improbable, some real connection exists, other disturbances so mask and distort the sun-spot effects that the evidence is thus far inconclusive, and it may be many years before the question can be finally decided. At present it is not certain whether the earth is warmer or cooler, more rainy or less so, at the time of sun-spot maximum.

It is certain that sun-spots cannot produce any sensible effects by their *direct* action in diminishing the heat and light of the sun, since they never cover as much as one-thousandth part of the solar surface. There seems to be, however, at present, according to Halm, a slight balance of statistical evidence in favor of the belief that on the whole the temperature of the earth is really slightly *higher* at or near a *spot minimum* than at a maximum.

CHAPTER IX

THE SUN (*Continued*)

The Spectroscope, the Solar Spectrum, and the Chemical Constitution of the Sun — The Doppler-Fizeau Principle — The Chromosphere and Prominences — The Corona — The Sun's Light — Measurement of the Intensity of the Sun's Heat — Theory of its Maintenance — The Age and Duration of the Sun — Summary as to the Constitution of the Sun

About 1860 the spectroscope appeared in the field as a new and powerful instrument of astronomical research, resolving at a glance many problems which before had seemed to be absolutely inaccessible to investigation. It is not extravagant to say that its invention has done almost as much for the advancement of astronomy as that of the telescope.

Astronomical importance of the spectroscope.

It enables us to study the light that comes from distant objects, to read therein a record, more or less complete, of their chemical composition and physical conditions, to measure the speed with which they are moving towards or from us, and sometimes, as in the case of the solar prominences, to see and observe at any time objects otherwise visible only on rare occasions.

The essential member of the spectroscope: the prism or grating.

244. The Spectroscope. — The *essential* part of the instrument is either a prism or train of prisms, or else a "diffraction grating," which is merely a piece of glass or speculum-metal, ruled with many thousand straight equidistant lines, from ten thousand to twenty thousand in each inch. Either the prism or the grating performs the office of "dispersing" the rays of different wave-length and color.

If with such a "dispersion piece," as it may be called (either prism or grating), one looks at a distant point of light, — a star, for instance, — he will see, instead of a point, a long streak, red at one end and violet at the other. If the object observed is not a point, but *a line of light* parallel to the edge of the prism or to the lines of the grating, then instead of a mere colored streak without width one gets a *spectrum*, — a colored *band* or *ribbon* of light, — which may show transverse markings that will give the observer most valuable information.

The slit and collimator.

It is usual to form this line of light by admitting the light through a narrow *slit* (seldom more than $\frac{1}{500}$ of an inch wide) placed at one end of a tube which carries at the other end an achromatic object-glass having the slit in its principal focus (*Physics*, p. 369). The rays from the slit after having passed the object-glass form a parallel beam, just as if they had come from a very distant object. This tube with slit and lens constitutes the *collimator*, so named because it is precisely the same as the instrument used with the transit-instrument to adjust its line of collimation.

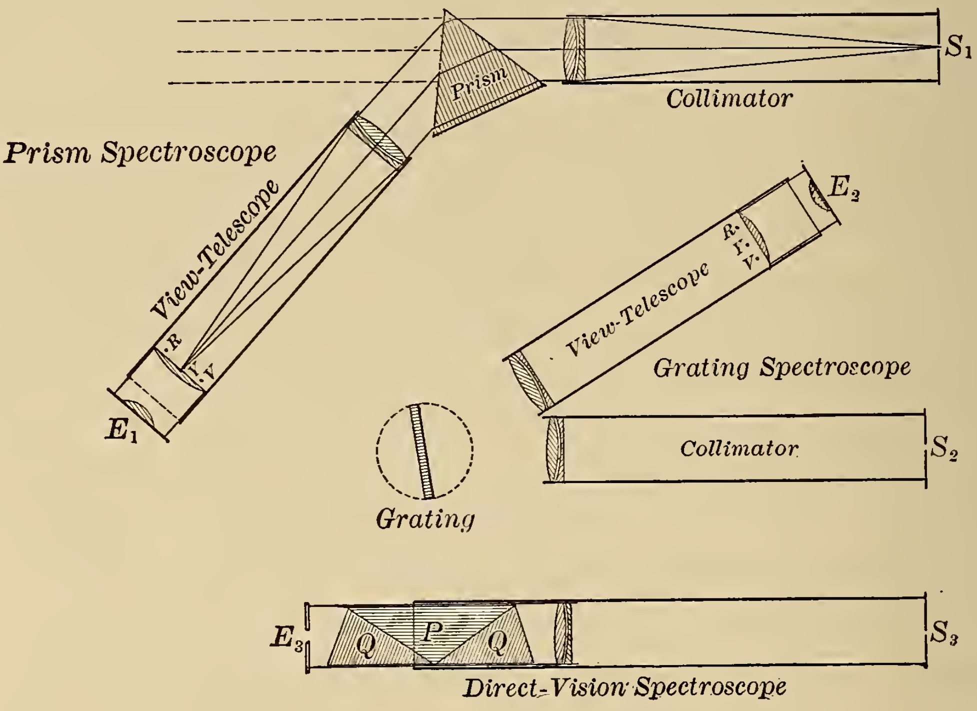
S_1
Prism
Collimator
Prism Spectroscope
View-Telescope
R
Y
V
E_1
E_2
R
Y
V
View-Telescope
Grating Spectroscope
Collimator
S_2
Grating
E_3
Q
P
Q
S_3
Direct-Vision Spectroscope

Fig. 90. — Different Forms of Spectroscope

The view-telescope.

Instead of looking at the spectrum with the naked eye it is better in most cases to magnify it by using a small *view-telescope* (so called to distinguish it from the large telescope to which the spectroscope is often attached).

The instrument, therefore, as usually constructed and shown in the diagram (Fig. 90), consists of three parts, — collimator, prism or grating, and view-telescope, — although in the "direct-vision" spectroscope, shown in the figure, the view-telescope is omitted.

The telespectroscope.

Fig. 91, from *The Sun*, by permission of Appleton & Co., represents a large "telespectroscope" (as the combination of telescope and spectroscope is called) arranged for photographic work.

How the spectrum is formed.

245. The Formation of the Spectrum. — If the slit S be illuminated by strictly "homogeneous light" (*i.e.*, all of one wave-length), a single image of it will be formed. If the light is yellow, a yellow image will appear at Y (Fig. 90). If at the same time light of a different wave-length and color — red, for instance — be also admitted, a *red* image will be formed at R, and the observer will then see a spectrum with two bright lines, *the lines being really nothing more than images of the slit.* If violet light also is admitted, a third (violet) image will be formed at V, and the spectrum will show three bright lines.

The spectrum a series of images of the slit.

The continuous spectrum.

If the light comes from a luminous *solid*, like the lime cylinder of a calcium light, or the filament of an incandescent lamp, or from an ordinary gas or candle flame (in which the light-giving particles are really bits of *solid carbon*), rays of all possible wave-lengths will be emitted and pass through the slit, and as a consequence we shall have an infinite number of these slit-images packed close together, like a picket-fence in which the pickets touch each other; we then get what is called a *continuous spectrum*, ranging in color from red at one end to violet at the other, but showing no transverse lines or markings.

Spectrum of bright lines.

If the light comes, however, from an electric discharge between two metallic balls, or in a so-called Geissler tube, or from a Bunsen-burner flame charged with the vapor of some volatile

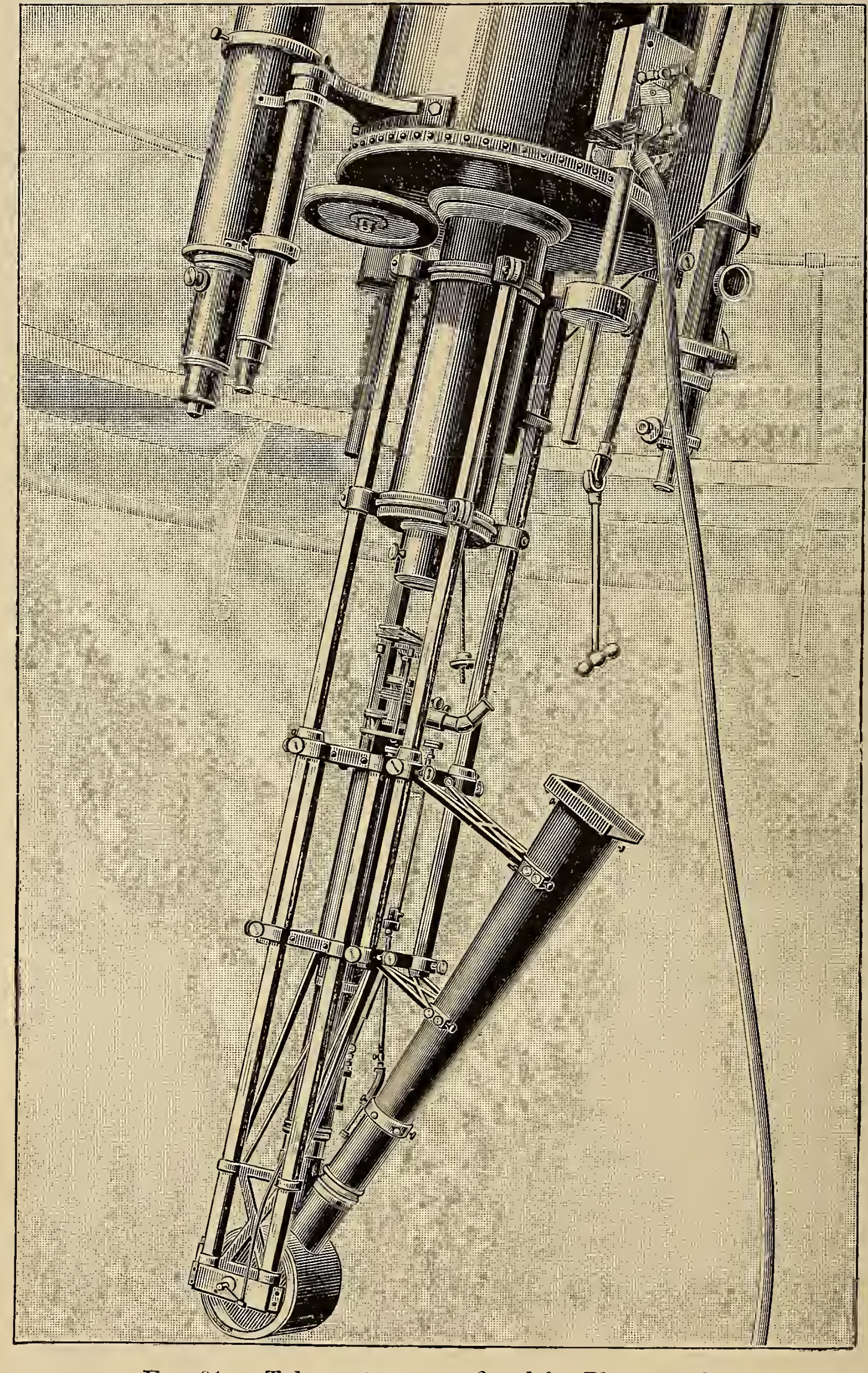

FIG. 91. — Telespectroscope, fitted for Photography
From *The Sun*, by permission of the publishers

metal, the spectrum will consist of a series of bright lines or bands of different colors and usually numerous.

246. The Solar Spectrum. — If we look at *sunlight*, either direct or reflected (as from a piece of paper or from the moon), we get a spectrum, continuous in the main but crossed by thousands of *dark* lines, or *missing slit-images*, known as the "Fraunhofer lines," because Fraunhofer was the first to map them (in 1814). To some of the more conspicuous lines he assigned letters of the alphabet which are still retained as designations: thus, A is a strong line at the extreme red end of the spectrum; C, one in the scarlet; D, one in the yellow; F, in the blue; and H and K are a pair at its violet extremity. Fig. 92 is from a photograph of a small portion of the violet region of the solar spectrum including the great H and K lines. The central strip is made by light from the very edge of the sun; the strips above and below, by light from the center of the disk; there are some notable differences in the appearance of some of the lines in the two cases.

The solar spectrum and its dark Fraunhofer lines.

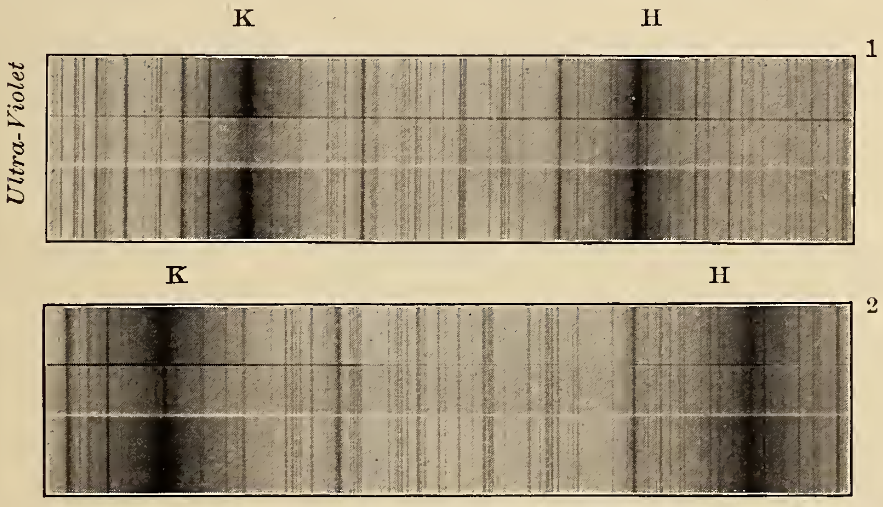

FIG. 92. — H and K Region of Solar Spectrum
From photograph by Jewell, Johns Hopkins University

On the scale of the lower band of this photograph the whole of the *visible* part of the solar spectrum would be about 20 feet long. Our present maps of the spectrum contain more than ten thousand lines, some strong and heavy, others so fine as to be hardly visible, but each as permanent a feature of the spectrum as rivers and cities on a geographical map.

The ultra-violet and infra-red invisible regions of the spectrum.

The *visible* portion of the spectrum is by no means the whole, — only a small part of it, indeed. Above H and K lies a long "ultra-violet" region consisting of rays whose wave-length is *too short* to affect our eyes, but crowded with dark lines and accessible to photography. At the other end, below A, there is an "infra-red" region some twenty times as long as the visible spectrum and consisting of rays, which, while they bring us a large part of all the heat we receive from the sun, have wave-lengths *too long* to produce vision. A small part of this infra-red spectrum can be photographed, but most of it is accessible only to such heat-measuring instruments as Langley's "bolometer" (*General Astronomy*, Arts. 343 and 344). This region also is full of *interspaces* of exactly the same nature as the dark lines in the visible spectrum.

Position of a spectrum line depends upon the wave-length of the ray to which it is due.

The position of each line in the spectrum depends entirely on the *wave-length* or *luminous pitch* of the ray which produces it, or rather (since the line is dark) has been *suppressed*, and is missing. The significance of the lines depends upon their *arrangement* and *characteristics*, just as the "sense" of a printed page lies in the letters and their grouping. As to the colors of the spectrum, the spectroscopist generally pays no more attention to them than the geographer to the colors on his map.

Significance of lines in spectrum depends on their arrangement and characteristics.

The explanation of the Fraunhofer lines remained a mystery for nearly fifty years, until cleared up by the discoveries of Kirchhoff and Bunsen in 1859.

247. Principles upon which Spectrum Analysis depends. — These (substantially), as announced by Kirchhoff in 1859, are the three following:

Kirchhoff's laws.

(1) A *Continuous Spectrum* is given by luminous bodies, which are so *dense* that the molecules interfere with each other in such a way as to prevent their free luminous vibration, *i.e.*,

by bodies which are *solid* or *liquid*, or, *if gaseous*, *are under high pressure.* Such bodies emit a jumble of all possible wave-lengths and colors.

Origin of the continuous spectrum.

Origin of the bright-lined spectrum.

(2) The spectrum of a *luminous gas under low pressure is discontinuous*, made up of *bright lines or bands*, and these lines are *characteristic; i.e.*, the same substance *under similar conditions always gives the same set of lines* and generally does so even under conditions considerably different; but it may (and many gases do) give two or more different spectra when the circumstances differ too widely.

Absorbing power proportional to radiating power.

(3) A gas or vapor *absorbs* from a beam of white light which passes through it *precisely those rays of which, when the gas is luminous, its own spectrum consists.* The spectrum of the transmitted light then exhibits a *reversed* spectrum, which shows upon a continuous background dark lines replacing the bright ones that characterize the gas.

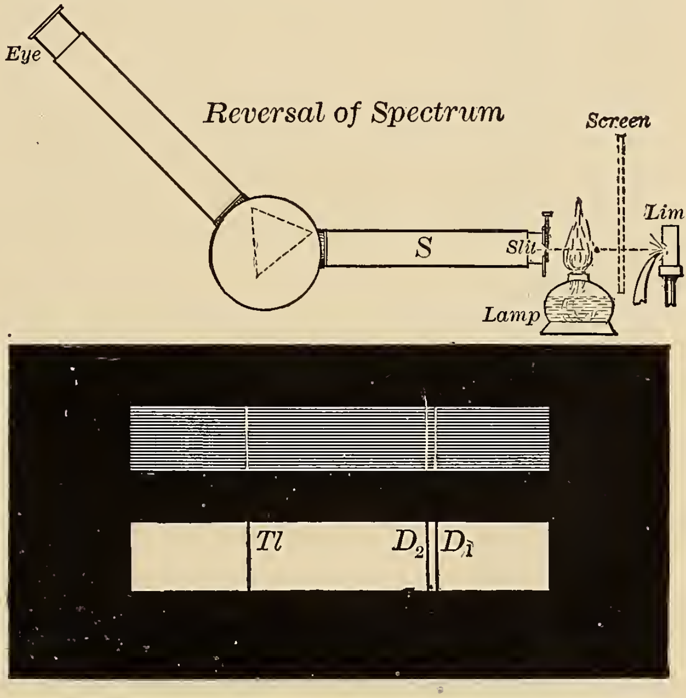

FIG. 93. — Reversal of the Spectrum

Reversal of lines shown experimentally.

This principle of *reversal* is illustrated by Fig. 93. In front of the slit of the spectroscope is placed a spirit-lamp or a Bunsen burner, with a little bead of carbonate of soda in the flame, and if we add a little salt of thallium, we shall then get a spectrum showing the two principal yellow lines of sodium and the green line of thallium, — all three bright. If now a lime-light or an electric arc be put in action behind the flame, we at once get the effect shown at the bottom of the figure, —

a brilliant continuous spectrum crossed by three *black*[1] lines which exactly replace the bright ones. Insert a screen behind the lamp flame and the lines immediately brighten again.

Fraunhofer lines due to absorption of rays by the atmospheres of the sun and earth.

The explanation of the Fraunhofer lines, therefore, is that they are mainly due to the absorbing action of the gases and vapors of the solar atmosphere upon the light transmitted through them from the liquid or solid particles which compose the clouds of the solar photosphere. Some of the dark lines of the solar spectrum, known as *telluric lines*, are, however, due to the gases and vapors of the *earth's* atmosphere, — to water vapor and oxygen especially.

248. Chemical Constituents of the Sun. — Numerous lines of the solar spectrum have been identified as due to the presence in the sun's atmosphere of known terrestrial elements in the state of vapor.

Determination of elements existing in the solar atmosphere.

To effect the comparison necessary for this purpose the observer's apparatus must be so arranged that he can confront the spectrum of sunlight with that of the substance to be examined, which must be *brought into the gaseous condition*, so that it can emit its characteristic spectrum of bright lines.

Experimental arrangements.

In the case of those substances which volatilize at a comparatively low temperature, as, for instance, sodium, calcium, thallium, and the alkaline metals generally, the flame of a spirit-lamp or Bunsen burner answers the purpose. A little piece of the metal or of one of its easily volatilized salts is inserted in the flame, and the bright lines or bands of its spectrum appear at once in the spectroscope.

If this flame is not hot enough, that of the oxyhydrogen blowpipe used for the calcium light may answer.

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[1] Their apparent darkening, however, when the brilliant light from the lime is transmitted through the flame, is only *relative*, not real. Their brightness is actually a *little* increased; but the brightness of the background is increased *immensely*, making it so much brighter than the three lines that, contrasted with it, they look black, as does an electric arc when interposed between the eye and the sun.

This failing, recourse is had to electricity. Most of the metals vaporize at once in the *electric arc* between carbon electrodes, but we may have to employ the still higher temperature of an *electric spark* produced between electrodes of the metal by an "induction coil"; and in passing it is to be noted that the spectrum of the metal produced by the spark usually presents notable differences from the arc spectrum.

Volatilization of substances by the electric arc and spark.

Finally, if we have a *permanent gas*, say hydrogen, to deal with, it is sealed up, usually much rarefied, in a glass *Geissler tube* (Fig. 94), 5 or 6 inches long, with metallic electrodes at each end, by means of which electrical discharges can be passed through the gas.

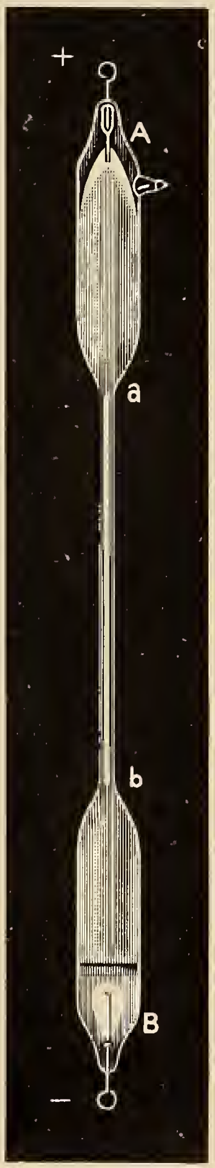

Fig. 94
Geissler Tube

249. Method of comparing Spectra. — In order to effect the comparison, half the slit is covered with a little reflector, or a so-called "comparison prism" which reflects into it the sunlight, while the other half of the slit receives directly the light from the luminous vapor. Upon looking into the spectroscope the observer will have the two spectra, of the sunlight and of the metal, side by side, and can at once see what bright lines of the metallic spectrum do or do not exactly coincide with the dark lines of the solar spectrum. If he finds that every one of the conspicuous bright lines matches a conspicuous dark line, he can be certain that the substance exists as vapor in the sun's atmosphere.

Elements detected by the coincidence of bright lines in their spectra with Fraunhofer lines in spectrum of the sun.

In such comparisons photography may be most effectively used instead of the eye. The slit of the spectroscope is so arranged that either half of its length can be used independently. An impression of the solar spectrum is then obtained by a few seconds' exposure to sunlight admitted through one

Use of photography in making the comparison.

half of the slit, which is then closed, and the room darkened. Immediately afterwards light from an electric arc containing the vapor of metal to be tested is admitted through the other half for a sufficient time. The plate, when developed, will then show the two spectra side by side. Fig. 95 is a half-tone reproduction, on a reduced scale, of a *negative* made by Professor Trowbridge in investigating the presence of iron in the sun. The lower half is part of the violet portion of the sun's spectrum (showing dark lines as bright), and the upper half that of an electric arc charged with the vapor of iron. In the original every line of the iron spectrum coincides exactly with a correlative in the solar spectrum, though in the engraving some of the coincidences fail to be obvious. There are, of course, on the other hand, certain lines in the solar spectrum which do not find any correlative in that of iron, being due to other elements.

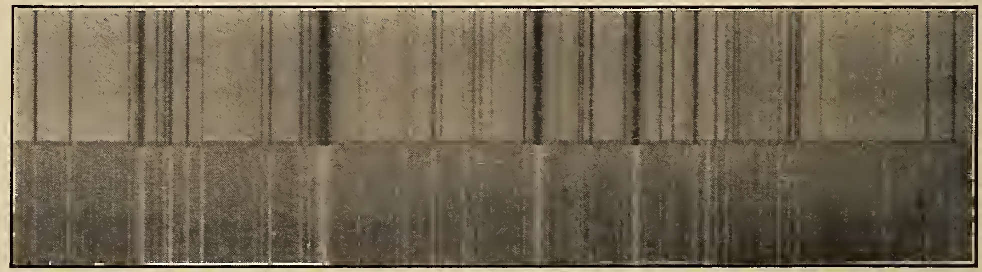

FIG. 95

List of thirty-six elements certainly detected in this way.

250. Elements known to exist in the Sun. — As the result of such comparisons, first made by Kirchhoff, but since repeated and greatly extended by late investigators, a large number of our chemical elements have been ascertained to exist in the solar atmosphere in the form of vapor.

Professor Rowland in 1890 gave the following preliminary list of thirty-six whose presence may be regarded as certainly established, and it is probable that further research will add a number of others. The elements are arranged in the list according to the *intensity* of the dark lines by which they are represented in the solar spectrum; the appended figures denote the rank which each element would hold if the arrangement had been based on the *number* instead of the intensity of the lines. In the case of iron the number exceeds two thousand.

* Calcium, 11.
* Iron, 1.
* Hydrogen, 22.
* Sodium, 20.
* Nickel, 2.
* Magnesium, 19.
* Cobalt, 6.
Silicon, 21.
Aluminium, 25.
* Titanium, 3.
* Chromium, 5.
* Manganese, 4.
* Strontium, 23.
* Vanadium, 8.
* Barium, 24.
* Carbon, 7.
Scandium, 12.
* Yttrium, 15.
Zirconium, 9.
Molybdenum, 17.
Lanthanum, 14.
Niobium, 16.
Palladium, 18.
Neodymium, 13.
Copper, 30.
* Zinc, 29.
* Cadmium, 26.
* Cerium, 10.
Glucinum, 33.
Germanium, 32.
Rhodium, 27.
Silver, 31.
Tin, 34.
Lead, 35.
Erbium, 28.
Potassium, 36.

An asterisk denotes that the lines of the element indicated appear often or always as bright lines in the spectrum of the chromosphere (Sec. 257).

Exceptional case of helium.

Helium was added in 1895, — peculiar in that it manifests its presence, not by dark Fraunhofer lines, but only by *bright* lines in the spectrum of the chromosphere. Certain observations of Rungé on lines in the infra-red portion of the spectrum seem to indicate that *oxygen* should also be included.

It will be noticed that all the bodies named in the list, carbon and hydrogen alone excepted, are *metals*, and that many of the most important terrestrial elements fail to appear; nitrogen, chlorine, bromine, iodine, sulphur, phosphorus, and boron are all missing, and oxygen gives only faint and as yet uncertain indications of its presence.

Negative conclusions unwarranted.

251. Unsafety of Negative Conclusions.—We must be cautious, however, in drawing *negative* conclusions. It continually happens that when a mixture of gases or vapors is examined with the spectroscope, certain ones only can be recognized; as long as these are present the others keep in hiding. Thus the presence of argon in atmospheric air cannot be detected by the spectroscope until nearly all the oxygen and nitrogen have been removed; and the other new gases of the atmosphere, *krypton*, *neon*, and *xenon*, are still more difficult to deal with.

It is quite conceivable also that the spectra of the missing elements may be, under solar conditions, so different from their spectra as presented in our laboratories that we cannot recognize them; for it is now unquestionable that many substances under different conditions give two or more widely different spectra, — nitrogen, for instance.

Lockyer's dissociation theory.

Lockyer thinks it more probable that the missing substances are not truly "elementary," but are decomposed or "dissociated" by intense heat, and so cannot exist on the sun, but are replaced by their components. He maintains, in fact, that none of our so-called "elements" are really elementary, but that all are decomposable and are to some extent actually decomposed in the sun and stars; and some of them by the electric spark in our own laboratories. Granting this, many interesting and remarkable spectroscopic facts find easy explanation. At the same time the hypothesis is encumbered with serious difficulties and has not yet been finally accepted by physicists and chemists.

The reversing layer.

252. The Reversing Layer. — According to Kirchhoff's theory, the dark lines are formed by the transmission of light emitted by the minute solid or liquid particles of which the photospheric clouds are supposed to be formed, through somewhat cooler vapors containing the substances which we recognize in the solar spectrum. If this be so, the spectrum of the gaseous envelope, which by its absorption causes the dark lines, should *by itself* show a spectrum of corresponding bright lines.

Reversal of the Fraunhofer lines at the instant of beginning or end of totality. The flash spectrum.

The opportunities are rare when it is possible to obtain the spectrum of this gas stratum separate from that of the photosphere; but at the time of a total eclipse, at the moment when the sun's disk has just been obscured by the moon and the sun's atmosphere is still visible beyond the moon's limb, the observer ought to get this bright line spectrum, if his spectroscope is carefully directed to the exact point of contact.

The actual observation was first made during the Spanish eclipse of 1870. The lines of the solar spectrum, which up to the time of the final obscuration of the sun had remained dark as usual (with the exception of a few belonging to the spectrum of the chromosphere), were suddenly reversed, and the whole field of view was filled with brilliant colored lines, which flashed out quickly and then gradually faded away, disappearing in two or three seconds, — a most beautiful thing to see.

The natural interpretation of this phenomenon is that the dark lines in the solar spectrum are, mainly at least, produced by a *very thin stratum* close down upon the photosphere, since the moon's motion in three seconds would cover a thickness of only about 800 miles. It was not possible, however, to be certain from such a mere glance that *all* the dark lines of the solar spectrum were reversed.

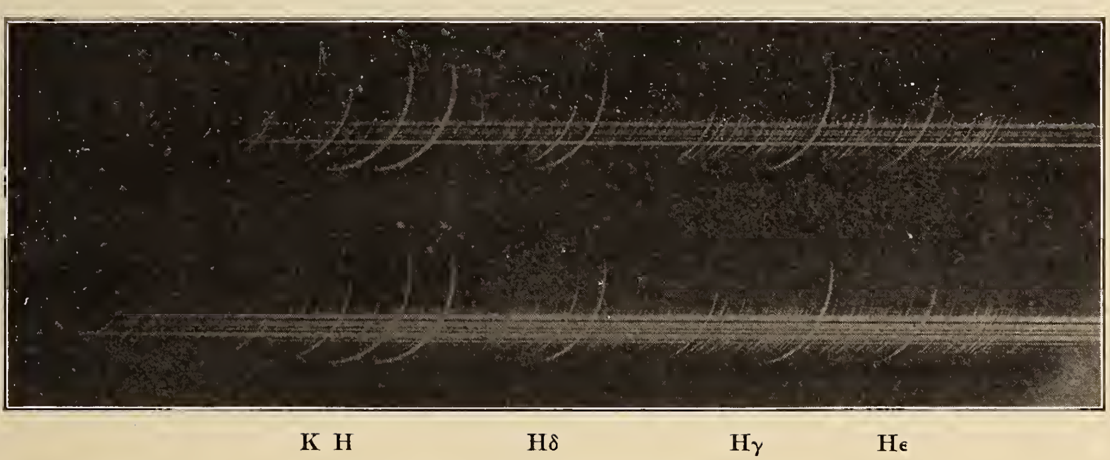

Fig. 96. — The Flash Spectrum

Photographs of the flash spectrum.

Several partial confirmations of the observation have since been *visually* obtained at eclipses, though none so complete as desirable; but the photographs of the "flash spectrum," as it is now called, obtained during the recent eclipses of 1896, 1898, 1900, and 1901, made with various forms of the "prismatic camera" (a camera of long focus, with a prism, a train of prisms, or a "grating" outside the object-glass), have fully corroborated it. Fig. 96 is a reproduction of one of the exquisite photographs of the flash spectrum obtained by Sir Norman Lockyer in India during the eclipse of 1898. The lines above (to the left of H and K) are in the *invisible* portion of the spectrum and are most of them due to hydrogen. Until these permanent records of the phenomena were obtained there was room to doubt whether the bright lines seen might not belong mainly to the spectrum of the "chromosphere" (Sec. 257), instead of being reversed Fraunhofer lines.

Sir Norman Lockyer has never admitted the existence of any such *thin* "reversing layer," maintaining that a large proportion of the dark lines are formed only in the regions of lower temperature, *high up* in the sun's atmosphere, and not close to the photosphere, *i.e.*, different lines of a given substance originate at very different elevations in the solar atmosphere.

253. Sun-Spot Spectrum. — The spectrum of a sun-spot differs from the general solar spectrum, not only in its diminished brightness, but in the great widening and intensification of certain dark lines and the thinning, and sometimes the *reversal*, of others, especially those of hydrogen and H and K of calcium, "the great twin brothers," as Miss Clerke calls them, which are also conspicuous in the solar prominences, and, we may remark in passing, are also always reversed in the spectrum of the faculæ, appearing as thin bright lines running through the center of the wide, black, hazy-edged bands. The majority of the Fraunhofer lines are, however, as a rule, quite unaltered; and in the case of those substances which show widened lines in the spot spectrum, only a few of their lines are thus affected.

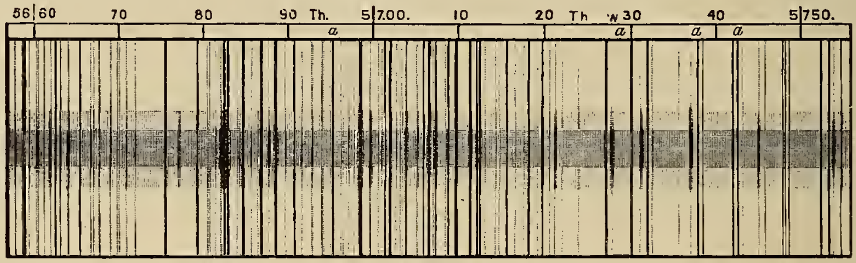

FIG. 96 A. — Portion of Sun-Spot Spectrum

From photograph of 1893

Peculiarities of the spectrum of a sun-spot.

Some substances which are very inconspicuous in the ordinary solar spectrum become obtrusive in the sun-spot spectrum, — *vanadium*, for instance. Fig. 96 A is from a photograph of the

Vanadium in sun-spots.

yellowish-green portion of a sun-spot spectrum and exhibits very well the leading characteristics.

The general darkness of the spectrum of a sun-spot, in the green portion at least, appears to be due to the presence of myriads of thin dark lines so closely packed, with here and there an interval, as to be resolvable only in instruments of high power. This indicates that the darkening is due, in part at least, to the *absorption* of light *by transmission through vapors*, rather than to a diminution of the emissive power of the surface from which the light comes.

Darkening due to absorption by vapors.

254. Displacement and Distortion of Lines. — Sometimes in the spectrum of an active sun-spot or of a prominence certain lines are displaced and broken, as shown in Fig. 97. These distortions can be explained as due to the swift motion towards or from the observer of the gaseous matter, which by its absorption produces the line observed. In the case illustrated in the figure hydrogen was the substance, and its motion was *away from* the earth at the rate of nearly *300 miles a second.*

Displacement and distortion of lines.

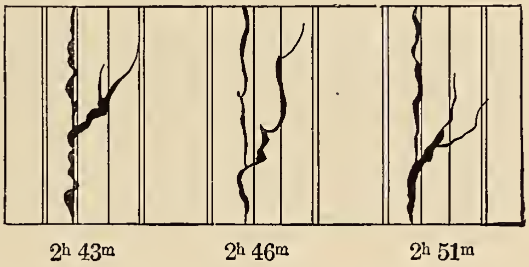

Fig. 97. — The C Line in the Spectrum of a Sun-Spot, Sept. 22, 1870

The general principle upon which the explanation of such phenomena depends was first enunciated by the German physicist Doppler in 1842, and has turned out to be one of extreme importance and wide application. It is this: *When the distance between the observer and a body which is emitting regular vibrations is increasing, then the number of vibrations received in a second is decreased and their wave-length, real or virtual, is correspondingly increased;* and *vice versa* if the distance is decreasing.

Doppler's principle.

Thus, in the case of recession, the pitch of an engine whistle suddenly drops when a whistling engine passes us and recedes;

and a light-ray (say the particular ray which produces the C line in the spectrum of hydrogen) has its wave-length *increased* and its refrangibility, which depends upon its wave-length, *diminished*, if the luminous object is receding, so that the C line and all the other hydrogen lines *are shifted toward the red end of the spectrum.* This effect of motion on the *lines of the spectrum* was first pointed out by Fizeau in 1848, so that in its astronomical application the principle is now usually referred to as the "Doppler-Fizeau" principle.

Effect of motion upon position of lines in the spectrum. The Doppler-Fizeau principle.

Fig. 98 illustrates the principle. The lower strip is a small piece of the yellow portion of the spectrum of a star (imaginary) which is rapidly approaching the earth, the two conspicuous dark lines being the D_1 and D_2 lines of *sodium*. The upper strip is the corresponding part of the spectrum of a flame or electric spark containing sodium vapor and showing its lines bright. The two spectra are confronted by a "comparison prism" (Sec. 249), and it is obvious that the lines of the star spectrum are shifted towards the blue end by about one fourth of the distance between the D lines, *i.e.*, by about 1.5 units of wave-length on the Rowland scale (the unit is one ten-millionth of a millimeter). As the wave-length of D_1 is 5896 units (nearly), it follows from the formula of the next article that the imaginary star must have been rushing towards us at the rate of nearly 48 miles a second, — pretty fast, but several real stars are swifter.

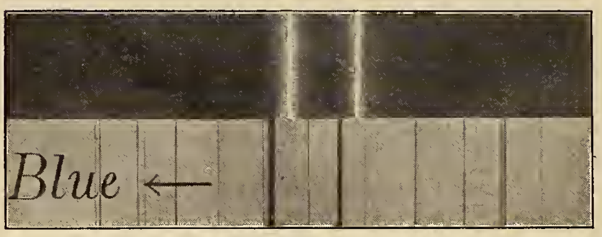

FIG. 98

Formula giving relation between the radial velocity of a luminous object and the shift of lines in its spectrum.

255. Formula of the Doppler-Fizeau Principle. — While the reasoning upon which the principle rests is simple, a general theoretical treatment for light-waves is difficult.

For the demonstration of the formulæ given below, the reader is referred to Frost's translation of Scheiner's *Astronomical Spectroscopy*, Part II, Chapter II.

If V is the velocity of light (186330 miles a second), r the speed with which the observer is receding from the object, s the speed with which the source of light itself is *receding*, λ

the normal wave-length of the given line in the spectrum, and λ' the *apparent* wave-length as affected by the two motions, we have the equation:

$$\lambda' = \lambda \times \frac{V+s}{V-r}. \qquad (1)$$

Subtracting λ from both sides of the equation, we get

$$\lambda' - \lambda, \text{ or } \Delta\lambda = \lambda \frac{r+s}{V-r}, \qquad (2)$$

which holds for all velocities, great or small.

Since, however, in all ordinary cases r is insignificant as compared with V, it may be dropped in the denominator, and we have

$$\Delta\lambda = \lambda \times \frac{r+s}{V}.$$

Finally, putting v for $r+s$, the total rate at which the distance between the object and the observer is *increasing*, we have

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{V}, \text{ or } v = V \times \frac{\Delta\lambda}{\lambda}, \qquad (3)$$

which is the usual formula employed in computing "motion in the line of sight" (or "radial velocity," as it is now usually called) from observations of the shift of lines in the spectrum. When the distance is *decreasing*, v becomes negative, and also $\Delta\lambda$, indicating a diminution of wave-length and a corresponding shifting of the line towards the *blue* end of the spectrum. At present motions of less than half a mile per second can be detected by the spectroscopes which are used in studying stellar spectra.

Other causes which produce a somewhat similar apparent shift of lines in the spectrum.

256. Other Causes of Displacement of Spectrum Lines. — It has been recently (1895) discovered by Humphreys and Mohler at Baltimore that the position of a line in the spectrum of a luminous vapor may also be shifted in a somewhat similar manner *towards the red* by great increase of pressure, — a pressure of 180 pounds to the square inch producing as great a displacement as a receding rate of some 2 miles a second; but the shift varies for different lines in the spectrum and does not follow the same law as in the case of motion.

In 1900 Professor Julius of Utrecht demonstrated how an apparent shift of spectrum lines may also follow from what is called "anomalous refraction" in the sun's atmosphere near sun-spots and solar prominences; and Michelson in a still more recent paper shows how rapid *changes of density* in the medium through which light comes to us may produce similar effects. It is quite possible, therefore, that some of the phenomena which have hitherto been explained on the Doppler-Fizeau principle as indicating tremendous velocities of moving matter may, on further examination, receive a different interpretation.

The chromosphere.

257. The Chromosphere and Prominences. — Outside the photosphere lies the *chromosphere*, of which the lower atmosphere, or "reversing layer," is only the densest and hottest portion. This chromosphere, or "color sphere," is so called because it is brilliantly scarlet, owing the color to hydrogen, which is its main, or at least its most conspicuous, constituent. The spectroscope shows it to be principally composed of hydrogen, helium, and calcium vapor. In structure it is like a sheet of flame overlying the surface of the photosphere to a depth of from 5000 to 10000 miles, and as seen through the telescope at a total eclipse of the sun has been aptly described as like "a prairie on fire."[1]

The prominences or protuberances.

At such a time, after the sun is fairly hidden by the moon, a number of scarlet star-like objects are usually seen blazing like rubies upon the contour of the moon's disk. In the telescope they look like fiery clouds of varying form and size, and, as we now know, they are projections from the chromosphere, or isolated clouds of chromospheric material. They were called *prominences* or *protuberances*, as a sort of non-committal name, while it was still uncertain whether they were appendages of the sun or of the moon.

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[1] There is, however, no real *burning* in the case, *i.e.*, no *chemical combination going on between the hydrogen and some other element like oxygen.* The hydrogen is too hot to burn in this sense, the temperature of the solar surface being above that of *dissociation*, — so high that any compound containing hydrogen would there be decomposed.

They were first proved to be solar during the eclipse of 1860, by means of photographs which showed that the moon's disk moved over them as it passed across the sun. Fig. 99 is from a photograph of the eclipse of April, 1893, by Schaeberle.

Their gaseous constitution demonstrated by the spectroscope in 1868.

Their real nature as clouds of incandescent *gas* was first revealed by the spectroscope in 1868, during the Indian eclipse of that year. On that occasion numerous observers recognized in their spectrum the bright lines of hydrogen along with another conspicuous yellow line, at first wrongly attributed to sodium but afterwards to a hypothetical element then unknown in our laboratories and provisionally named "helium," its yellow line being known as D_3 (D_1 and D_2 being the *sodium* lines which lie close by).

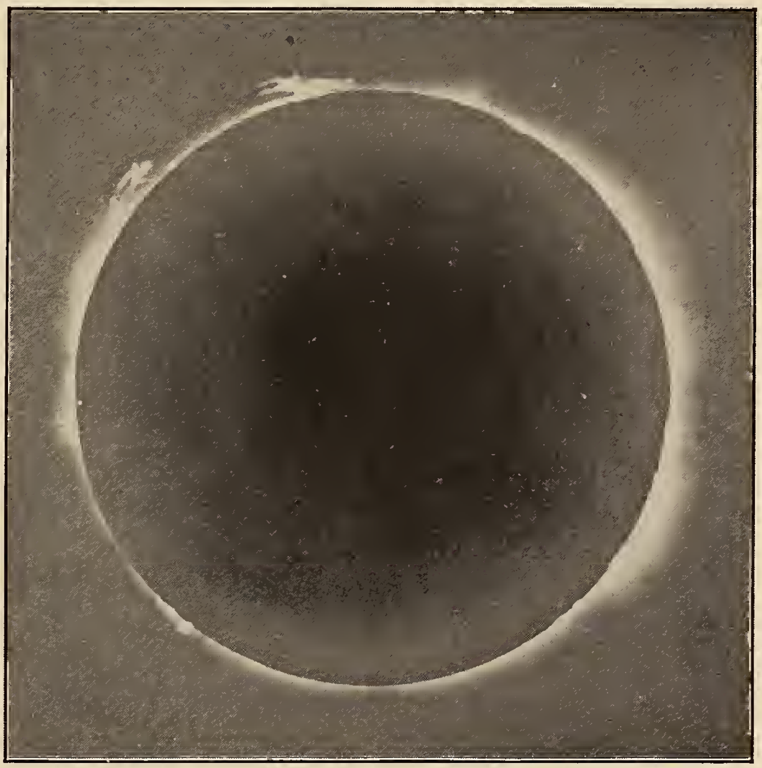


Fig. 99. — Prominences, 1893

The identification of helium as a terrestrial element.

Helium was discovered as a terrestrial element in April, 1895, by Dr. Ramsay, one of the discoverers of argon. In examining the spectrum of the gas extracted from a specimen of cleveite, a species of pitch-blende, he found the characteristic D_3 line along with certain other unidentified lines which appear in the spectrum of the chromosphere and prominences. The same gas has since been found in a number of other minerals and mineral waters and also in *meteoric iron*. Its density turns out to be about double that of hydrogen, but *less than that of any other known element*, and it resists liquefaction more stubbornly than any other gas, — indeed, it is the only one not yet subdued, excepting possibly some of the new gases (neon, etc.) not yet obtained in sufficient quantity to permit investigation on this line. Chemically, it is extremely inert, refusing to enter into combination with other elements (as hydrogen does so freely), and therefore exists on the earth only in minute quantities. It seems, however, to be abundant in certain stars and nebulæ, where its lines are conspicuous along with those of hydrogen. The D_3 line is not the only helium line, but the chromosphere spectrum contains at least three others that are always observable, besides a dozen or more that occasionally make their appearance.

The H and K lines of calcium are also, like those of hydrogen and helium, *always* present as *bright lines* in the chromosphere; and several hundred lines of the spectra of iron, strontium, magnesium, sodium, etc., have been observed in it now and then. Fig. 100 shows the appearance of the calcium lines in the chromosphere spectrum, and also the hydrogen line (Hϵ), which is close to the H line, as well as Hζ, to the left of K.

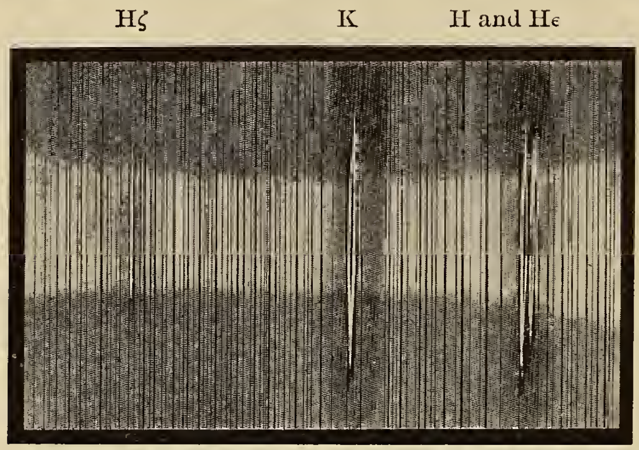


Fig. 100. — H and K Lines in Chromosphere Spectrum

Prominences observable with the spectroscope without an eclipse.

258. The Prominences and Chromosphere observable at Any Time with the Spectroscope.

— During the eclipse of 1868 Janssen was so struck with the brightness of the hydrogen lines in the spectrum of the prominences that he believed it possible to observe them in full daylight, and the next day he found it to be so. He also found that by a proper management of his instrument he could make out the forms and structure of the prominences which he had seen the day before during the eclipse. Lockyer, in England, a few days later, but quite independently, made the same discovery and ascertained that the prominences were mere extensions from a hydrogen envelope completely surrounding the sun, and it was he who gave to this envelope the now familiar name of "chromosphere." His name is always, and justly, associated with that of Janssen as a co-discoverer. A little later Huggins showed that by simply opening the slit of the spectroscope the form and structure of the prominence, if not too large, could be observed as a whole, and not merely by piecemeal as before. Within the last few years it has become possible even to *photograph* them by an instrument called a *Spectroheliograph.*

259. How the Spectroscope enables us to see the Chromosphere and Prominences without an Eclipse. — The reason why we cannot see them by simply screening off the sun's disk is that the brilliant illumination of our own atmosphere near the sun drowns them out, as daylight does the stars.

Explanation of the principle by which the spectroscope makes the prominences visible. It reduces the brightness of the background, but not that of the prominences.

When we point the telespectroscope so that the sun's image falls as shown in Fig. 101, with its limb just tangent to the edge of the slit, then, if there be a prominence at that point, we shall get two overlying spectra: one, the spectrum of the illuminated air; the other, superposed upon this background, is that of the prominence itself. Now the latter is a spectrum consisting of bright lines, or, if the slit be opened a little, of *bright images* of whatever part of the prominence may fall between the jaws of the slit, and the *brightness of these lines or images is independent of the dispersive power of the spectroscope;* increase of dispersion merely sets the images farther apart, without making them fainter (except as light is lost by the transmission through a greater number of prisms). The spectrum of the aërial illumination, on the other hand, is that of sunlight, — a *continuous spectrum* showing the usual Fraunhofer lines; and this spectrum is *made faint by great dispersion.* Moreover, it presents dark lines or spaces just at the very places in the spectrum where the bright images of the prominences fall. They therefore become easily visible.

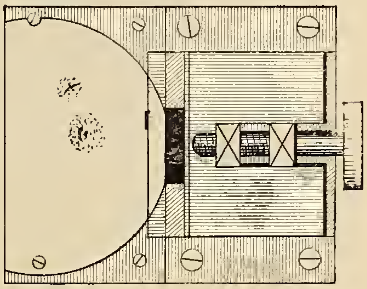


FIG. 101. — Spectroscope Slit adjusted for Observation of the Prominences

A grating of ordinary power attached to a telescope of no more than 2 or 3 inches aperture gives a very satisfactory view of these beautiful and interesting objects. The red image, which corresponds to the C line of hydrogen, is by far the best for visual observations. When the instrument is properly adjusted, the slit slightly opened, and the image of the sun's limb brought exactly to its edge, the observer at the eyepiece of the spectroscope will see things about as we have attempted to represent them in Fig. 102, — as if he were looking at the clouds in an evening sky from across the room through a slightly opened window blind.

260. Different Kinds of Prominences. — The prominences may be broadly divided into two classes, — the "quiescent" or "diffuse," and the "eruptive" or, as Secchi calls them, the "metallic," because they show in their spectrum the lines of many of the metals in addition to the lines of hydrogen, helium, and calcium.

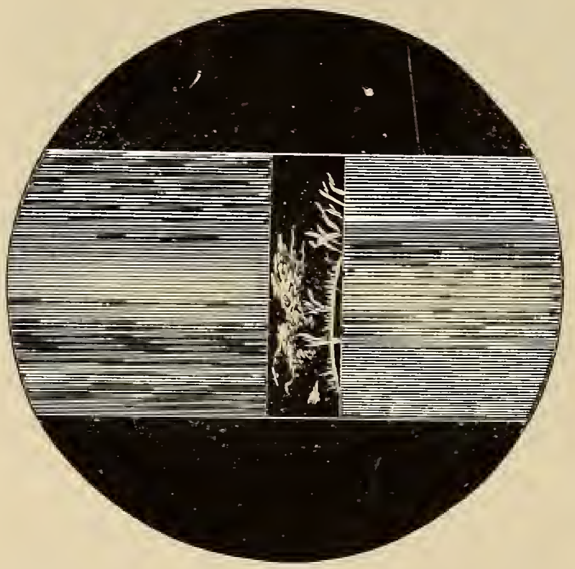


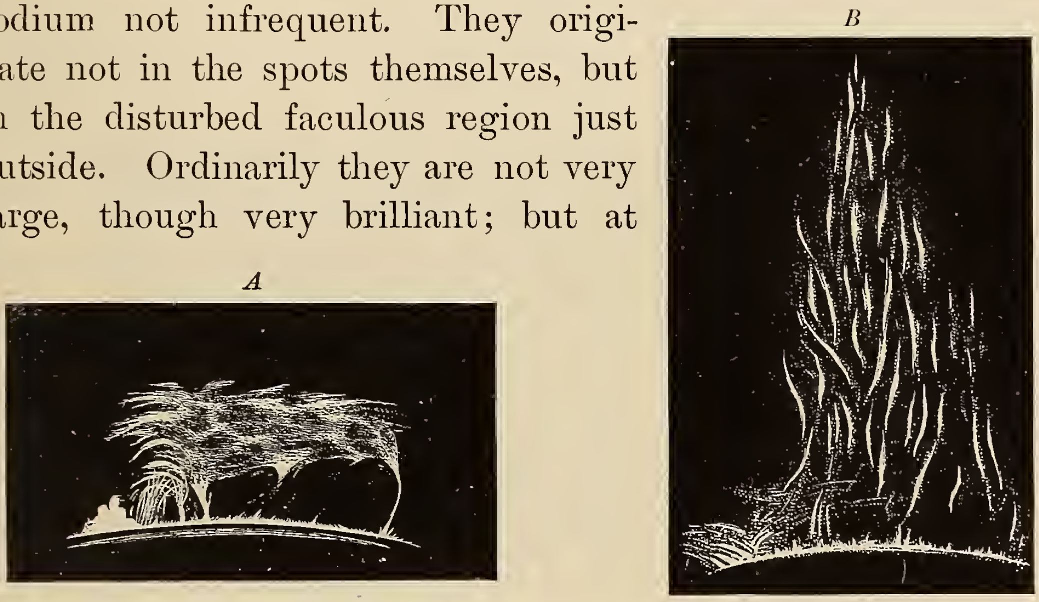
FIG. 102. — The Chromosphere and Prominences seen in the Spectroscope

Quiescent prominences.

The *Quiescent Prominences* are immense clouds, often from 50000 to 100000 miles in height and of corresponding horizontal dimensions, either resting directly upon the chromosphere as a base, or connected with it by stems and columns, as shown in Fig. 103 *A*, though in some cases they appear to be entirely detached. They are not very brilliant and ordinarily show no lines in their spectrum except those of hydrogen and helium and H and K of calcium (which are often *doubly* reversed, as shown in Fig. 100); nor are their changes usually rapid, but they often continue sensibly unaltered sometimes for days together, *i.e.*, as long as they remain in sight in passing around the limb of the sun. All their forms and behavior indicate that, like the clouds in our own atmosphere, they exist, and float, not in a vacuum, but in a medium having a density comparable with their own, *though not giving bright lines in its spectrum*, and for that reason not visible in the spectroscope. They are found on all portions of the sun's disk, not being confined to the sun-spot zones.

Eruptive, or metallic, prominences

The *Eruptive Prominences*, on the other hand, appear only in the spot zones, and as a rule in connection with active spots. Their spectrum usually contains the bright lines of various metals, magnesium and iron being especially conspicuous, and sodium not infrequent. They originate not in the spots themselves, but in the disturbed faculous region just outside. Ordinarily they are not very large, though very brilliant; but at times they become enormous, in one instance under the writer's own observation reaching an elevation of more than 400000 miles. They are most fascinating objects to watch, on account of the rapidity of their changes. Sometimes the actual motion of their filaments can be perceived directly, like that of the minute-hand of a clock, and this implies a velocity of at least 250 miles a second in the moving mass. In such cases the lines of the spectrum are also, of course, greatly displaced and distorted.



Prominences, Sept. 7, 1871, 12.30 P.M.

Same at 1.15 P.M.

FIG. 103. — A Solar Explosion

Rapidity of change.

Fig. 103 represents a prominence of this sort at two times, separated by an interval of three quarters of an hour. The large quiescent prominence

Solar explosions.

of Fig. *A* was completely blown to pieces, as shown in *B*, by the "explosion," as it may be fairly called, which occurred beneath it, — the first case in which such a phenomenon was actually observed. See also Fig. 104, showing successive stages of an eruptive prominence photographed at Professor Hale's private observatory in 1895. A considerable number of similar occurrences have been recorded by various observers during the last thirty years, but they are by no means every-day affairs.

Number of prominences.

The number of prominences of both kinds visible at one time on the circumference of the sun's disk ranges from one or two to twenty-five or thirty; the eruptive prominences being numerous only near the times of sun-spot maximum.

Photography of prominences by simple spectroscope.

261. Photography of Prominences; the Spectroheliograph. — It is *possible*, but not very satisfactory, to photograph a small prominence by the same arrangement as for visual observation, merely putting a sensitive plate at the focus of the "view-telescope" (Sec. 244), using the F line of the spectrum, or, better, the K line, instead of the red C, which requires too long an exposure.

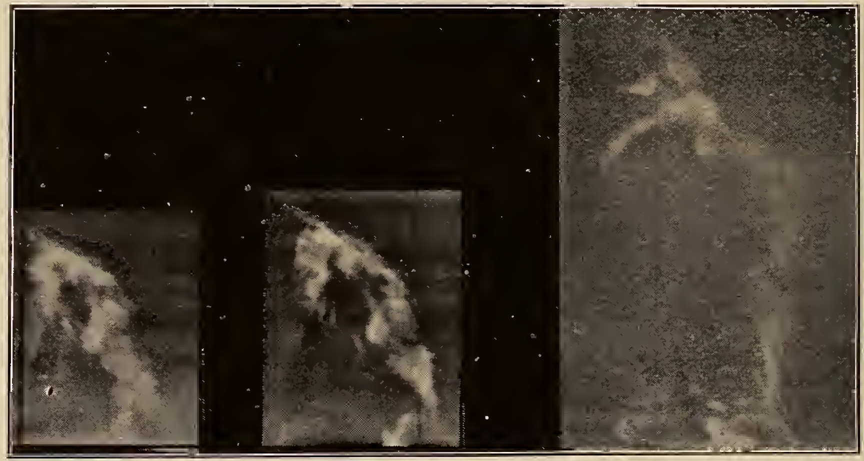

10.34 10.40 10.58

Fig. 104. — Photographs of Prominences, March 25, 1895

After Hale

Photography by the spectroheliograph.

A much better plan is to use a "spectroheliograph," — independently devised by Professor Hale of Chicago and Deslandres of Paris. A detailed description would take too much space, but the essential feature is a narrow slit moving in front of the sensitive plate in exact accordance with the motion of the collimator slit; so that as the latter is moved across the image of the prominence, while the latter moves before the plate, the bright K line of each portion of the prominence shines through upon the plate and so photographs the object *in sections*, by its "K-light," if the expression may be allowed.

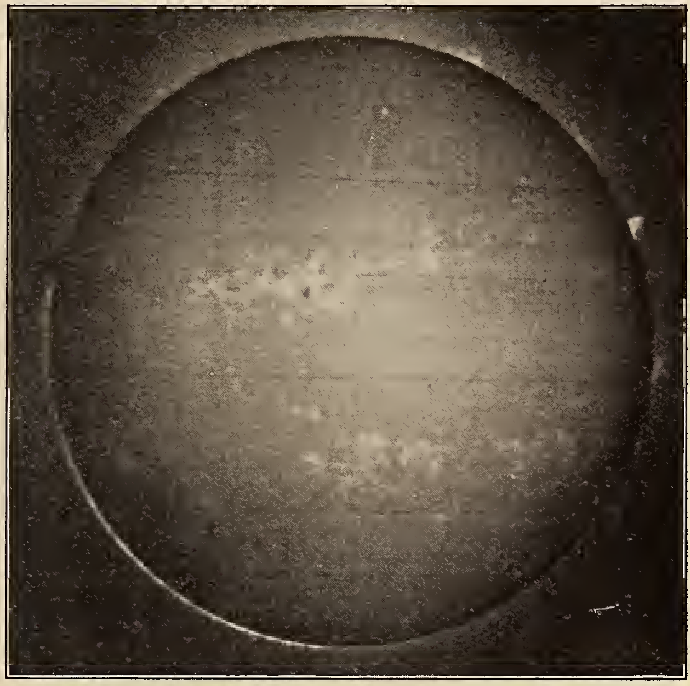


Fig. 104 A. — Spectroheliogram of Entire Sun
After Hale

Fig. 104 is from a negative thus made at Professor Hale's private Kenwood observatory on March 25, 1895, with the then newly invented spectroheliograph, — three exposures on an ascending prominence at intervals of eight and eighteen minutes. During this time the height of the prominence increased from 135000 miles to 281000. Fig. 104 A exhibits the whole sun with its spots and faculæ and the surrounding chromosphere, — made by the same instrument. A screen covers the sun's image while the chromosphere and prominences are first photographed by a slow motion of the slit, and then, the screen being removed, the slit is drawn back rapidly across the sun's image, thus producing the picture of the solar surface.

The corona: its general appearance.

262. The Corona. — The *corona* is a halo or glory of light which surrounds the sun at the time of a total eclipse and has been known from remote antiquity as one of the most beautiful and impressive of all natural phenomena. The portion near the sun is dazzlingly bright and of a pearly tinge, which contrasts finely with the scarlet prominences. It is made up of filaments and rays which, roughly speaking, diverge radially, but are strangely curved and intertwined. At a little distance from the edge of the sun the light becomes more diffuse, and the outer boundary of the corona is not very well defined, though certain dark rifts extend through it clear to the sun's surface.

Often the filaments are longest in the sun-spot zones, giving the corona a roughly quadrangular form. This seems to be specially the case in eclipses which occur near the time of a sun-spot maximum. In eclipses which occur near the sun-spot *minimum*, on the other hand, the equatorial rays predominate, forming streamers, sometimes fan-like and sometimes pointed, extending several millions of miles from the solar surface. Near the poles of the sun there are usually tufts of sharply defined threads of light, which curve both ways from the pole.

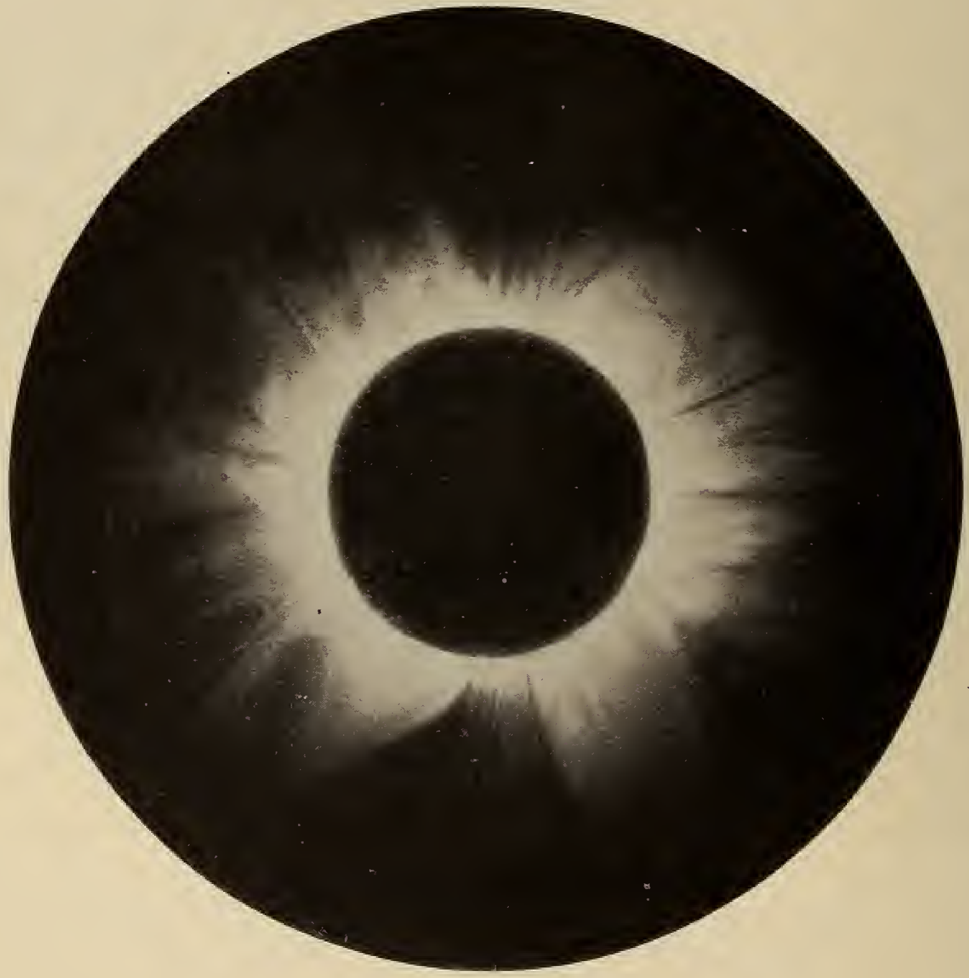


Fig. 105. — Corona of 1871

Difference between the corona at a sun-spot maximum and at a minimum.

The corona varies greatly in brightness at different eclipses, according to the apparent diameter of the moon at the time, and with the sun-spot period. Its total light is always at least two or three times as great as that of the full moon.

Light given by the corona.

Drawings and Photographs of the Corona. — There is very great difficulty in getting accurate representations of this phenomenon. The two or three minutes during which only it is usually visible at any given eclipse do not allow time for trustworthy hand-work; at any rate, drawings of the same corona made even by good artists, sitting side by side, differ very much, — sometimes ridiculously. Photographs are better, so far as they go, but hitherto they have not succeeded in bringing out many details of the phenomenon which are easily visible to the eye; nor do the pictures which show well the outer portions of the corona generally bring out the details near the sun's limb, though an ingenious device of Burckhalter, which, by a whirling screen of peculiar form in front of the sensitive plate, gives a much longer exposure to the outer regions than to the parts near the sun, has greatly improved the results.

Drawings and photographs.

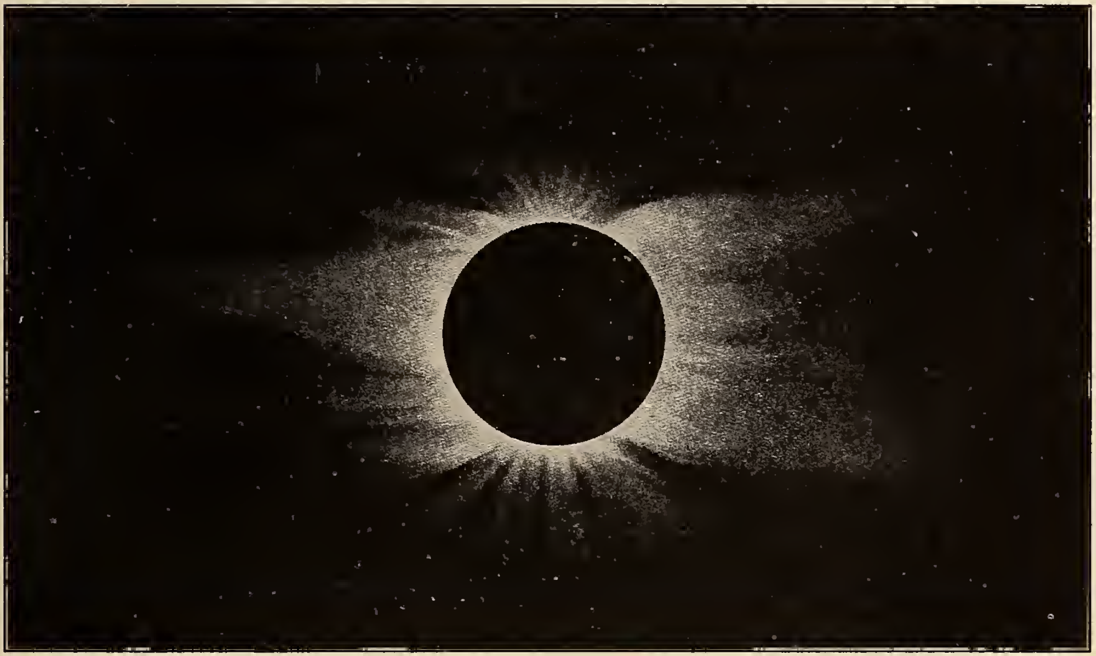


Fig. 106. — Corona of Eclipse of 1900, Wadesboro, N.C.

Fig. 105 is copied from an engraving made by combining several photographs of the eclipse of 1871. Fig. 106 is reduced from a drawing by Wesley of the corona of the eclipse of May, 1900, made from a combination of the sketches and photographs obtained by the members of the various eclipse parties of the British Astronomical Association. It is an admirable representation of what the writer saw at Wadesboro, N.C., except that the curved wings on the west and the long, pointed, eastern streamer could all be traced much farther by the eye, — to a distance fully three and a half times the moon's diameter, or at least 3 000000 miles.

The spectrum of the corona. The green line, $\lambda = 5304$.

263. Spectrum of the Corona. — The characteristic feature of the visual spectrum is a bright green line which lies very near, and was long supposed to be the "reversal" of, a dark line in the solar spectrum, — known as the "1474 line," because its position is at 1474 on the scale of Kirchhoff's map of the spectrum, generally used in 1869, when the corona line was first discovered. This identification, for which the writer was mainly responsible, turns out, however, to be erroneous, the wave-length of "1474" being about 5317 on Rowland's scale, while the wave-length of the real corona line, as first discovered from the photographs in 1898 (and since confirmed), is 5304.

The "1474" line (probably of iron) is always present in the *chromosphere* spectrum as a bright line, and at an eclipse when the corona first appears it is much the brightest line in the green part of the spectrum; but, as we now know, it fades out shortly, while the true corona line, which is much fainter, remains, of course, visible through the whole totality.

Coronium probably a gas of extremely low density.

The unknown substance which produces this corona line has been provisionally named "coronium," just as "helium" was named twenty-seven years before it was identified as a terrestrial element. It is probably a gas of extremely low density, — perhaps even lighter than hydrogen.

Besides this conspicuous green line there are several others, — five at least and probably more, — all in the violet or ultraviolet, all probably due to the same substance, and showing, like the principal line, but much more faintly, *as rings* on photographs made by the prismatic camera during the recent eclipses.

The hydrogen and helium lines, and H and K of calcium, have also been photographed as bright lines during eclipses and attributed to the corona; the later observations prove, however, that they are not really coronal, but are due to reflection (in our own atmosphere) of light coming from the chromosphere and prominences.

Hydrogen and helium not found in the corona.

The light of the corona is distinctly polarized: on one or two occasions observers have also reported in its visual spectrum the presence of dark Fraunhofer lines, and these have now at last been successfully photographed by the Lick observers during the Sumatra eclipse of 1901. The corona therefore unquestionably contains some matter which reflects sunlight.

Part of the light of the corona due to reflection.

264. Nature of the Corona. — The corona is proved to be a true solar appendage and not a mere optical phenomenon, nor due to either the atmosphere of the earth or moon, by two unquestionable facts: first, its spectrum as described above is not that of mere reflected sunlight, but of a glowing gas; and second, photographs of the corona made at widely different stations on the track of an eclipse show, in the main, details that are identical as seen at stations thousands of miles apart, and exhibit the motion of the moon across the coronal filaments. In a sense, then, the corona is a phenomenon of the sun's atmosphere, though this solar "atmosphere" is very far from bearing to the sun the same relations that are borne towards the earth by the air. The corona is not a simple spherical envelope of gas comparatively at rest and held in equilibrium by gravity, but other forces than gravity are dominant in it, and matter that is not gaseous probably abounds.

The corona a solar appendage.

Not related to the sun as our own atmosphere is related to the earth.

Its phenomena are not yet satisfactorily explained and remind us far more of auroral streamers and comets' tails than of anything that occurs in the lower regions of our terrestrial atmosphere. Indeed, there are many features which warrant something more than a mere suspicion that it is more or less analogous to Roentgen and cathode rays, due, as Professor

Speculations as to its nature and the forces which determine its form.

Bigelow suggests, to *ions* driven off from the molecules of the solar gases and controlled in their motions by electric and magnetic forces emanating from the sun. (See also Sec. 502.)

Its extreme rarity.

That the corona is composed of matter excessively rarefied is shown by the fact that in a number of cases comets are known to have passed directly through it (as, for instance, in 1882) without the slightest perceptible disturbance or retardation of their motion. Its mean density must, therefore, be almost inconceivably less than that of the best vacuum we are able to make by artificial means.

No method yet found for observing the corona at other times than eclipses.

Numerous attempts have been made to find a way of observing the corona without an eclipse, but thus far without any certain result. The spectroscopic method, which succeeds with the chromosphere and prominences, fails with the corona, because the lines of its spectrum are not bright enough; and, moreover, there are, in the ordinary solar spectrum, no dark lines to match them and help in forming a background. Furthermore, since the *streamers* of the corona are probably not entirely gaseous, but partly of dust-like constitution (giving, therefore, the spectrum of reflected sunlight), they at least would not be observable by this method.

THE SUN'S LIGHT AND HEAT

The sun's candle-power.

265. The Sun's Light. — By photometric methods (*Physics*, p. 328) we can compare the sun's light with that of a "standard candle" (*Physics*, p. 327), and we find that the sun gives us 1575 billions of billions (1575×10^{24}) times as much light as a standard candle would give at that distance.

Experiment shows that when the sun is overhead sunlight illuminates a white surface about 65000 times as brightly as a standard candle at one meter distance, or certainly not less than 70000 times, if we allow for the absorption of sunlight in our air. Multiplying this 70000 by the square of the sun's distance in meters $(15 \times 10^{10})^2$, we get the sun's "candle-power" as stated above. But the determination cannot claim any minute accuracy, because of the continual variations in the purity of our atmosphere and the difficulty in determining the losses of the sunlight before it reaches the photometric screen.

Sunlight compared with light from other bodies.

The light received from the sun is about 600000 times that of the moon, about 7000 000000 times that of Sirius, the brightest of the fixed stars, and about 50000 000000 times that of Vega or Arcturus.

Intensity of sunlight.

The *intensity* of sunlight is the amount of light emitted by each square unit of its surface, — a very different matter from its total quantity. From the best data at present obtainable (only rather rough approximations are possible) the sun's surface appears to be about 190000 times as bright as a candle flame, and about 150 times as bright as the calcium light. The brightest part of the electric arc-light — its "crater" — comes nearer to the brilliance of the solar surface than anything else we know of, being from one half to one fourth as bright.

266. Relative Brightness of Different Parts of the Sun's Disk. — As already stated (Sec. 233), the sun's disk is brightest at the center, but the variation is very slight until near the edge, where the brightness falls off very rapidly, so that at the limb itself it is not more than one third of the brightness at the center. The color is modified also, verging towards chocolate, because the blue and violet rays are much more affected than the red and yellow; this is the reason why the darkening at the limb of the sun is so much more conspicuous in the photographs than in the telescope.

Darkening of the limb of the sun and modification of color.

The sun stripped of its atmosphere would be blue and much more brilliant.

The darkening is unquestionably due to the general absorption of light by the lower parts of the sun's atmosphere, though it is difficult to determine just how much the sun's brightness is diminished by it. Different estimates vary considerably, but according to Langley, if the sun's atmosphere were removed, we should receive from two to five times as much light as now, and, moreover, its tint would be strongly *blue*, more blue even than that of an electric arc.

The sun's heat: its quantity expressed in calories. The solar constant.

267. The Quantity of Solar Heat; the Solar Constant. — *The solar constant is the number of heat-units which a square unit of the earth's surface, unprotected by any atmosphere, and exposed perpendicularly to the sun's rays, would receive in a unit of time.* (It is a somewhat doubtful assumption that this quantity is really "constant" under all the changing conditions of the solar system; but if not, the fact has not yet been shown by observation.) The heat-unit most used at present, by engineers at least, is the *calory*, which is the amount of heat required to raise the temperature of one kilogram of water 1° C. A smaller unit, only $\frac{1}{1000}$ part as great, is much used in scientific work, substituting in the definition a *gram* of water for the kilogram. This heat-unit is called the "small calory," — it might perhaps be named the "calorette."

The engineer's calory and the small calory, or calorette.

As the result of the best observations thus far made, it is found that the *solar constant* is between twenty-five and thirty *engineer's calories* per *square meter per minute*, or 3.0 "calorettes" per square *centimeter* per minute.[1]

Values obtained for the solar constant range from 19 to 40.

In what follows we shall use 30 as the solar constant, although Scheiner in 1899 sets it still higher, — at 40. The different determinations, since that first made by Pouillet and Herschel in 1838, range all the way from 19 to 40, — an indication of the extreme difficulty of the subject. At the earth's surface a square meter seldom actually receives more than fifteen calories in a minute, fully fifty per cent being lost, or diverted, in its passage through the atmosphere.

Method of determining the solar constant.

268. Method of determining the Solar Constant. — The principle is simple, though the practical difficulties are very great, and so far have prevented any high degree of accuracy.

The determination is made by *admitting a beam of sunlight, of*

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[1] There would be some advantage in stating the solar constant on the C.G.S. system, by dividing 3.0 by 60, the number of seconds in a minute. According to this, the solar constant equals 0.050 C.G.S., or $\frac{1}{20}$ of a "calorette," received on a square *centimeter* in *one second.*

known cross-section, to fall upon a known quantity of water, for a known time, and measuring the consequent rise of temperature.

It is necessary, however, to determine and allow for all heat lost by the apparatus during the experiment or received from other sources, and especially to take into account the effect of atmospheric absorption. This is the most difficult and uncertain part of the operation, since the absorption is continually changing, — capriciously with the meteorological conditions, and regularly with the changing altitude of the sun.

Difficulties and uncertainties.

It should be noted that the rays absorbed by the atmosphere, though diverted from the instrument which is endeavoring to measure their amount, *are not lost to the earth.* The *air* is illuminated and warmed by them, and the earth gets the benefit of the effect at second hand, so to speak.

For a description of the pyrheliometer and actinometer, with which the heat radiation is measured, and of the bolometer, with which the percentage of loss is determined for rays of differing wave-length, the student is referred to the *General Astronomy*, Arts. 340–345.

269. The Solar Heat at the Earth's Surface expressed in Terms of Melting Ice.—Since it requires 79¼ calories of heat to melt a kilogram of ice with a specific gravity of 0.92, it follows that, taking the solar constant at 30, the heat received from a vertical sun would melt in an hour a sheet of ice 24.7 millimeters, or very nearly an inch, in thickness. From this it is easily computed that the amount of heat received by the earth from the sun in a year is sufficient to melt a shell of ice 225 feet thick on the earth's equator, or 177 feet over the earth's entire surface, if the heat were equally distributed over all latitudes.

Solar heat expressed in terms of melting ice.

270. Solar Heat expressed as Energy. — Since according to the known value of the "mechanical equivalent of heat" (*Physics*, p. 175) a horse-power (33000 foot-pounds per minute) can easily be shown to be equivalent to about 10.7 calories per minute, it follows that each square meter of the earth's surface

Expressed as energy and in horse-power.

perpendicular to the sun's rays ought to receive about 2.8 horse-power continuously. Atmospheric absorption cuts this down to about 1¾ horse-power, of which about one eighth can be realized by a suitable machine, such as Ericsson's solar engine (Fig. 107), which for several years was exhibited annually in New York.

The solar engine.

The solar heat was concentrated by the large reflector, 18 feet in diameter, upon the *boiler*, which was a 6-inch iron tube. A "head" at the upper end (removed when the photograph was taken) received the steam, and a pipe connected it with the 4-horse-power engine shown underneath. When the sun shone it worked well.

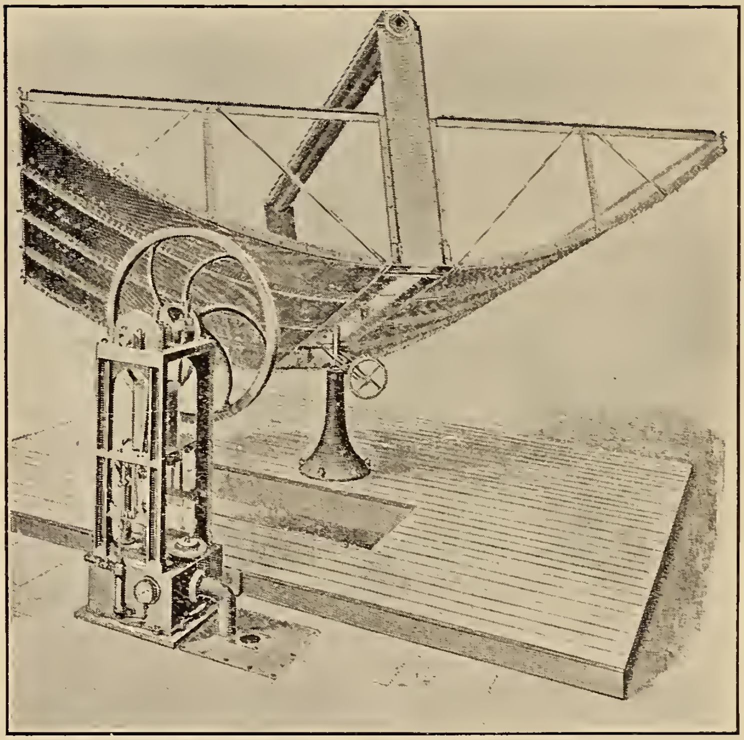

FIG. 107.—Ericsson's Solar Engine

The energy annually received from the sun by the whole of the earth's surface aggregates nearly 100 mile-tons to each square foot. That is, the average amount of heat annually received by each square foot of the earth's surface, if utilized in a theoretically perfect heat engine, would hoist nearly 100 tons to the height of a mile.

Solar radiation at the sun's surface.

271. Solar Radiation at the Sun's Surface.—If now we estimate the amount of radiation at the sun's surface itself, we come to results which are simply amazing. We must multiply the solar constant observed at the earth by the *square* of the ratio between 93 000000 miles (the earth's distance from the sun) and 433250 (the radius of the sun). This square is about 46000. In other words, the amount of heat emitted in a minute by a square meter of the sun's surface is about 46000 times as great as that received by a square meter at the earth. Carrying out the figures, we find that this heat radiation at the sun's surface amounts to *1400000 calories per square meter per minute;* that if *the sun were frozen over completely, to a depth of 64 feet*, the heat emitted would melt the shell in *one minute;* that if a bridge of ice could be formed from the earth to the sun by a column of ice 2½ miles square and 93000000 long, and if in some way the entire solar radiation could be concentrated upon it, it would be melted *in one second*, and in *seven more* would be dissipated in vapor.

Expressed in ice melting.

Expressing it as *energy*, we find that the solar radiation is nearly *130000 horse-power continuously for each square meter* of the sun's surface.

Expressed in horse-power.

So far as we can see, only a minute fraction of the whole radiation ever reaches a resting place. The earth intercepts about $\frac{1}{2200\,000000}$, and the other planets of the solar system receive in all perhaps from ten to twenty times as much. Something like $\frac{1}{100\,000000}$ seems to be utilized within the limits of the solar system. As for the rest, science cannot yet give any certain account of it.

Question as to what becomes of heat radiated into space.

All the conclusions stated in the two preceding sections are based on the assumption *that the sun radiates heat in all directions alike*, whether the rays do or do not meet a material surface. No reason can be assigned why this assumption should not be true, but it cannot be said to be *proved* as yet, either by experiment or by the nature of the case. If it should ever be shown to be incorrect, certain difficulties in the theory of planetary evolution would be greatly mitigated.

272. The Sun's Temperature; Effective Temperature. — As to the *temperature* of the sun's surface we have no exact knowledge, except that it must be higher than any artificial heat which we are able to produce. Indeed, it is only "by courtesy," so to speak, that the sun can be said to have *a* temperature,

The temperature of the sun: widely different at different points.

since the temperature at different elevations above and beneath the surface must differ enormously; nor, probably, is it the same in a sun-spot as in the faculæ, though we note in passing that observations indicate no systematic difference depending on position upon the sun's surface, *i.e.*, on solar latitude and longitude.

Effective temperature about 12000° F.

When we speak of the temperature of the sun we mean what is called the "effective temperature," *i.e.*, the temperature that a surface of *standard radiating power* (lampblack is the standard) would require in order to radiate heat at the same rate as the sun. If the actual surface of the sun has a radiating power inferior to that of the standard, as is probably the case, then the actual mean temperature must be higher than the effective, and *vice versa*.

If we knew absolutely the law which connects the radiation rate of a surface with its temperature, we could compute the effective temperature from the solar constant.

If we accept as correct Stefan's Law, which is borne out by the most trustworthy recent laboratory work, viz., that the rate of radiation is proportional to the *fourth power* of the *absolute temperature*,[1] the sun's "effective temperature" comes out about 7000° C. or 12000° F.

The highest temperatures artificially obtained in the electric arc are in the neighborhood of 4000° C.

The assumption of various other laws of radiation has led to a ridiculously wide range of computed solar temperatures, all the way from 1500° C., by Pouillet, to the millions of Secchi and Ericsson. And the result, as stated above, is doubtful by at least 500° C.

The burning lens.

273. The Burning Lens. — A most impressive demonstration of the intensity of the sun's heat lies in the fact that in the focus of a powerful burning lens all known substances melt and vaporize. Now at the focus of a lens *the temperature can never more than equal that of the source from which the heat comes.* Theoretically, the limit is that temperature which would be produced by the sun's direct radiation at a distance such that the sun's apparent diameter would just equal that of the lens viewed from its focus.

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[1] The absolute temperature is the temperature reckoned from the *absolute zero*, − 273° C. or − 449.4 F.

Highest temperature reached by burning lens.

The temperature produced at F (Fig. 108) would, if there were no losses, be just the same as that of a body placed so near the sun that the sun's angular diameter equals LFL'. Now, in the case of the most powerful lenses hitherto made, about 4 feet in diameter, a body at the focus was thus virtually carried to within about 240000 miles from the sun's surface (the distance of the moon from the earth), and here, as has been said, the most refractory substances succumb immediately.

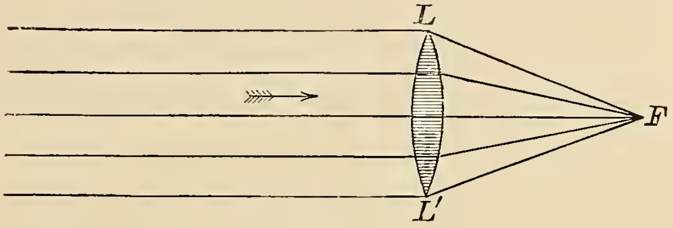

FIG. 108

A corroboration of the evidence of the burning lens is found in the great extension of the solar spectrum into the ultra-violet region and in the *penetrating* power of the solar rays. Rays coming from a source of comparatively low temperature — from a stove, for instance — are almost wholly absorbed by a plate of glass, while those of the sun pass almost without loss.

Constancy of the sun's heat: no sensible change for the past two thousand years, though temporary fluctuations are possible.

274. Constancy of the Sun's Heat.—It is an interesting question, as yet unanswered, whether the total amount of the sun's radiation does or does not perceptibly vary. There may be considerable fluctuations in the quantity of heat hourly received from the sun without our being able to detect them surely with our present means of observation, but as far as observations go there is no evidence that the total amount varies very much.

As to any steady, progressive increase or decrease of solar heat, it is quite certain that no important change of that kind has been going on for the past two thousand years, because the distribution of plants and animals on the earth's surface is practically the same as in the days of Pliny; it is, however, rather probable than otherwise that the general climatic changes which geology indicates as having formerly taken place on the earth — the glacial and carboniferous epochs, for instance — may ultimately be traced to changes in the sun's condition.

Maintenance of the sun's heat.

275. Maintenance of Solar Heat. — One of the most interesting and important problems of modern science relates to the explanation of the method by which the sun's heat is maintained. We cannot here discuss the subject fully, but must content ourselves with saying, —

Not to be accounted for by combustion or fall of meteoric matter.

(1) *Negatively*, that the phenomenon cannot be accounted for on the supposition that the sun is a hot, solid, or liquid body *simply cooling*, nor by *combustion*, nor (adequately) *by the fall of meteoric matter on the sun's surface*, though this cause undoubtedly operates to a limited extent.

Probably due to slow contraction of the sun's diameter.

(2) *Positively*, the solar radiation *can be* accounted for on the hypothesis, proposed first by Helmholtz, *that the sun is shrinking slowly but continuously.* It is a matter of demonstration that an annual shrinkage of about 300 feet in the sun's diameter would liberate heat sufficient to keep up its present observed radiation without any fall in its temperature. If the shrinkage were more than 300 feet, the sun would be hotter at the end of a year than it was at the beginning.

Shrinkage too small to be detected by observation as yet.

It is not possible to exhibit this hypothetical shrinkage as a fact of observation, since this diminution of the sun's diameter would amount to a mile only in 17.6 years, and nearly eight thousand years would be spent in reducing it by a single second of arc. No change much smaller than 1″ could be certainly detected even by our most modern instruments.

We can only say that while no other theory yet proposed meets the conditions of the problem, this appears to do so perfectly, and therefore has high probability in its favor, especially as it appears to be a mere continuation of the process by which the present solar system was evolved.

276. Lane's Law. — It was first pointed out by Lane of Washington in 1870 that a gaseous body, losing heat by radiation and contracting under its own gravity, *must rise in temperature* until it ceases to be a "perfect gas," either by beginning to liquefy, or by reaching a density at which the laws of "perfect gases" cease to hold. In a mass of perfect gas the "work" due to its shrinkage (like the work done by a descending clock weight) is more than sufficient to replace the loss of temperature due to its radiation, and it therefore becomes hotter. This is not the case with a mass of solid or liquid, which, as it loses heat and begins to liquefy or solidify, diminishes in temperature as well as dimensions, and grows colder.

Lane's Law: a mass of gas, contracting under its own gravity from loss of heat, rises in temperature until it ceases to be a perfect gas.

It appears that in the sun at present the relative proportion of true gases and liquids (the droplets which form the photospheric clouds) is such as to keep the temperature nearly stationary, — a condition which may endure for thousands of years.

CONSTITUTION OF THE SUN

The now generally received opinion on this subject may be summed up substantially as follows:

277. The Central Nucleus. — As to the condition of this we cannot claim certain knowledge, but many considerations lead to the conclusion that it is purely *gaseous* and has a temperature immensely higher than that of the solar surface even. But this central mass, while gaseous in that it follows essentially the characteristic laws of Dalton, Boyle, and Charles (*Physics*, p. 274), must be greatly condensed by the enormous pressure due to solar gravity; denser than water, and *viscous*, so to speak, like tar or pitch in resisting rapid motion within it.

The central nucleus gaseous.

Certain phenomena, however, such as the tendency of photospheric disturbances (sun-spots and faculæ) to break out repeatedly in the same region (Sec. 237), suggest something like a *quasi-solidity*, sufficient to lead to the definite localization of certain conditions at certain points below the photosphere; and there are other phenomena which rather tend in the same direction. Indeed, there are some who still hold to a solid or liquid nucleus for the sun.

The photosphere a cloud sheet.

278. The Photosphere. — The photosphere is believed to be a sheet of *luminous clouds*, constituted mechanically like terrestrial clouds, *i.e.*, of minute solid or liquid particles floating in gas. These photospheric clouds are supposed to be formed (just as clouds of rain and snow are formed in our own atmosphere) by the cooling and condensation of vapors at the solar surface where they are exposed to the cold of outer space, and they float in the permanent gases of the solar atmosphere in the same way that our own clouds do on our own atmosphere.

We do not know precisely what materials constitute the photosphere, but naturally suppose them to be those indicated by the Fraunhofer lines, *i.e.*, chiefly the metals, with *carbon* and its chemical congeners.

Difficulties of the cloud-sheet theory.

But this cloud theory of the photosphere is not without its difficulties. It is embarrassed by the fact that we know of no substance that remains solid or liquid, even at a temperature anywhere near that which seems to prevail at the solar surface. *Carbon*, perhaps the most refractory of known substances, vaporizes completely at a temperature of about 7000° F. Still the temperature of 12000° F. ascribed to the solar surface is only the "effective" average temperature, and possibly is not inconsistent with the hypothesis that the "granules" of the photosphere are due to *local coolings* caused by explosive expansion of vapors forced up from below by tremendous pressure into or through a gaseous envelope of much higher temperature.

Some are disposed to evade the difficulty by invoking an electrical action of some kind, but as yet in too vague a manner to permit intelligent criticism.

Schmidt's optical theory of the photosphere.

We merely mention an ingenious theory proposed a few years ago by Prof. A. Schmidt of Stuttgart, viz., that the photosphere is a purely optical phenomenon, a sort of *mirage* so to speak, the sun itself being entirely gaseous. The theory, based wholly on optical principles, has some strong points; but it ignores many spectroscopic facts, and the fundamental laws of physics seem to make it certain that a globe containing iron and the other metals in the state of vapor *must* inevitably form a photospheric shell of "cloud" in the outer portions exposed to radiation, thus "clothing itself with light as with a garment."

279. The Solar Atmosphere; the Reversing Layer, Chromosphere, and Prominences. — As has just been said, the photospheric clouds float in, and under, an atmosphere containing a considerable quantity of the same vapors out of which they themselves have been formed. This vapor-laden atmosphere, probably comparatively shallow, constitutes the *reversing layer*. By its *general* absorption it produces the peculiar darkening at the limb of the sun, and by its *selective* absorption it produces the dark Fraunhofer lines or solar spectrum. It will be remembered that Sir Norman Lockyer and others have been disposed to question the existence of any such shallow absorbing stratum, but that the photographs made at the recent eclipses seem to establish its reality.

The reversing layer: the part of the solar atmosphere immediately enveloping the photospheric clouds.

The *chromosphere* and *prominences* are chiefly composed of permanent gases, mainly hydrogen, helium, and calcium, which, near the photosphere, are mingled with the vapors of the reversing stratum, but rise to far greater elevations than those vapors. The appearances are for the most part as if the chromosphere were formed of jets of heated gases, ascending through the interspaces between the photospheric clouds, like flames playing over a coal fire.

The chromosphere and prominences composed mainly of permanent gases.

280. The Corona also rests at its base on the photosphere, and the characteristic green line of its spectrum is brightest just at the surface of the photosphere, in the reversing stratum, and in the chromosphere itself; but it extends far beyond even the loftiest prominences to distances sometimes of several millions of miles. It seems to be not entirely gaseous, but to contain, in addition to the mysterious coronium, *dust and fog* of some sort, very likely of meteoric origin. Many of the phenomena of the corona are still unexplained, and since thus far it has

The corona still to a great extent a mystery.

been observable only during the brief moments of solar eclipses, progress in its study has been necessarily slow. No observer has yet seen the corona for a sum total of time amounting to fifteen minutes.

Fig. 109 (from *The Sun*, by permission of D. Appleton & Co.) presents the theory stated above, though the distinction between the photospheric cloud shell and the chromosphere is hardly brought out as clearly as desirable; nor is it certain that all the spots are *cavities*, as represented.

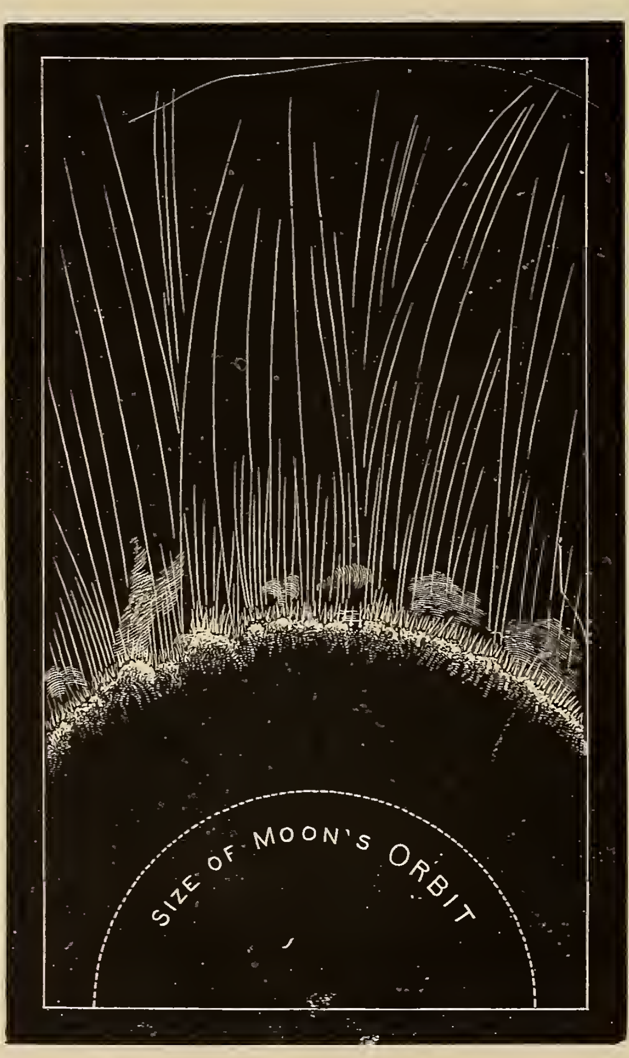

FIG. 109. — Constitution of the Sun
From *The Sun*, by permission of the publishers

EXERCISES

1. Assuming Faye's equation (Sec. 230) for the solar rotation, what are the rotation periods at the sun's equator, in solar latitude 30°, in latitude 45°, and at the pole?

$$\textit{Ans.}\begin{cases}\text{At equator, } & 25.06 \text{ days.}\\ \text{Lat. } 30^\circ, & 26.49 \quad \text{``}\\ \text{Lat. } 45^\circ, & 28.09 \quad \text{``}\\ \text{At pole, } & 31.95 \quad \text{``}\end{cases}$$

2. Assuming Spoerer's equation (Sec. 230), what would be the results?

3. What would be the synodic or apparent time of rotation for a spot in latitude 45° according to Faye's equation? *Ans.* 30.43 days.

4. If the diameter of the sun were doubled, its density remaining unchanged, what would be the force of gravity at its surface?

5. If the sun were expanded into a homogeneous sphere, with a radius equal to the distance of the earth from the sun, its mass remaining unchanged, what would be the force of gravity at its surface? *Ans.* $\frac{1}{1659}$ of g.

6. In this case, what change, if any, would result in the orbit of the earth? *Ans.* None.

7. In the neighborhood of a sun-spot a point is found in its spectrum where a portion of the C line ($\lambda = 6563.0$) is deflected to 6566.0. What is the velocity (in the line of sight) of the hydrogen at that point? (See Sec. 255.) *Ans.* 85.17 miles receding.

8. How great is the displacement of the hydrogen line F ($\lambda = 4861.5$) at that point? *Ans.* 2.22 units (of wave-length).

9. How great a displacement is produced in the line D ($\lambda = 5896.16$) by a velocity of 100 miles a second? *Ans.* 3.16 units.

10. If a luminous body were moving towards us with a velocity one fourth that of light, what would be the effect upon the apparent length of the portion of the spectrum included between two given lines, — say C and F?

11. What if it were moving towards us with the speed of light, and what if it were receding at that rate?

12. What if the *observer* were receding with the speed of light, and what if he were moving towards it at that rate?

13. If the diameter of the sun is decreasing at the rate of 300 feet a year, how long before its apparent diameter will have decreased by 1″?
Ans. 7927 years.

14. If the rate of shrinkage be assumed to continue uniform (*i.e.*, 300 feet a year — an impossible assumption), how long would it be before its diameter is diminished by 1%? *Ans.* Over 150000 years.

15. How much would its mean density then be increased?
Ans. About 3%.

16. Taking the "calory" as equivalent to 428 kilogrammeters of energy, what weight falling 100 meters to the surface of the earth would, at the end of its fall, possess an energy equal to that of the solar radiation received in an hour upon 10 square meters of the earth's surface, admitting a loss of 50% absorbed by the air? *Ans.* 38520 kgm.

17. Assuming that sunlight at the earth equals 70000 times that of a standard candle at a distance of 1 meter, at what distance would the light of the sun equal that of a 2000 candle-power electric arc 10 meters distant? *Ans.* About 59 times the earth's distance.

18. How does the illumination of a surface by an arc-light of 2000 candle-power at a distance of 1 meter compare with its illumination by sunlight? *Ans.* $\frac{1}{30}$.

CHAPTER X

ECLIPSES

Form and Dimensions of Shadows — Eclipses of the Moon — Solar Eclipses — Total, Annular, and Partial — Ecliptic Limits and Number of Eclipses in a Year — Recurrence of Eclipses and the Saros — Occultations

Eclipses caused by shadows.

281. The word "eclipse" is a term applied to the darkening of a heavenly body, especially of the sun or moon, though some of the satellites of other planets besides the earth are also "eclipsed." An eclipse of the *moon* is caused by its passage through the shadow of the earth; eclipses of the *sun*, by the interposition of the moon between the sun and the observer, or, what comes to the same thing, by the passage of the moon's shadow over the observer.

Shadows of earth and moon are cones.

The shadow which causes an eclipse is the space from which sunlight is excluded by an intervening body; geometrically speaking, it is a *solid*, not a *surface*. If we regard the sun and the other heavenly bodies as spherical, these shadows are *cones* with their axes in the line joining the centers of the sun and the shadow-casting body, the point being always directed away from the sun.

Dimensions of the shadow of the earth.

282. Dimensions of the Earth's Shadow. — The length of the earth's shadow is easily found. In Fig. 110 we have, from the similar triangles, OED and ECa,

$$OD : OE :: Ea : EC, \text{ or } L.$$

OD is the difference between the radii of the sun and the earth, $= R - r$. $Ea = r$, and OE is the distance of the earth from the sun $= D$. Hence,

$$L = D\left(\frac{r}{R - r}\right) = \frac{1}{108.5} D.$$

Length of earth's shadow 857000 miles.

(The fraction 108.5 is found by simply substituting for R and r their values, R being $109.5 \times r$.) This gives 857000 miles for the length of the earth's shadow when D has its mean value of 93 000000 miles. The length varies about 14000 miles on each side of the mean, in consequence of the variation of the earth's distance from the sun at different times of the year.

From the cone aCb all sunlight is excluded, or would be were it not for the fact that the atmosphere of the earth by its refraction bends some of the rays into this shadow. The effect of this atmospheric refraction is to increase the diameter of the shadow about two per cent where the moon crosses it, but to make it less perfectly dark.

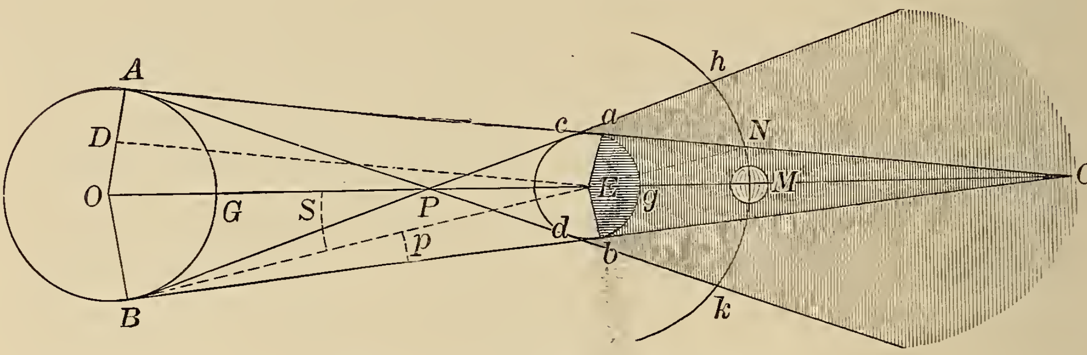

Fig. 110. — The Earth's Shadow

The penumbra.

283. Penumbra. — If we draw the lines Ba and Ab (Fig. 110), crossing at P, between the earth and the sun, they will bound the *penumbra*, within which a part, but not the whole, of the sunlight is cut off; an observer outside of the shadow but within this cone frustum, which tapers towards the sun, would see the earth as a black body encroaching on the sun's disk.

Boundaries of shadow and penumbra optically indefinite, though geometrically definite.

While the boundaries of the shadow and penumbra are perfectly definite *geometrically*, they are not so optically. If a screen were placed at M, perpendicular to the axis of the shadow, no sharply defined lines would mark the boundaries of either shadow or penumbra. Near the edge of the shadow the penumbra would be very nearly as dark as the shadow itself, only a mere speck of the sun being there visible; and at the outer edge of the penumbra the shading would be still more gradual.

284. Eclipses of the Moon. — The axis of the central line of the earth's shadow is always directed to a point directly opposite the sun. If, then, at the time of the full moon, the moon happens to be near the ecliptic (*i.e.*, not far from one of the nodes of her orbit), she will pass through the shadow and be eclipsed. Since, however, the moon's orbit is inclined to the ecliptic at an average angle of 5° 8′, lunar eclipses do not happen very frequently, — seldom more than twice a year. Ordinarily the full moon passes north or south of the shadow without touching it.

Why eclipses of moon do not occur at every full moon.

Lunar eclipses are of two kinds, — partial and total: total when she passes completely into the shadow, partial when she only partly enters it, going so far to the north or south of the center of the shadow that only a portion of her disk is obscured.

Partial and total eclipses of the moon.

285. Size of the Earth's Shadow at the Point where the Moon crosses it. — Since *EC*, in Fig. 110, is 857000 miles, and the distance of the moon from the earth is on the average about 239000 miles, *CM* must average 618000 miles, so that *MN*, the semidiameter of the shadow at this point, will be $\frac{618}{857}$ of the earth's radius. This gives $MN = 2854$ miles, and makes the whole diameter of the shadow a little over 5700 miles, — about two and two-thirds times the diameter of the moon. But this quantity varies considerably with the moon's distance; the shadow, where she crosses it, is sometimes more than three times her diameter, sometimes hardly more than twice.

Diameter of earth's shadow where the moon crosses it.

An eclipse of the moon, when *central*, *i.e.*, when the moon crosses the center of the shadow, may continue *total* for about two hours, the interval from the first contact to the last being about two hours more. This depends upon the fact that the moon's hourly motion is nearly equal to its own diameter.

Duration of a lunar eclipse.

The duration of a non-central eclipse varies, of course, according to the part of the shadow traversed by the moon.

Lunar ecliptic limits: 9° 30′ to 12° 15′.

286. Lunar Ecliptic Limit. — The lunar *ecliptic limit* is the greatest distance from the node of the moon's orbit at which the sun can be at the time of a lunar eclipse. This limit depends upon the inclination of the moon's orbit, which is somewhat variable, and also upon the distance of the moon from the earth at the time of the eclipse, which is still more variable. Hence, we recognize two limits, — the major and minor.

If the distance of the sun from the node at the time of full moon exceeds the major limit, an eclipse is impossible; if it is less than the minor, an eclipse is inevitable. The major limit is found to be 12° 15′; the minor, 9° 30′.

Since the sun, in its annual motion along the ecliptic, travels 12° 15′ in less than thirteen days, it follows that every eclipse of the moon must take place within thirteen days from the time when the sun crosses the node.

Phenomena of a lunar eclipse.

287. Phenomena of a Total Lunar Eclipse. — Half an hour or so before the moon reaches the shadow, its limb begins to be sensibly darkened by the penumbra, and the edge of the shadow itself when it first attacks the moon appears nearly black by contrast with the bright parts of the moon's surface. To the naked eye the outline of the shadow looks reasonably sharp; but even with a small telescope it is found to be indefinite, and with a large telescope and high magnifying power it becomes entirely indistinguishable, so that it is impossible to determine within about half a minute the time when the boundary of the shadow reaches any particular point on the moon. After the moon has wholly entered the shadow her disk is usually

Moment of beginning not accurately observable.

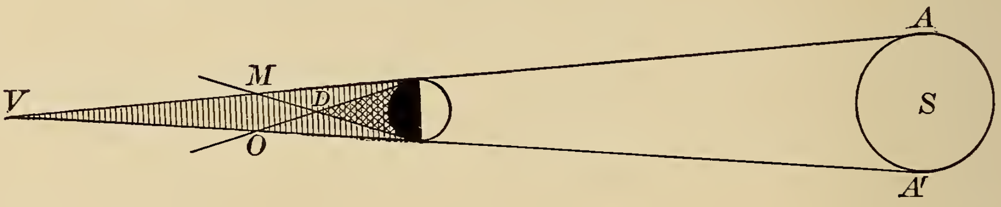

Fig. 111. — Light bent into Earth's Shadow by Refraction

distinctly visible, illuminated with a dull, copper-colored light, which is sunlight, deflected around the earth into the shadow by the refraction of our atmosphere, as illustrated by Fig. 111.

Even when the moon is exactly central in the largest possible shadow, an observer on the moon would see the disk of the earth surrounded by a narrow ring of brilliant light, colored with sunset hues by the same vapors which tinge terrestrial sunsets, but acting with double power because the light has traversed a double thickness of our air. If the weather happens to be clear at this portion of the earth (upon its *rim*, as seen from the moon), the quantity of light transmitted through our atmosphere is very considerable, and the moon is strongly illuminated. If, on the other hand, the weather happens to be stormy in this region of the earth, the clouds cut off nearly all the light. In the lunar eclipse of 1884 the moon was absolutely invisible for a time to the naked eye, — a very unusual circumstance on such an occasion.

Color of the eclipsed moon. Illumination caused by light deflected into the shadow by our atmosphere.

The heat radiation of the moon, according to the observations of Lord Rosse, falls off during the progress of the eclipse, almost in the same ratio with the light. At the moment when the eclipse becomes total fully ninety-eight per cent of the heat has disappeared, and half of the remaining two per cent is lost during the totality. As the light returns the heat rises almost as rapidly as it fell, showing that the moon's surface has very little power of *storing* heat, — a natural consequence of its airlessness; but it is several hours before the heat radiation recovers fully the value it had before the eclipse.

Effect upon the moon's heat radiation.

If the eclipse is well visible in both hemispheres, arrangements are usually made to observe as many *star occultations* as possible during the totality, for the purpose of determining the moon's place and parallax, and for other purposes also.

288. Computation of a Lunar Eclipse. — Since all the phases of a lunar eclipse are seen everywhere at the same absolute instant wherever the moon is above the horizon, it follows that a single computation giving the Greenwich times of the different phenomena is all that is needed. Such computations are made and published in the Nautical Almanac. Each

Why the calculation of a lunar eclipse is simple.

observer has only to correct the predicted time by simply adding or subtracting his longitude from Greenwich, in order to get the true local time. The computation of a lunar eclipse is not at all complicated.

For the method of projecting and computing a lunar eclipse, see Appendix, Secs. 703 and 704.

ECLIPSES OF THE SUN

Length of the moon's shadow.

289. Dimensions of the Moon's Shadow. — By the same method as that used for the shadow of the earth (Sec. 282) we find that the length of the *moon's* shadow at any time is very nearly $\frac{1}{400}$ of its distance from the sun, and *averages* 232150 miles. It varies not quite 4000 miles each way, ranging from 228300 to 236050 miles.

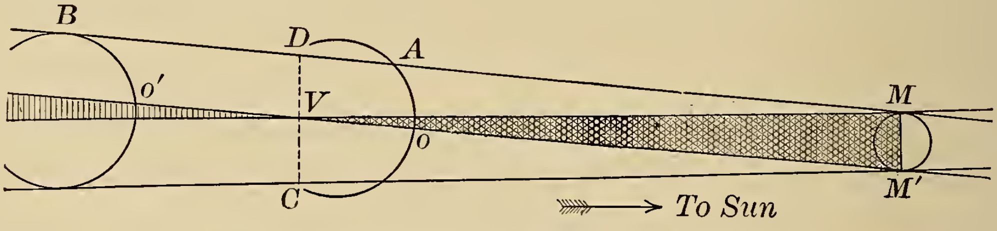

FIG. 112. — The Moon's Shadow on the Earth

Since the mean length of the shadow is less than the mean distance of the moon from the earth (238800 miles), it is evident that *on the average* the shadow will not reach the earth.

Maximum diameter of cross-section of moon's shadow on the earth 168 miles.

On account of the eccentricity of the moon's orbit, she is much of the time considerably nearer than at others and may come within 221600 miles from the earth's center, or about 217650 miles from its surface. If at the same time the shadow happens to have its greatest possible length, its point may reach nearly 18400 miles beyond the earth's surface. In this case the cross-section of the shadow where the earth's surface cuts it squarely (at *o* in Fig. 112) will be about *168 miles in diameter, which is the largest value possible.* If, however, the shadow strikes the earth's surface obliquely, the shadow spot will be oval instead of circular, and the extreme *length* of the oval may much exceed the 168 miles.

Maximum diameter of the shadow produced 196 miles.

Since the distance of the moon may be as great as 252970 miles from the earth's center, or nearly 249000 miles from its surface, while the shadow may be as short as 228300 miles, we may have the state of things indicated by placing the earth at *B* in Fig. 112. The vertex of the shadow, *V*, will then fall 21000 miles short of the surface, and the cross-section of the *shadow produced* will have a diameter of 196 miles at *o′*, where the earth's surface cuts it. When the shadow falls near the edge of the earth the breadth of this cross-section may be as great as 230 miles.

Total and annular eclipses.

290. Total and Annular Eclipses. — To an observer within the true shadow cone (*i.e.*, between *V* and the moon in Fig. 112) the sun will be *totally* eclipsed. An observer in the "produced" cone beyond *V* will see the moon smaller than the sun, leaving an uneclipsed ring around it, and will have what is called an *annular*, or "ring-formed," eclipse. These annular eclipses are considerably more frequent than the total, and now and then an eclipse is annular in part of its course across the earth and total in part. (The point of the moon's shadow extends in this case beyond the nearest part of the surface of the earth, but does not reach as far as its center.)

Width of the belts of partial eclipse on each side of the central line.

291. The Penumbra and Partial Eclipses. — The penumbra can easily be shown to have a diameter on the line *CD* (Fig. 112) of a trifle more than twice the moon's diameter. An observer situated within the penumbra has a *partial* eclipse. If he is near the cone of the shadow, the sun will be mostly covered by the moon; but if near the outer edge of the penumbra, the moon will only slightly encroach on the sun's disk. While, therefore, *total and annular* eclipses are visible as such only by an observer within the narrow path traversed by the shadow spot, the same eclipse will be visible as a *partial* one everywhere within 2000 miles on each side of the path. The 2000 miles is to be reckoned perpendicularly to the axis of the shadow, and may correspond to a much greater distance on the spherical surface of the earth.

292. Velocity of the Shadow and Duration of Eclipses. — Were it not for the earth's rotation, the moon's shadow would pass an observer at the rate of nearly 2100 miles an hour on the average. The earth, however, is rotating towards the east in the same general direction as that in which the shadow moves, and at the equator its surface moves at the rate of about 1040 miles an hour. An observer, therefore, on the earth's equator with the moon at its mean distance from the earth and near the zenith would, on the average, be passed by the shadow with a speed of about 1060 miles an hour (2100 – 1040), — about equal to that of a cannon-ball. In higher latitudes, where the surface velocity due to the earth's rotation is less, the relative speed of the shadow is higher; and where the shadow falls very obliquely, as it does when an eclipse occurs near sunrise or sunset, the advance of the shadow on the earth's surface may become very swift, — as great as 4000 or 5000 miles an hour.

Velocity of the moon's shadow over the earth's surface.

A *total* eclipse of the sun observed at a station near the equator, under the most favorable conditions possible, may continue total for 7^m58^s. In latitude 40° the duration can barely equal $6\frac{1}{4}^m$. The greatest possible excess of the apparent semidiameter of the moon over that of the sun is only $1' 19''$.

Maximum possible duration of a total solar eclipse 7^m58^s.

At the equator an *annular* eclipse may last for 12^m24^s, the maximum width of the ring of the sun visible around the moon being $1' 37''$.

Maximum for an annular eclipse 12^m24^s.

In the observation of an eclipse four contacts are recognized: the *first* when the edge of the moon first touches the edge of the sun, the *second* when the eclipse becomes total or annular, the *third* at the cessation of the total or annular phase, and the *fourth* when the moon finally leaves the solar disk. From the first contact to the fourth the time may be a little over four hours.

293. The Solar Ecliptic Limits. — It is necessary, in order to have an eclipse of the sun, that the moon should encroach on

Solar ecliptic limits.

the cone *ACBD* (Fig. 113), which envelops the earth and sun. In this case the true angular distance between the centers of the sun and moon, *i.e.*, their distance as seen from the center of the earth, would be the angle *MES*.[1] This angle may range from 1° 34′ 13″ to 1° 24′ 19″, according to the changing distance of the sun and moon from the earth. The corresponding distances of the sun from the node, taking into account also the variations in the inclination of the moon's orbit, give 18° 31′ and 15° 21′ for the *major* and *minor* ecliptic limits.

Limits for partial eclipse 15° 21′ to 18° 31′.

In order that an eclipse may be *central* (total or annular) at any part of the earth, it is necessary that the moon should lie *wholly inside* the cone *ACBD*, as *M′*, and the corresponding *major* and *minor central* ecliptic limits come out 11° 50′ and 9° 55′.

Limits for central eclipse 9° 55′ to 11° 50′.

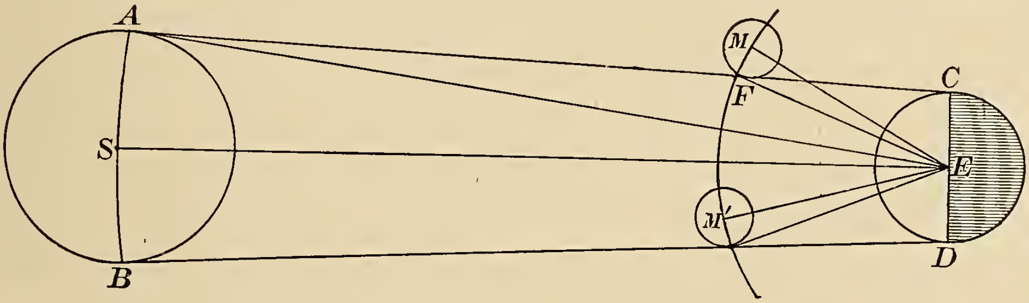

FIG. 113. — Solar Ecliptic Limits

294. Phenomena of a Solar Eclipse. — There is nothing of special interest until the sun is nearly covered, though before that time the shadows cast by the foliage begin to be peculiar.

Phenomena of a total solar eclipse.

The light shining through every small interstice among the leaves, instead of forming as usual a *circle* on the ground, makes a little *crescent*, — an image of the partly covered sun.

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[1] *MES* equals the sun's angular semidiameter *SEA* + the moon's semidiameter *MEF* + the angle *AEF*; and *AEF* equals the difference between *EFC*, the moon's parallax, and *CAE*, the parallax of the sun; hence, as usually written, *MES*, the "radius of the shadow," $= S + S' + P - p$, P being the parallax of the moon, and p that of the sun.

Effect on color of sunlight just before and after totality.

About ten minutes before totality the darkness begins to be felt, and the remaining light, coming, as it does, from the *edge* of the sun alone, is much altered in quality, being very deficient in the *blue* and *violet*, so that it produces an effect very like that of a calcium light rather than sunshine. Animals are perplexed, and birds go to roost. The temperature falls, and sometimes dew appears. In a few moments, if the observer is so situated that his view commands the distant horizon, the moon's shadow is seen coming, much like a heavy thunderstorm, and advancing with almost terrifying swiftness. Just before the shadow reaches the observer, quivering, ripple-like bands appear on every white surface; and immediately on its arrival, and sometimes a little before, the corona and prominences become visible, while the brighter planets and the stars of the first two or three magnitudes make their appearance. The suddenness with which the darkness falls is startling. The sun is so brilliant that even the small portion which remains visible up to within a very few seconds of the total obscuration so dazzles the eye that it is unprepared for the sudden transition. In a few moments, however, vision adjusts itself, and it is then found that the darkness is not really very intense.

Advance of the shadow, and the shadow bands.

Appearance of the corona and prominences and of the stars.

Darkness usually not very intense.

If the totality is of short duration (that is, if the diameter of the moon exceeds that of the sun by less than a minute of arc), the corona and chromosphere, the lower parts of which are very brilliant, give a light at least three or four times that of the full moon. Since, moreover, in such a case the shadow is of small diameter, a large quantity of light is also sent in from the surrounding air, where, 30 or 40 miles away, the sun is still shining. In such an eclipse there is not much difficulty in reading an ordinary watch face. In an eclipse of long duration, say five or six minutes, it is much darker, and lanterns become necessary.

Observations to be made.

295. Observation of an Eclipse. — A total solar eclipse offers opportunities for numerous observations of great importance which are possible at no other time, besides certain others which can also be made during a partial eclipse. We mention:

(*a*) Times of the four contacts, and direction of the line joining the "cusps" of the partially eclipsed sun. These observations determine with extreme accuracy the relative positions of the sun and moon at the moment. (*b*) The search for intra-mercurial planets. (*c*) Observations of certain peculiar dark fringes, the so-called "shadow bands," which appear upon the surface of the earth for about a minute before and after totality. (*d*) Photometric measurement of the intensity of light at different stages of the eclipse. (*e*) Telescopic observations of the details of the prominences and of the corona. (*f*) Spectroscopic observations (both visual and photographic), upon the "flash spectrum" and upon the spectra of the lower atmosphere of the sun, of the prominences and of the corona. (*g*) Observations with the polariscope upon the polarization of the light of the corona. (*h*) Drawings and photographic pictures of the corona and prominences. (*i*) Miscellaneous observations upon meteorological changes during the progress of the eclipse, — barometer, thermometer, wind, etc., — and effects upon the magnetic elements.

296. Calculation of a Solar Eclipse. — The calculation of a solar eclipse cannot be dealt with in any such summary way as that of a lunar eclipse, because the times of contact and other phenomena are different at every different station. Moreover, since the phenomena of a solar eclipse admit of extremely accurate observation, it is necessary to take account of numerous little details which are of no importance in lunar eclipses. The Nautical Almanacs give, three years in advance, a chart of the track of every solar eclipse, and with it data for the accurate calculation of the phenomena at any given place.

The calculation of a solar eclipse much more laborious than that of a lunar, because the circumstances depend on the place of the observer.

T. Oppolzer, a Viennese astronomer, no longer living, published a few years ago a remarkable book, entitled *The Canon of Eclipses*, containing the elements of all eclipses (8000 solar and 5200 lunar) occurring between the year 1207 B.C. and A.D. 2162, with maps showing the approximate track of the moon's shadow on the earth. It indicates total eclipses visible in the United States in 1918, 1923, 1925, 1945, 1979, 1984, and 1994.

Number of eclipses in a year.

297. Number of Eclipses in a Year.—The least possible number is *two*, both of the sun; the largest *seven*, five solar and two lunar or four solar and three lunar. The *most usual* number of eclipses is four.

The eclipse months and the eclipse year.

The eclipses of a given year always take place at two opposite seasons (which may be called the *eclipse months* of the year), near the times when the sun crosses the nodes of the moon's orbit. Since the nodes move westward around the ecliptic once in about nineteen years (Sec. 192), the time occupied by the sun in passing from a node to the same node again is only 346.62 days, which is sometimes called the *eclipse year*.

In an *eclipse year* there can be but *two lunar* eclipses, since twice the maximum lunar ecliptic limit (2 × 12° 15′) is less than 29° 6′, the distance the sun moves along the ecliptic in a synodic month; the sun therefore cannot possibly be near enough the node at *both* of two successive full moons; on the other hand, it is possible for a year to pass without any lunar eclipse, the sun being too far from the node at all four of the full moons which occur nearest to the time of its node passage.

Lunar eclipses in a year range from none to three.

In a *calendar* year (of 365¼ days) it is, however, possible to have *three* lunar eclipses. If one of the moon's nodes is passed by the sun in January, it will be reached again in December, the other node having been passed in the latter part of June, and there may be a lunar eclipse at or near each of these three node passages. This actually occurred in 1852 and 1898, and will happen again in 1917.

As to solar eclipses, it is sufficient to say that the solar ecliptic limits are so much larger than the lunar that there *must be at least one solar* eclipse at each node passage of the year, at the new moon next before or next after it; and there may be *two*, one before and one after, thus making four in the eclipse year.

(When there are two solar eclipses at the same node, there will always be a lunar eclipse at the full moon between them.) In the *calendar* year a fifth solar eclipse may come in if the first eclipse month falls in January. Since a year with five *solar* eclipses in it is sure to have two lunar eclipses in addition, they will make up *seven* in the calendar year. This will happen next in 1935; but in 1917 there will also be seven eclipses, — four of the sun and three of the moon.

Solar eclipses range from two to five.

Greatest possible number of eclipses in a year seven; least number two, both of the sun.

298. Frequency of Solar and Lunar Eclipses. — Taking the whole earth into account, the solar eclipses are the more numerous, nearly in the ratio of *three to two. It is not so, however, with those which are visible at a given place.* A solar eclipse can be seen only from a limited portion of the globe, while a lunar eclipse is visible over considerably more than half the earth, — either at the beginning or the end, if not throughout its whole duration; and this more than reverses the proportion between lunar and solar eclipses for any given station.

Relative frequency of solar and lunar eclipses.

Solar eclipses that are total somewhere or other on the earth's surface are not very rare, averaging one for about every year and a half. But *at any given place* the case is very different; since the track of a solar eclipse is a very narrow path over the earth's surface, averaging only 60 or 70 miles in width, we find that in the long run a total eclipse happens at any given station only once in about 360 years.

Rareness of total solar eclipses at any given station.

During the nineteenth century seven shadow tracks traversed the United States, and there will be the same number in the twentieth.[1]

299. Recurrence of Eclipses; the Saros. — It was known to the Chaldeans, even in prehistoric times, that eclipses occur at a regular interval of $18^{y}11\frac{1}{3}^{d}$ ($10\frac{1}{3}$ days, if there happen to be *five* leap-years in the interval). They named this period the *Saros.* It consists of 223 synodic months, containing 6585.32 days, while 19 *eclipse years* contain 6585.78. The difference

The Saros.

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[1] This does not take into account our insular possessions.

is only about 11 hours, in which time the sun moves on the ecliptic about 28′.

Number of eclipses in one Saros.

If, therefore, a solar eclipse should occur to-day with the sun *exactly* at one of the moon's nodes, at the end of 223 months the new moon will find the sun again close to the node (28′ *west* of it), and a very similar eclipse will occur again; but the track of this new eclipse will lie about 8 hours of longitude further west on the earth, because the 223 months exceed the even 6585 days by $\frac{32}{100}$ of a day. The usual number of eclipses in a Saros is about seventy-one, varying two or three one way or the other.

Star occultations.

300. Occultations of Stars. — In theory and computation the occultation of a star is identical with a total solar eclipse, except that the shadow of the moon cast by the star is sensibly *a cylinder* instead of a cone, and has no penumbra. Since the moon always moves eastward, the star disappears at the moon's eastern limb, and reappears on the western. Under all ordinary circumstances both disappearance and reappearance are instantaneous, indicating not only that the moon has no sensible atmosphere, but also that the (angular) diameter of even a very bright star is less than 0″.02. Observations of occultations determine the place of the moon in the sky with great accuracy, and when made at a number of widely separated stations they furnish a very precise determination of the moon's parallax and also of the difference of longitude between the stations.

Suddenness of the disappearance and reappearance of the star.

Occasionally the star, instead of disappearing suddenly when struck by the moon's limb (faintly visible by "earth-shine"), appears to cling to the limb for a second or two before vanishing. In a few instances it has been reported as having reappeared and disappeared a second time, as if it had been for a moment visible through a rift in the moon's crust. In some cases the anomalous phenomena have been explained by the subsequent discovery that the star was double, but many of them still remain mysterious; it is quite likely that they were often *illusions* due to physiological causes in the observer.

Anomalous phenomena sometimes observed at occultations.

CHAPTER XI

CELESTIAL MECHANICS

The Laws of Central Force — Circular Motion — Kepler's Laws, and Newton's Verification of the Theory of Gravitation — The Conic Sections — The Problem of Two Bodies — The Parabolic Velocity — Exercises — The Problem of Three Bodies and Perturbations — The Tides

It is out of the question to attempt here an extended treatment of the theory of the motions of the heavenly bodies, but there are certain fundamental facts and principles easily understood and so important, and indeed essential, to an intelligent comprehension of the mechanism of the solar system that we cannot pass them without notice.

Motion of body not acted on by force.

301. Motion of a Body not acted upon by Any Force. — According to the first law of motion, *a moving body left to itself describes a straight line with a uniform speed.* When, therefore, we find a body so moving we may infer that it is acted on by *no* force whatever or, at least, that if any forces are acting, they exactly balance each other, their resultant being zero, and absolutely without effect upon the motion of the body.

No projectile force required to maintain free motion.

It is a common blunder to speak of such a body as actuated by a "projectile force," — a survival of the Aristotelian idea that rest is more natural to a body than motion, and that "force" must operate to *keep a body moving.* This is not true: mere motion implies no acting force. *Change* of motion only, either in speed or in direction, implies such action.

With the notion referred to there usually goes another, — that a moving body must have been *put in motion* by some force, as if all bodies were once at rest — say at the moment of creation — and acquired their motion later; in respect to which we have no knowledge.

Motion of body under force acting in the line of motion.

Motion under force acting across line of motion.

302. Motion under the Action of a Force. — If the motion of a body is in a *straight* line but with a *varying* speed, we infer a force acting directly in the line of motion, either accelerating or retarding. If the body *a* moves in a curve (Fig. 114), we know that some force is acting *crosswise* to the motion and towards the *concave side* of the curve. If the speed increases, we know that the acting force pulls not only crosswise, but forward, as *ab*, making an angle of less than 90° with the "line of motion," *at*, tangent to the curve at *a*; and *vice versa* if the motion is retarded.

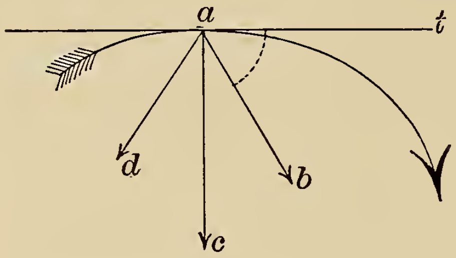

FIG. 114. — Curvature of an Orbit

Condition of constant speed.

If the speed keeps constant, we know that at *a* the force acts along *ac*, always *exactly perpendicular* to the line of motion.

Only a single force needed to explain curvilinear motion.

It is not unusual to find curvilinear motion spoken of as due necessarily to *two* forces; one, the "projectile force," imagined to act along the line of motion, while the second force draws sidewise. There may have been a projectile force acting in the past, but if so it is "ancient history"; we need, at present, in order to explain the facts, the action of only a single force, operating to *change* the direction or the speed, or both, of the body's motion. *From a curved path* we can infer the necessary existence *of but one force*. This force may be, and often is, the "resultant" of several; but then they act as one, and only one is needed.

Law of motion under a central force.

303. Laws governing the Motion of a Body moving under the Action of a Force directed to a Fixed Center; Law of Areas. — In this case it is obvious that the path of the body will be a *curve*, concave towards the center of force, and all lying in one plane with that center.

It is easy to prove, further, that it will move in such a way that its radius vector will describe equal areas in equal times around that point.

Demonstration that an impulse

Imagine a body moving uniformly along the straight line *AB* (Fig. 115), so that *AB*, *BC*, *CL* (the spaces described in successive seconds) are all equal; then, wherever O may be, the triangles AOB, BOC, COL, etc., are all equal, having equal bases and the common vertex O. A body in *uniform rectilinear motion* therefore describes with its radius vector equal areas in equal times *around any point whatever.*

directed towards a point does not alter the area described by the radius vector around that point.

Suppose, now, that when the body reaches C a blow or impulse directed towards O is given, imparting a velocity which would carry it to K in one second if it had been at rest when struck. The resultant of the original motion CL, combined with the newly imparted motion CK, is CD, found according to the "parallelogram of velocities" (*Physics*, p. 18) by drawing KD and LD parallel, respectively, to CL and CK, so that at the end of a second the body will arrive at D instead of going to L, and its velocity will have become CD instead of BC.

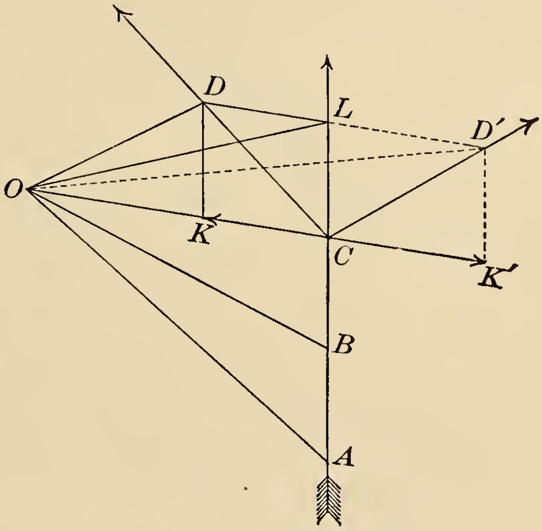

FIG. 115

Now the area of the triangle COD equals that of COL, because they have the common base CO, and their vertices are on a line, DL, *parallel to that base*, making their "altitude" the same. But $COL = BOC$; therefore $COD = COB$. It follows, therefore, that *when a moving body receives an impulse directed towards a given point the area described by the radius vector in a second around that point remains unchanged by that impulse.*

If the impulse had been directed *from* the point O, towards K' instead of K, the result would have been the same. The same reasoning shows that the area COD' is equal to COB.

But if K were *not on the radius vector* CO, the area would be changed, increasing if CK lay between CO and CL, and decreasing if between CO and CB.

Hence follows the general principle of uniform description of areas.

304. Furthermore, since a continuous force, like attraction, directed towards or from a fixed center, O, may be regarded as an uninterrupted succession of little impulses, each directed along the radius vector, we have the perfectly general law that *whenever a body moves under the sole action of a force directed along the radius vector drawn from the body to a center, the radius vector will describe around that center areas proportional to the time.* It makes no difference according to what law the *intensity* of the force varies: it may be attractive or repulsive, continuous or intermittent, may vary as gravity does or with complete irregularity; but so long as it never acts except along the radius vector the "areal velocity," as it is called (*i.e.*, the number of square feet or acres or square miles described by the radius vector in a unit of time), remains absolutely constant.

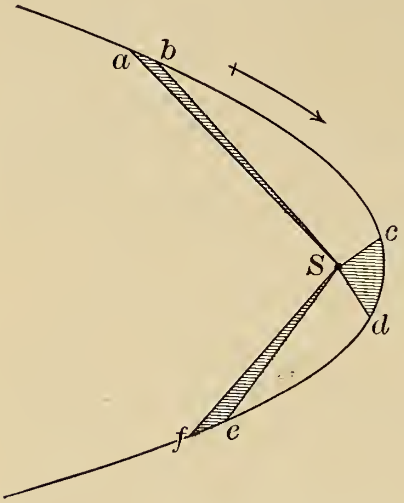

FIG. 116. — The Law of Equal Areas

Thus, in Fig. 116, representing part of a comet's orbit around the sun, if the arcs *ab*, *cd*, *ef* are each described in the same time, then the shaded areas are all equal.

The converse theorem is also easily proved, viz., that if a body moves in a curve in such a way that its radius vector drawn to a given point describes equal areas in equal times around that point, then the force that acts upon the body is always directed to that point.

Areal, linear, and angular velocities defined.

305. Areal, Linear, and Angular Velocities. — *Areal velocity* has just been defined. The *linear velocity* of a body is the number of linear units (feet, meters, miles) which it moves over in a unit of time, — say a second. Its symbol is usually V. The *angular velocity* is the number of units of angle (radians, degrees, seconds) swept over by the radius vector in a unit of time. The usual symbol for this is ω.

In Fig. 117 if AB is the length of the path described in a unit of time, AB is the *linear velocity* V; the angle ASB is the *angular velocity*, ω; and the area ASB is the *areal velocity*, which is constant. Calling this A and regarding the sector as a triangle (which it is nearly enough), we have $A = \frac{1}{2} V \times p$, p being the line Sb drawn from the center of force perpendicular to the line of motion; so that if we regard AB as the base of the triangle, p is its altitude. Hence, we have the equation

Formulæ for linear and angular velocities in terms of areal velocity.

$$V = \frac{2A}{p}. \qquad (1)$$

Also, $A = \frac{1}{2} r_1 r_2 \sin ASB$. Since in a second of time the angle ASB, or ω, is so small that it may be taken equal to its sine, and $r_1 r_2$ equals (sensibly) r^2, we have

$$\omega = \frac{2A}{r^2}. \qquad (2)$$

In every case, therefore, of motion under a central force, (1) the *areal velocity* (square miles per second) *is constant* in all parts of the orbit; (2) the *linear velocity* (miles per second) *varies inversely as* p, the perpendicular drawn from the center to the line of motion; (3) the *angular velocity* (radians or degrees per second) *varies inversely as the square of the radius vector.*

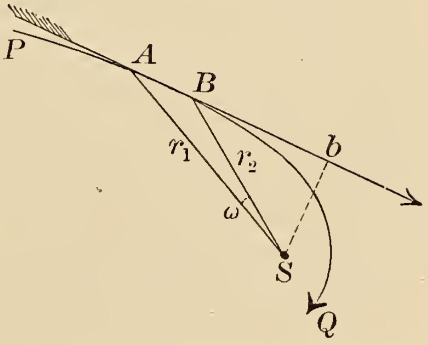

FIG. 117.—Linear and Angular Velocities

These three statements are not independent laws, but simply different geometrical equivalents for one law. They hold good regardless of the nature of the force, requiring only that *when it acts* it acts *directly towards, or from, the center*, along the line of the radius vector.

Extreme generality of the laws.

306. Circular Motion.—In the special case when the path of a body is a circle described under the action of a force directed to its center, both the linear and angular velocities are constant,

Central force in case of circular motion.

as is also the force, which is given by the familiar formula already several times used:

$$f = \frac{V^2}{r} \quad (a); \qquad \text{or} \qquad f = 4\,\pi^2 \frac{r}{t^2} \quad (b),$$

obtained by substituting for V in equation (a) its value, $2\pi r$ (the circumference of the circle), divided by t, the time of revolution. As the orbits of the principal planets are all nearly circular, these formulæ will find frequent application.

307. Kepler's Laws. — Early in the seventeenth century Kepler discovered, as unexplained facts, three laws which govern the motions of the planets,—laws which still bear his name. He worked them out from a discussion of the observations which Tycho Brahe had made through many preceding years upon the planets, Mars especially. They are as follows:

Kepler's laws stated.

(1) The orbit of each planet *is an ellipse with the sun in one of its foci.* (See Sec. 160.)

(2) *The radius vector of each planet describes equal areas in equal times.*

(3) *The squares of the periods of the planets are proportional to the cubes of their mean distances from the sun;* i.e., $t_1^2 : t_2^2 :: a_1^3 : a_2^3$. This is the so-called "Harmonic Law."

Examples illustrating the Harmonic Law.

308. To make sure that the student apprehends the meaning and scope of this third law, we add a few simple examples of its application:

1. What would be the period of a planet having a mean distance from the sun of one hundred astronomical units, *i.e.*, a distance a hundred times that of the earth?

$$1^3 : 100^3 = 1^2(\text{year}) : X^2;$$

$$\text{whence, } X \text{ (in years)} = \sqrt{100^3} = 1000 \text{ years.}$$

2. What would be the distance from the sun of a planet having a period of 125 years?

$$1^2(\text{year}) : 125^2 = 1^3 : X^3; \text{ whence } X = \sqrt[3]{125^2} = 25 \text{ astron. units.}$$

3. What would be the period of a satellite revolving close to the earth's surface?

$$(\text{moon's dist.})^3 : (\text{dist. of satellite})^3 = (27.3 \text{ days})^2 : X^2,$$

$$\text{or, } 60^3 : 1^3 = 27.3^2 : X^2;$$

$$\text{whence, } X = \frac{27.3 \text{ days}}{\sqrt{60^3}} = 1^{\text{h}}24^{\text{m}}.$$

Modification of Harmonic Law taking account of planets' masses.

The Harmonic Law as it stands in Sec. 307 is not *strictly* true: it would be so if the planets were mere *particles*, infinitesimal as compared with the sun; but this is not the case, though the difference is so slight that Kepler did not detect it. The accurate statement, as Newton showed, is

$$t_1^2(M + m_1) : t_2^2(M + m_2) = r_1^3 : r_2^3,$$

in which M is the sun's mass, and m_1 and m_2 are the masses of the two planets compared. In the case of Jupiter the correction makes a difference of about two days in its period; *i.e.*, its period is about two days shorter than that of a *particle* moving in the same orbit would be.

Newton proved that the law of gravitation follows from Kepler's laws.

309. For fifty years these laws remained an unexplained mystery. Many surmises, partly correct, were early made as to their physical meaning. Several persons "guessed" that the explanation would be found in a force directed to the sun; Newton proved it. He first demonstrated substantially, as given in Sec. 303, the law of equal areas and its converse as being in the case of central motion a necessary consequence of the three fundamental laws of motion, which he had been the first to formulate. He also proved by a demonstration a little beyond the scope of this book that if a planet moves in an *ellipse* with the center of force at its focus, then the force acting upon the body at different points in its orbit must *vary inversely as the square of the radius vector at those points;* and, finally, he proved that, granting the Harmonic Law, the force from planet to planet must also vary according to the same law of inverse squares.

Demonstration for circular orbits.

310. For *circular* orbits the proof is very simple. From equation (*b*) (Sec. 306) we have, for the first of two planets,

$$f_1 = 4\,\pi^2 \frac{r_1}{t_1{}^2},$$

in which f_1 is the central force (measured as an acceleration in feet per second), and r_1 and t_1 are, respectively, the planet's distance from the sun and its periodic time.

For a second planet,

$$f_2 = 4\,\pi^2 \frac{r_2{}^2}{t_2{}^2}.$$

Dividing the first equation by the second, we get

$$\frac{f_1}{f_2} = \frac{r_1}{r_2} \times \left(\frac{t_2{}^2}{t_1{}^2}\right).$$

But, by Kepler's third law,

$$t_1{}^2 : t_2{}^2 = r_1{}^3 : r_2{}^3; \text{ whence } \frac{t_2{}^2}{t_1{}^2} = \frac{r_2{}^3}{r_1{}^3}.$$

Substituting this value of $\frac{t_2{}^2}{t_1{}^2}$ in the preceding equation, we have

$$\frac{f_1}{f_2} = \frac{r_1}{r_2} \times \frac{r_2{}^3}{r_1{}^3} = \frac{r_2{}^2}{r_1{}^2};$$

i.e., $f_1 : f_2 = r_2{}^2 : r_1{}^2$, which is the law of inverse squares.

In the case of *elliptical* orbits the proposition is equally true if for r we substitute a, the semi-major axis of the orbit; but the demonstration is much more complicated.

Inferences from Kepler's laws as to the force which acts on the planets.

311. Inferences from Kepler's Laws.—From Kepler's laws we may therefore infer, as Newton proved: *First* (from the law of areas), that *the force which determines the orbits of the planets is directed towards the sun.*

Second (from the first law), that *the force which acts upon any given planet varies inversely, at different points in the orbit, as the square of the radius vector.*

Gravitation depends upon mass and distance

Third (from the Harmonic Law), that *the force which acts upon one planet is the same that it would be for any other planet put in the place of the first;* in other words, the attracting force depends only on *the mass* and *distance* of the bodies concerned, and is *independent of their physical condition*, such as temperature, chemical constitution, etc. It makes no difference *yet detected* in the motion of a planet around the sun, whether it is hot or cold, made of hydrogen or of iron; but it would be going too far to say that the future may not yet show some slight differences depending upon such circumstances.

only, and is sensibly independent of all other circumstances.

312. Newton's Test of his Theory of Gravitation by the Motion of the Moon. — When Newton first conceived the idea of universal gravitation in 1665, he saw at once that the moon's motion around the earth ought to furnish a test. Since the moon's distance (as was well known even then) is about sixty times the radius of the earth, the distance it should fall towards the earth in a second ought to be, if his idea of gravitation was correct, $\frac{1}{60^2}$, or $\frac{1}{3600}$, of 193 inches (the distance which a body falls in a second at the earth's surface), provided we assume that the earth attracts as if its mass were all collected at its center, — to prove which gave Newton much trouble, and became possible only after his invention of "fluxions."

Test of the theory of gravitation by means of the moon's motion.

Now $\frac{1}{3600}$ of 193 inches is 0.0535 inches. Does the moon fall towards the earth, *i.e.*, deflect from a straight line, by this amount each second?

The deflection of the moon towards the earth is just what it should be according to the law of gravitation.

According to the law of central forces, considering the moon's orbit as circular,

$$f = 4\pi^2 \times \frac{r}{t^2},$$

and the *deflection* is one half of this, viz., $2\pi^2 \frac{r}{t^2}$. If we compute the result, making $r = 238840$ miles reduced to inches, and t the number of seconds in a sidereal month, the deflection comes out 0.0534 inch, a difference of only $\frac{1}{10000}$ of an inch, — practically a complete accordance.

Unfortunately for Newton, when he first made this test, the distance of the moon *in miles* was not known, because the size of the earth had not then been determined with any accuracy. The length of a degree was

Why the test appeared to fail when Newton first applied it.

supposed to be about 60 miles instead of 69, as it really is. Newton computed the radius of the earth on this erroneous basis and, multiplying it by 60, obtained for *r*, the distance of the moon, a quantity about sixteen per cent too small; from this he calculated a corresponding deflection of only about 0.044 inch. The discordance between this and 0.0535 was too great, and he loyally abandoned the theory as contradicted by facts.

Six years later, in 1671, Picard's measurement of an arc of the meridian in France corrected the error in the size of the earth, and Newton on hearing of it at once repeated his calculation, or tried to, for the story goes that he was too excited to finish it, and a friend completed it for him. The accordance was now satisfactory, and he resumed the subject with zeal and soon established the correctness of his theory.

The test not sufficient to demonstrate the correctness of the theory.

It is to be noted that while *discordance* in even a single case would be fatal to the theory, *accordance in a single case* does not prove it, but only makes it more or less probable. The demonstration of the law of gravitation lies in its entire accordance, not with one or two selected facts, but with a countless multitude, and in its freedom from a single contradiction shown by the most refined observations.

Its proof lies in its agreement with all facts thus far observed.

Apparent contradictions have now and then cropped out, but all have found explanation, except, perhaps, one slight divergence at present outstanding (in the motion of the apsides of the planet Mercury) which thus far baffles the mathematicians, but will, in all probability, sooner or later disappear like its predecessors.

The inverse problem: to determine what the orbit must be if the law is correct.

313. The Inverse Problem. — Newton did not rest with merely showing that the motion of the planets and of the moon could be explained by the law of gravitation; but he also investigated and solved the more general *inverse* problem and determined *what kind of motion is necessary according to that law.* He found that the orbit of a body moving around a central mass under the law of gravitation need not be a circle, nor even an ellipse of *slight eccentricity* like the planetary orbits. But it must be a *Conic.* Whether it will be a circle, ellipse, parabola, or hyperbola depends on circumstances.

The two conics.

314. The Conics. — (1) The *ellipse* is the section of a cone made by a plane which cuts completely across it, as *EF* in Fig. 118. The ellipse varies in form and size, according to the position and inclination of the cutting plane, the circle being simply a special case when the section is perpendicular to the axis of the cone.

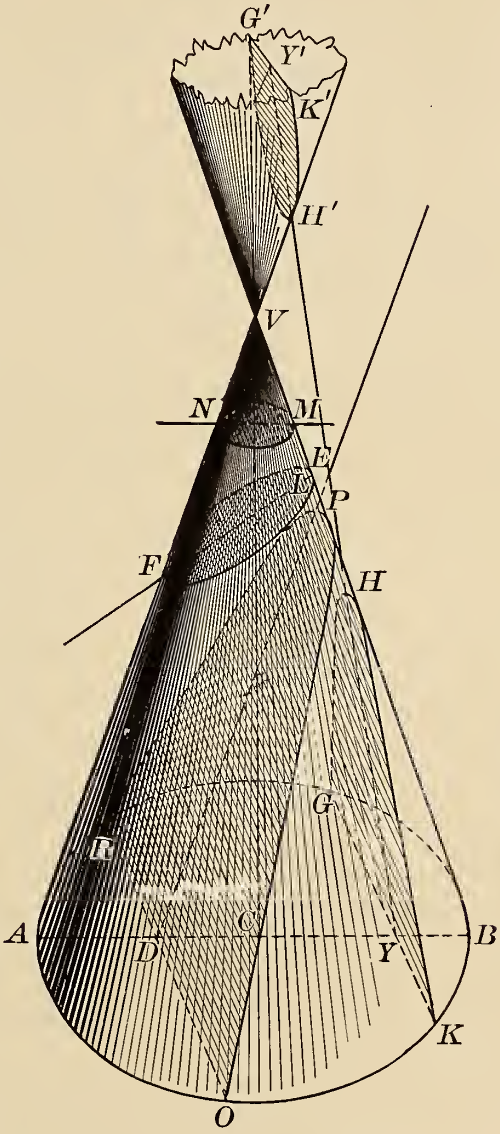

Fig. 118. — The Conics

(2) *The Hyperbola.* When the cutting plane makes with the axis an angle less than *BVC* (the *semiangle of the cone*) it plunges continually deeper and deeper into the cone and *never comes out on the other side.* The section in this case is an *hyperbola*, *GHK.* If the cutting plane be produced upward, it encounters the other nappe of the cone (the "cone produced"), cutting out from it a second hyperbola, *G'H'K'*, exactly like the first, but turned in the opposite direction. The pair of twin curves, *GHK* and *G'H'K'*, are considered as two parts of the same hyperbola, the axis of which, *HH'* in the figure, lies between the two branches and outside of both, and is therefore always reckoned as *negative.* The *center* of the pair of twin hyperbolas is the middle point of this axis.

(3) *The Parabola.* When the cutting plane is *parallel to the side of the cone*, as *PRO*, it never cuts in deeper, nor, on the other hand, does it run across the cone. The section in this

The parabola: the bounding curve between ellipses and hyperbolas.

case is called a *parabola*, which, so to speak, is the boundary or partition between the ellipses and hyperbolas which can be cut from a given cone by changing the inclination of a given plane. The least deflection of the cutting plane outward from the parallel changes the parabola into an ellipse, and into an hyperbola, if inward.

All parabolas identical in form, differing only in size.

All parabolas, of whatever size, and cut from whatever cone, *are of the same shape*, as all circles are, — a fact by no means obvious without demonstration, though we cannot give the proof here. This does not mean, however, that *an arc* of one parabola is of the same shape as *any arc of another parabola* (taken from a different part of the curve), but that the *complete* parabolas, cut out from infinitely extended cones, are all similar, whether the cone be sharp or blunt, or whether the plane cuts it near to or far from its vertex.

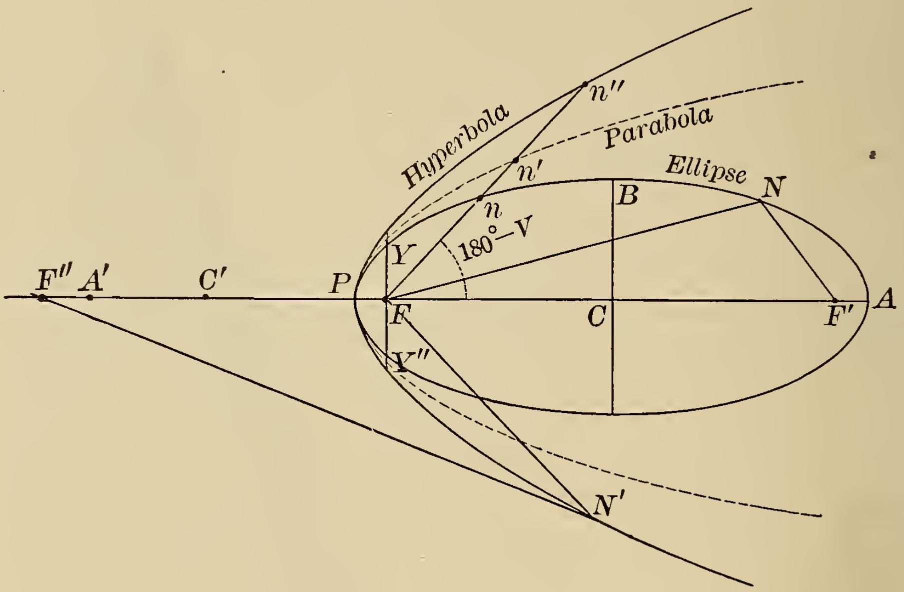

Fig. 119. — The Relation of the Conics to Each Other

The ellipse, hyperbola, and parabola compared as curves upon a plane.

315. The Ellipse, Parabola, and Hyperbola. — Fig. 119 shows the appearance and relation of these curves as drawn upon a plane. The *Ellipse* is a "closed curve" returning into itself, and in it the sum of the distances of any point, N, from the two foci equals the major axis; *i.e.*, $FN + F'N = PA$.

The *Hyperbola* does not return into itself, but the two branches PN' and Pn'' go off into infinity, becoming ultimately nearly straight and *diverging* from each other at a definite angle. In the hyperbola the *difference* of two lines drawn from any point on the curve to the two foci equals the major axis; *i.e.*, $F''N' - FN' = PA'$, $C'P$ being the semi-major axis, a, of the hyperbola.

The *Parabola*, like the hyperbola, fails to return into itself, but its two branches, instead of diverging, become more and more nearly parallel. It has but one accessible focus and may be regarded either as an *ellipse* with its second focus, F', removed to an infinite distance, and therefore having an *infinite* major axis; or, with equal correctness it may be considered as an *hyperbola*, of which the second focus, F'', is pushed indefinitely far in the opposite direction, so that it has an infinite (*negative*) major axis.

In the ellipse the eccentricity $\left(\frac{FC}{PC}\right)$ is *less* than unity.
In the hyperbola it is *greater* than unity $\left(\frac{FC''}{PC'}\right)$.
In the parabola it is *exactly unity;* in the circle, *zero.*

The eccentricity of ellipse is less than unity; that of hyperbola is greater than unity; that of parabola is exactly unity.

The eccentricity of a conic determines its form. All parabolas, therefore, are of the same form, as already said, as are all circles. Of ellipses and hyperbolas there is an infinite variety of forms, from such as are so narrow as to be only a line or a pair of diverging lines, to those that are broad as compared with their length.

The problem of two bodies.

316. The Problem of Two Bodies.—This problem, proposed and completely solved by Newton, may be thus stated:

Given the masses of two spheres and their positions and motions at any moment; given also the law of gravitation: required the motion of the bodies ever afterwards and the data necessary to compute their place at any future time.

Motion of their common center of gravity unaffected by their mutual attraction.

The mathematical methods by which the problem is solved require the use of the calculus and must be sought in works on analytical mechanics and theoretical astronomy, but the general results are easily understood.

In the first place, the motion of the *center of gravity* of the two bodies is not in the least affected by their mutual attraction.

The size of their orbits inversely proportional to their masses.

In the next place, the two bodies will describe as orbits around their common center of gravity two *curves precisely similar in form, but of size inversely proportional to their masses*, the form and dimensions of the two orbits being determined by the masses and velocities of the two bodies.

The relative orbit of the smaller with respect to the larger.

If, as is generally the case in the solar system, the two bodies differ greatly in mass, it is convenient to ignore the center of gravity entirely and to consider simply the *relative* motion of the smaller one around the center of the other. It will move *with reference to that point* precisely as if its own mass, m, had been added to the principal mass, M, while it had become itself a mere particle. This *relative orbit* will be precisely like the orbit which m actually describes around the center of gravity, except that it will be magnified in the ratio of $(M + m)$ to M; *i.e.*, if the mass of the smaller body is $\frac{1}{100}$ of the larger one, its *relative* orbit around M will be just one per cent larger than its actual orbit around the common center of gravity of the two.

The relative orbit a conic, the species and size of which depend upon the masses, their initial positions, and velocities.

317. *Finally*, the orbit will always be a "conic," *i.e.*, *an ellipse* or *an hyperbola;* but which of the two it will be depends on three things, viz., the united *mass* of the two bodies $(M + m)$, the *distance*, r, between m and M at the initial moment, and the *velocity*, V, of m relative to M.

Criterion for species.

If this velocity, V, be *less* than a certain critical velocity, U, which depends only on $(M + m)$ and r and is called the "parabolic velocity" or "velocity from infinity," the orbit will be an *ellipse;* if greater, it will be an *hyperbola.* If, however, V and U should happen to be *exactly* equal, the orbit would be a parabola; but such exact equality is extremely improbable,— the chances are infinity to one against it.

The *direction* of the motion of m with respect to M, while it has influence upon the *form* of the orbit (its "eccentricity"), has nothing to do with determining its *species* and *semi-major axis* nor with its *period* in case the orbit is elliptic; these are all independent of the *direction* of m's motion.

The form depends partly upon direction of initial motion.

The problem is completely solved. From the necessary initial data corresponding to a given moment we can determine the position of the two bodies for any instant in the eternal past or future, *provided only that no force except their mutual attraction acts upon them in the time covered by the calculation.*

Definition of the parabolic velocity or velocity from infinity.

318. The Parabolic Velocity. — The parabolic velocity at the distance r is also called the "velocity from infinity," because it is the speed which would be acquired by the particle m in falling towards the mass M from an infinite distance to the distance r from M, — assuming, of course, that M is fixed and that m starts from rest and during its fall is not acted upon by any force excepting the attraction between itself and M. It might be supposed that this velocity would be infinite, but it is not so unless r becomes absolutely zero. It is given by the formula

U_r[1] $= \kappa\sqrt{\dfrac{M+m}{r}}$, or simply $\kappa\sqrt{\dfrac{M}{r}}$, (1)

when m is infinitesimal as compared with M. (For a demonstration of this formula the reader is referred to works on analytical mechanics.)

Formula for the parabolic velocity.

In this formula κ is a constant which depends on the mass of M and on the units of measurement employed. If we take the mass of the sun as the unit of mass and the radius of the earth as the unit of distance for r, it becomes 26.156 miles per second, and we have for the parabolic velocity due to the sun's attraction on a particle falling from infinity to the distance r,

$$U_r \text{ (miles per second)} = 26.156\sqrt{\frac{1}{r}}, \qquad (2)$$

and

$$U_r^2 = \frac{684.14}{r}. \qquad (2')$$

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[1] U_r signifies "parabolic velocity at distance r."

If the mass of the sun were *four* times as great, the coefficient would be *doubled*, since, according to equation (1), U varies with the square root of M. At a distance one fourth that of the earth from the sun, r would become one fourth and the parabolic velocity would also be doubled. At the distance of Neptune, where $r = 30.05$, U is only 4.77 miles per second.

Parabolic velocity at surfaces of sun, earth, and moon.

319. Formula (1) enables us to compute the parabolic velocity *at the surface of any body* whose mass and radius are known. In the case of the sun $M = 1$ and $r = \frac{1}{214.66}$ (*i.e.*, 433250 ÷ 93 000000), so that at the sun's surface $U = 383.2$ miles per second; if a body were ejected from the sun with a speed exceeding this, it would go off and never return.

For the *earth*, $M = \frac{1}{332000}$ and $r = \frac{1}{23467}$ (Sec. 225), and from equation (1) we find U at the *earth's* surface equals 6.9 miles per second.

At the surface of the *moon* a similar computation gives U as only 1.48 miles, or less than 8000 feet per second. A body projected from the moon with a speed greater than this would never return, and it will be recalled that in this fact probably lies the explanation why the moon has lost her atmosphere.

Relation between the parabolic velocity and the species of the orbit.

320. Relation between the Parabolic Velocity and the Nature of the Orbit of a Body revolving around the Sun.

—From theoretical astronomy (Watson, p. 49) we have the equation

$$a = \mu \frac{r}{2\mu - rV_r^2}, \qquad (3)$$

a being the semi-major axis of the orbit of a body, V_r the velocity of the body in its orbit at a point whose radius vector is r, and μ a constant which equals $\frac{1}{2}M\kappa^2$, — the κ of equation (1). From equation (1), $M\kappa^2 = r \times U_r^2$, so that $\mu = \frac{1}{2} rU_r^2$. Substituting this value of μ in equation (3), we find at once

$$a = \frac{r}{2}\left(\frac{U_r^2}{U_r^2 - V_r^2}\right). \qquad (4)$$

The semi-major axis of orbit depends

This equation is of great importance, since it shows that the *species* of the orbit is determined solely by the difference between U^2 and V^2.

upon the difference between U^2 and V^2 at distance r.

If the denominator of the fraction is *positive*, the value of a will be positive and the orbit will be an *ellipse.* This is the case when V_r, the *orbital velocity* at the distance r, is *less* than U_r, the *parabolic* velocity at that distance.

If, on the other hand, V_r is greater than U_r, the denominator becomes *negative*, and so does a, and the orbit is an *hyperbola.*

If V_r *exactly equals* U_r, the denominator becomes zero, a becomes infinite, and the orbit is a *parabola.* This explains why U is called the "parabolic velocity": at every distance from the sun the velocity of a body *moving in a parabola* is precisely what it would have acquired in falling to that point from an infinite distance under the sun's attraction.

If the orbit is an *ellipse*, the velocity at every point in the orbit is less than the parabolic velocity, and greater if the orbit is an *hyperbola.*[1]

Condition for motion in a circle.

In order that a planet may move *in a circle* around the sun, as the principal planets do very nearly, a must equal r, and equation (4), by substituting r in place of a, gives

$$r = \frac{r}{2} \times \frac{U^2}{U^2 - V^2}, \text{ or } \frac{U^2}{U^2 - V^2} = 2;$$

whence, $V^2 = \frac{1}{2} U^2$ and $V = U\sqrt{\frac{1}{2}} = 0.7071 \times U$; *i.e.*, the *velocity of a body moving in a circular orbit is equal to the parabolic velocity multiplied by* $\sqrt{\frac{1}{2}}$.

Vice versa, $U = V\sqrt{2}$, and hence the parabolic velocity at distance unity (that of the earth from the sun) equals the earth's

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[1] The expression for the eccentricity is more complicated than that for the semi-major axis, since it involves the angle γ between the radius vector and the tangent drawn at its extremity. The equation is

$$e^2 = 1 - 4\frac{V^2}{U^2}\left(\frac{U^2 - V^2}{U^2}\right)\sin^2\gamma.$$

The eccentricity is therefore greater than, less than, or equal to unity, according as $(U^2 - V^2)$ is positive or negative. It will be noticed also that no linear quantity (r or a) enters into the expression, which determines only the *form* and not the *size* of the orbit.

orbital velocity, 18.5 miles × $\sqrt{2} = 26.16$ miles per second; and this is the way in which the constant κ is usually computed.

Expression for planet's period.

321. The Expression for a Planet's Period. — From theoretical astronomy (Watson, p. 46) we have the equation

$$t = 2\,\pi \times \frac{a^{\frac{3}{2}}}{\sqrt{\mu}}, \qquad (5)$$

where t is the periodic time. This embodies Kepler's third law, and shows that all planets moving in ellipses and having the same *major axis* will have the same period, notwithstanding differences in the eccentricity of their orbits.

Also that if a is infinite, as in the *parabola*, the period is also infinite.

Also that in the *hyperbola* (in which a is negative) the period, since it involves the square root of the negative quantity a^3, is *imaginary*, *i.e.*, in this case *impossible*.

Effect of changes of velocity upon a body's orbit.

When a body is moving in a parabola ($V^2 = U^2$) the least decrease of V by a disturbing action will transform the orbit into an *ellipse* with a definite period, or an increase of velocity will make it an *hyperbola*.

Again, if a planet moving in a circular orbit should have its speed increased in a ratio greater than that of the square root of 2 to 1, say one and one-half times, it would go off in an hyperbolic arc and never return.

Behavior of the fragments of an exploded planet.

Finally, if a planet were to *explode* at any point in its orbit, all the pieces, except those which had a velocity greater than the parabolic velocity at the point of explosion, would move around the sun in ellipses, and at every revolution would pass through the point where the explosion occurred; moreover, any fragments which were thrown off with equal velocities would have the same *period* and after a single circuit around the sun would arrive there simultaneously.

EXERCISES

1. Given a comet moving in an ellipse with the eccentricity 0.5. Compare the velocities, both linear and angular, at the perihelion and aphelion.

Ans. { Lin. vel. at perihelion is *three* times that at aphelion.
Ang. vel. " " " *nine* " " " "

2. What would be the result if the eccentricity were $\frac{1}{3}$? what if it were $\frac{3}{4}$?

3. What would be the periodic time of a small body revolving in a circle around the sun close to its surface? (Apply Kepler's Harmonic Law.)

Ans. $2^{h}47^{m}.4$.

4. What would be its velocity? *Ans.* 270.8 miles a second.

5. If the earth had a satellite with a period of 8 months, what would its distance be? *Ans.* Four times that of the moon.

6. If Jupiter were reduced to a mere particle, how much would its period be lengthened? (Consider its mass to be $\frac{1}{1048}$ of the sun's, and see Sec. 308.)

Solution. Let x be the new period; then,

$$x^2 : t^2 \frac{1049}{1048} = r^3 : r^3 = 1 : 1,$$ since r is not changed. Whence,

$$x = t\sqrt{\frac{1049}{1048}} = t\left(1 + \tfrac{1}{2} \times \tfrac{1}{1048} + \text{etc.}\right) = t\left(1 + \tfrac{1}{2096}\right),$$ very nearly.

But $t = 4332.6$ days, and $(x - t) = \dfrac{4332.6}{2096} = 2.067$ days. *Ans.*

7. How much longer would the earth's period be if it were a mere particle? *Ans.* $\frac{1}{660000}$ of a year, or 47.8 sec.

8. If the sun's mass were a hundred times greater, what would be the parabolic velocity at the earth's distance from it (Sec. 318)?

Ans. Ten times its present value, *i.e.*, 261.6 miles a second.

9. If the sun's mass were reduced 50 per cent, what would be the parabolic velocity at the distance of the earth? *Ans.* 18.5 miles a second.

10. If the sun's mass were to be suddenly reduced by 50 per cent or more, what would be the effect upon the now practically circular orbits of the planets? (See Sec. 320, last paragraph.)

11. What would be the effect upon the orbit of the earth if the sun's mass were suddenly doubled?

Ans. It would immediately become an eccentric ellipse, with its aphelion near the point where the earth was when the change occurred.

12. Let V_r be the velocity in an orbit at a point where the radius vector is r, and let U_r and U_{2a} be *the parabolic velocities* at distances r and $2a$ from the sun, a being the semi-major axis of the orbit. Show that

$$V_r^2 = U_r^2 \pm U_{2a}^2.$$

The *plus* sign applies if the orbit is an hyperbola, the minus sign if it is an ellipse.

In words this may be stated thus (since the *energy* of a moving body is proportional to the square of its velocity):

The energy of a body moving in an orbit under gravitation, when at a distance r *from the center of attraction, equals the energy it would have acquired by falling to* r *from infinity* ± *the energy it would have acquired by falling from infinity to the distance* 2 a, *the major axis of the orbit.*

THE PROBLEM OF THREE BODIES: PERTURBATIONS AND THE TIDES

322. As has been said, the problem of two bodies is completely solved; but if instead of *two* spheres attracting each other we have *three* or more, the general problem of determining their motions and predicting their positions transcends the present power of human mathematics.

Statement of the general problem of three bodies.

"The problem of three bodies" is in itself as determinate and capable of solution as that of two. Given the initial data, *i.e.*, *the masses, positions, and motions of the three bodies at a given instant;* then, assuming the law of gravitation, their motions for all the future and the positions they will occupy at any given date are absolutely predetermined. *The difficulty is with our mathematics.*

The general problem intractable, but special cases solved.

But while the *general* problem of three bodies is intractable, nearly all the *particular cases* of it which arise in the consideration of the motions of the moon and of the planets have already been practically solved by special devices, Newton himself leading the way; and the strongest proof of the truth of the theory of gravitation lies in the fact that it not only accounts for the *regular* elliptic motions of the heavenly bodies, but also for their apparent *irregularities*.

323. It is quite beyond the scope of this work to discuss the methods by which we can determine the so-called "disturbing forces" and the effects they produce upon the otherwise elliptical motion of the moon or of a planet. We make only two or three remarks.

Disturbing force depends upon the difference of attractions upon the bodies disturbed.

First, that the "disturbing force" of a third body upon two which are revolving around their common center of gravity *is not the whole attraction* of the third body upon either of the two, but is generally only a *small component* of that attraction. It depends upon the ***difference** of the two attractions exerted by the third body upon each of the pair whose relative motions it disturbs*, — a difference either in *intensity*, or in *direction*, or in *both*.

Disturbing force of sun upon moon only one ninetieth of its attraction.

If, for instance, the sun attracted the moon and earth *alike and in parallel lines*, it would not disturb the moon's motion around the earth in the slightest degree, however powerful its attraction might be. The sun always attracts the moon more than twice as powerfully as the earth does; but the sun's *disturbing force* upon the moon when at its very maximum is only one ninetieth of the earth's attraction.

The tyro is apt to be puzzled by thinking of the earth as *fixed* while the moon revolves around it; he reasons, therefore, that at the time of new moon, when the moon is between the earth and sun, the sun would necessarily pull her away from us, if its attraction were really double that of the earth; and it would do so *if the earth were fixed.* We must think of the earth and moon as both *free to move,* like chips floating on water, and of the sun as attracting them both with nearly equal power, — the nearer of the two a little more strongly, of course.

Disturbed bodies never move strictly in an ellipse.

324. *Second*, it is only by a mathematical fiction that the "disturbed body" is spoken of as "moving in an ellipse"; it never does so exactly. The path of the moon, for instance, never returns into itself.

But it is a great convenience for the purposes of computation to treat the subject as if the orbit were a *material wire* always of truly elliptical form, having the moving body strung upon it like a bead, this "orbit" being continually pulled about and changed in form and size by the action of the disturbing forces, taking the body with it, of course, in all these changes. This imaginary orbit at any moment is for that moment a true *instantaneous ellipse* of determinable form and position, but is constantly changing. It is in this sense that we speak of the eccentricity of the moon's orbit as continually varying and its lines of apsides and nodes as revolving.

Convenient fiction of the instantaneous ellipse.

The student must be careful, however, not to let this *wire* theory of orbits get so strong a hold upon the imagination that he begins to think of the "orbits" as material things, liable to collision and damage. An orbit is simply, of course, the path of a body, like the track of a ship upon the ocean.

Disturbances such only technically.

325. *Third*, the "disturbances" and "perturbations" are such only in a technical sense. Elliptical motion is no more *natural* or *proper* to the moon or to a planet than its actual motion is; nor in a philosophical sense is the pure elliptical motion any more *regular* (*i.e.*, "rule-following") than the so-called "disturbed" motion.

We make the remark because we frequently meet the notion that the "perturbations" of the heavenly bodies are imperfections and blemishes in the system. One good old theologian of our acquaintance used to maintain that they were a consequence of the fall of Adam.

Perturbations of moon due solely to the sun.

326. **Lunar Perturbations.** — The sun is the only body which sensibly disturbs the moon; the planets are too small and too distant to produce directly any effect which can be noticed, though *indirectly* by their effects on the orbit of the earth they make themselves slightly felt — at second-hand, so to speak.

The disturbing force due to the solar attraction can be easily computed at any moment by methods indicated in the *General*

Astronomy, but we shall not enter into that subject. This force is continually changing in amount and direction, and the student can readily understand that the accurate calculation of the summed-up effects of such a variable force in changing the orbit of the moon and her place in the orbit must be extremely difficult. For the most part, however, the disturbances are *periodic*, running through their phases and repeating themselves at regular intervals, so that they can be expressed by trigonometrical series. Over one hundred of these separate "inequalities," as they are called, are now recognized and taken account of in the construction of the Nautical Almanac.

Disturbing force easily computed. Computation of its effect complicated.

We mention a few only of the moon's disturbances, — those which are largest and most important, two or three of which, especially those which affect the time of eclipses, were discovered before the time of Newton, though not explained.

327. Effect on the Length of the Month; Revolution of the Line of Apsides; Regression of the Nodes. — (1) *Effect on Length of the Month.* On the whole, the action of the sun tends to lessen the effect of the earth's attraction on the moon by about $\frac{1}{360}$ part, *i.e.*, it virtually diminishes μ in equation (5) (Sec. 321), and this increases t, the period or length of the month, by about $\frac{1}{720}$ part. The month is *nearly an hour longer* than it otherwise would be at the moon's present distance from the earth.

Lengthening of the month.

(2) *Revolution of the Line of Apsides.* According to the "age" of the moon at the time when it passes the perigee or apogee, the sun shifts the line of apsides for that month, sometimes forward (eastward) and sometimes backward; but in the long run the forward motion predominates, and the line moves *eastward* and completes a revolution in 8.855 years.

Advance of apsides.

(3) *The Regression of the Nodes.* This has already been repeatedly mentioned. Speaking generally, the action of the sun on the whole *tends* to draw the plane of the moon's orbit towards the ecliptic; but, much as in the case of precession, the effect is not felt in any permanent change of the inclination

Regression of nodes.

of the orbit, but shows itself in a *westward shifting* of the node, which carries it around once in 18.6 years.

The evection.

328. Periodic Inequalities. — (4) *The Evection.* This is an irregularity which at the maximum puts the moon forward or backward in its orbit about 1¼°, and has for its period about one and one-eighth years, the time required for the sun to complete a revolution from the line of apsides of the moon's orbit to the same line again. It is the largest of the moon's periodic perturbations and was discovered by Hipparchus about 150 B.C. It was the only perturbation known to the ancients and may affect the time of an eclipse by nearly six hours, making it from three hours early to three hours late.

It depends upon an *alternate increase and decrease of the eccentricity of the moon's orbit*, which is always a maximum when the sun is passing the line of apsides, and a minimum when halfway between them.

The variation.

(5) *The Variation.* This is an inequality with a period of one synodic month and reaches its maximum of about 40′ at the *octants*, *i.e.*, the points 45° from new and full moon. At the octants *following* the new and full the moon is about 1^h20^m *ahead of time*, and at the octants *preceding* as much *behind time.*

This inequality was detected by Tycho Brahe about 1580, though there is some reason to suppose that it had been discovered some five hundred years before by an Arabian astronomer (Aboul Wefa) and lost sight of. It becomes zero at the full and new moon, and therefore does not affect the time of eclipses. For this reason it was missed by the Greek astronomers.

Annual equation.

(6) *The Annual Equation.* This is the one remaining inequality which affects the moon's place by an amount perceptible to the naked eye. At the maximum it is about 11′, with a period of one "anomalistic year" (Sec. 182).

It depends upon the fact that when the earth is nearest the sun, in January, the sun's disturbing effect on the moon is greater than the average, and the month is lengthened a little more than usual; and *vice versa* when the sun is most distant, in July. For half the year, therefore, from October to April, the moon keeps *falling behind*, while in the other half of the year the month is slightly shortened and the *moon gains*.

For more detailed geometrical explanations the student is referred to the *General Astronomy* and to Herschel's *Outlines of Astronomy*, or to works on celestial mechanics for their analytical discussion.

Secular acceleration.

329. The Secular Acceleration of the Moon's Mean Motion. — Among the multitude of lesser inequalities of the moon's motion this is of special interest theoretically and is still in some respects a "bone of contention" among astronomers. It was discovered by Halley about two hundred years ago. From a comparison of ancient with modern eclipses, he found that the month is now certainly shorter than it was in the days of Ptolemy, and that the shortening has been progressive, the moon at present being about a degree, or two hours in time, in advance of the position it would occupy if it had kept its motion unchanged since the Christian era. So far as astronomers could see at the time of the discovery, the process would continue indefinitely, — *in secula seculorum;* hence the name.

Its cause the diminishing eccentricity of the earth's orbit.

Laplace about 1800 showed that this effect can be traced to the change in the eccentricity of the earth's orbit, which is at present diminishing (Sec. 164). Since the major axis remains unaffected, decrease of eccentricity implies an increase of the *breadth* (minor axis) of the ellipse, of its *area* also, and therefore of the *average* distance of the earth from the sun during the year. From this increased distance between earth and sun follows a *decreased lengthening of the month* by the sun's disturbing action (Sec. 327). This practically amounts to the *shortening* of the month, which shortening will continue as long as the eccentricity of the earth's orbit continues to diminish, — about 24000 years, when the effect will cease and be reversed.

The theoretical amount of this acceleration of the moon's mean motion is about 6″ in a century, while the *actual* value, Discrepancy between computed and observed amount.

according to different estimates depending on comparison of modern with the much less accurate ancient observations, is decidedly larger, — 8″.09, according to Stockwell. The discrepancy is now generally ascribed to a *slight lengthening* of the day, diminishing the *number of seconds* in a month and so making the month *apparently* shorter, as containing a smaller number of seconds. Such a lengthening of the day could be accounted for by a retardation of the earth's rotation due to the friction of the tides (Sec. 345), but the actual difference which ought to be ascribed to this action is as yet very uncertain.

Possibly explained by tidal friction.

For an excellent non-technical account of the matter, see Newcomb's *Popular Astronomy*, p. 292.

330. The Tides. — Just as the disturbing force of the sun modifies the intensity and direction of the earth's attraction on the moon, so the disturbing forces due to the attractions of the sun and moon act upon the liquid portions of the earth to modify the intensity and direction of gravity and generate the *tides*.

The tides defined.

These consist in a regular rise and fall of the ocean surface, generally twice a day, the average interval between corresponding high waters on successive days at any given place being 24^h51^m. This is precisely the same as the average interval between two successive passages of the moon across the meridian, and the coincidence, maintained indefinitely, makes it certain that there must be some causal connection between the moon and the tides; as some one has said, the odd fifty-one minutes is "the moon's earmark."

Evidence that they are mainly due to action of the moon.

That the moon is largely responsible for the tides is also shown by the fact that when the moon is in *perigee*, *i.e.*, at the nearest point to the earth, they are nearly twenty per cent higher than when she is in apogee. The highest tides of all happen when the *new* or *full moon* occurs at the time when the moon is in perigee, especially if this perigeal new or full moon occurs about the first of January, when the earth is also nearest to the sun.

Definition of terms connected with the tides; spring and neap tides.

331. Definitions. — While the water is rising it is *flood*-tide; when falling it is *ebb*. It is *high water* at the moment when the water-level is highest, and *low water* when it is lowest. The *spring-tides* are the largest tides of the month, which occur near the times of new and full moon, while the *neap tides* are the smallest and occur at half-moon, the relative heights of spring and neap tides being about as 7 to 3.

Priming and lagging.

At the time of the spring-tides the interval between the corresponding tides of successive days is less than the average, being only about 24^h38^m (instead of 24^h51^m), and then the tides are said to *prime*. At the neap tides the interval is greater than the mean, — about 25^h6^m, — and the tide *lags*.

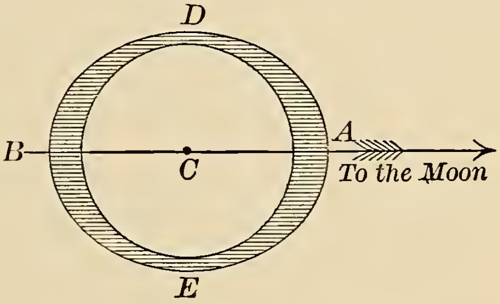

FIG. 120. — The Tides

The establishment.

The *establishment* of a port is the *mean interval* between the time of high water at that port and the next preceding passage of the moon across the meridian. The "establishment" of New York, for instance, is 8^h13^m; *i.e.*, on the average, high water occurs 8^h13^m after the moon has passed the meridian; but the actual interval varies fully half an hour on each side of this mean value at different times of the month, and under varying conditions of the weather.

The tide-raising force.

332. The Tide-Raising Force. — If we consider the moon alone, it appears that the effect of her attraction upon the earth, regarded as a liquid globe, is a tendency to distort the sphere into a slightly lemon-shaped form, with its long diameter pointing to the moon, raising the level of the water about *2 feet*, both directly under the moon and on the opposite side of the earth (at *A* and *B*, Fig. 120), and very slightly depressing it on the whole great circle which lies half-way between *A* and *B*. *D* and *E* are two points on this circle of depression.

Why tide is raised on side opposite the moon.

Students seldom find any difficulty in seeing that the moon's attraction ought to raise the level at *A*; but they often do find it very hard to understand why the level should also be raised at *B*. It seems to them that it ought to be more depressed just there than anywhere else. The mystery to them is how the moon, when directly underfoot, can exert a *lifting* force such as would diminish one's weight.

The earth not fixed while attracted by moon.

The trouble is that the student thinks of the solid part of the earth as *fixed* with reference to the moon, and the water alone as free to move. If this were the case, he would be entirely right in supposing that at *B* gravity would be increased by the earth's attraction instead of diminished; the earth, however, is not fixed, but perfectly free to move.

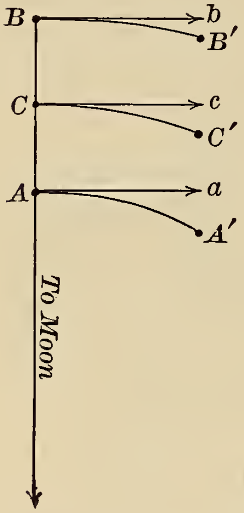

Fig. 121. — The Tide-Raising Force

Why gravity is diminished both under moon and on opposite side of earth.

333. Explanation of the Diminution of Gravity at the Point opposite the Moon. — Consider three particles (Fig. 121) at *B*, *C*, and *A*, moving with equal velocities, *Aa*, *Bb*, and *Cc*, but under the action of the moon, which attracts *A* more powerfully than *C* and *B* less so. Then, if the particles have no bond of connection, at the end of a unit of time they will be at *B*′, *C*′, and *A*′, having followed the curved paths indicated. But since *A* is nearest the moon, its path will be the most curved of the three, and that of *B* the least curved. It is obvious, therefore, that the *distances of both* B *and* A *from* C *will have been increased;* and if they were connected to *C* by an elastic cord, *the cord would be stretched*, both *A* and *B* being relatively *pulled away* from *C* by practically the same amount. We say *relatively*, because *C* is really pulled away from *B*, rather than *B* from *C*, — *C* being more attracted by the moon than *B* is; but the moon's attraction tends to *separate* the two all the same, and that is the point.

334. The Amount of the Moon's Tide-Raising Force. — When the moon is either in the zenith or nadir the weight of a body at the earth's surface is *diminished* by about one part in eight and a half millions, or one pound in 4000 tons.

Gravity diminished about $\frac{1}{8\,500\,000}$.

At a point which has the moon on its horizon it can be shown that gravity is *increased* by just half as much, or about one seventeen millionth.

The computation of the moon's lifting force at A and B (Fig. 120) is as follows: The distance of the moon from the earth's center is 60 earth radii, so that the distances from A and B are 59 and 61, respectively. The moon's mass is about $\frac{1}{80}$ of the earth's. Taking g for the force of gravity at the surface of the earth, we have, therefore, attraction of moon on $A = \frac{g}{80 \times 59^2}$, attraction on $C = \frac{g}{80 \times 60^2}$, and attraction on $B = \frac{g}{80 \times 61^2}$. From this we find

$$(A - C) = \frac{g}{8\,424000}, \text{ and } (C - B) = \frac{g}{8\,835000}.$$

Several attempts have been made within the last twenty years to detect this variation of weight by direct experiment, but so far unsuccessfully. The variations are too small.

The moon's attraction also produces everywhere, except at A, B, D, and E (Fig. 120), a *tangential force* which urges the particles along the surface towards the line AB and powerfully coöperates in the tide-making.

Moon's tangential force.

335. The Sun's Tide-Producing Force. — The sun acts precisely as the moon does, but, being nearly 400 times as far away,[1] its tidal action, notwithstanding its enormous mass, is less than that of the moon in the proportion of 2 to 5 (nearly). At new and full moon the tidal forces of the sun and moon *conspire*, and we then have the *spring-tides*, while at quadrature *they are opposed*, and we get the *neap tides*, their relative heights being as (5 + 2) to (5 − 2). The *priming* and *lagging* of the tides (Sec. 331) is also due to the sun's influence.

Tidal influence of the sun about two fifths that of the moon.

336. Condition for Permanent Tides. — If the earth were wholly composed of water, and if it kept always the same

---

[1] It can be proved that the "tide-producing force" of a body varies inversely as the *cube* of its distance, and directly as its mass.

Permanent tides if earth's day were a month long.

face towards the moon (as the moon does towards the earth), so that every particle on the earth's surface were always subjected to the same disturbing force from the moon, then, leaving out of account the sun's action for the present, a *permanent tide* would be raised upon the earth, as indicated in Fig. 120. The difference between the level at *A* and *D* would in this case be a *little less than 2 feet.*

Effect of earth's rotation.

337. Effect of the Earth's Rotation. — Suppose, now, the earth to be put in rotation. It is easy to see that the two tidal waves *A* and *B* would move over the earth's surface, following the moon at a certain angle dependent on the inertia of the water and *tending* to move with a westward velocity precisely equal to that of the earth's eastward rotation, — about 1000 miles an hour at the equator. The sun's action would produce similar tides superposed upon the lunar tides, and about two fifths as large; and at different times of the month these two pairs of tides would be differently related, as has already been explained, sometimes conspiring and sometimes opposed.

Tide crest 90° from moon if earth were covered by deep water.

If the earth were entirely covered with deep water, the tide-waves would run around the globe regularly, and if the depth of water were *not less than 13 miles*, the tide crests, as can be shown (though we do not undertake it here), *would follow the moon at an angle of just 90°.* It would be high water precisely where it might at first be supposed we should get low water, the place of high water being shifted 90° by the rotation of the earth.

Tide-wave cannot keep up with moon, since water is not deep enough and continents interfere.

If the depth of the water were, as it really is, much less than 13 miles, the tide-wave in the ocean could not keep up with the moon, and this would complicate the result. Moreover, the continents of North and South America, with the southern antarctic continent, make a barrier almost complete from pole to pole, leaving only a narrow passage at Cape Horn. Consider also the varying depth of the water of the different oceans and the irregular contours of the shores, and it is evident that the whole combination of circumstances makes it quite impossible to determine by theory what the course and character of the tide-waves must be. We are obliged to depend upon observations, and observations are more or less inadequate, because, with the exception of a few islands, our only possible tide stations are on the shores of continents where local circumstances largely control the phenomena.

338. Free and Forced Oscillations. — If the water of the ocean is suddenly disturbed, as, for instance, by an earthquake, and then left to itself, a "free wave" is formed, which, if the horizontal dimensions of the wave are large as compared with the depth of the water, will travel at a rate *depending solely on the depth.*

Free waves in the ocean; their velocity.

Its velocity is equal, as can be proved, to the *velocity acquired by a body in falling through half the depth of the ocean; i.e.*, $v = \sqrt{gh}$, where h is the depth of the water.

Observations upon waves caused by certain earthquakes in South America and Japan have thus informed us that between the coasts of those countries the Pacific averages between $2\frac{1}{2}$ and 3 miles in depth.

Now, as the moon in its apparent diurnal motion passes across the American continent each day and comes over the Pacific Ocean, it starts such a "parent" wave in the Pacific, and a second one twelve hours later. These waves, once started, move on *nearly* (but not exactly) like a free earthquake wave, — not *exactly*, because the velocity of the earth's rotation being about 1050 miles an hour at the equator, the moon moves (relatively) westward faster than the wave can naturally follow it, and so for a while the moon slightly accelerates the wave. The tidal wave is thus, *in its origin*, a "forced oscillation"; in its subsequent travel it is very nearly, but not entirely, "free."

339. Cotidal Lines. — Cotidal lines are lines drawn upon the surface of the ocean connecting points which have their *high water at the same moment of Greenwich time.* They mark the

Cotidal lines defined.

crest of the tide-wave for every hour, and if we could map them with certainty, we should have all necessary information as to the actual motion of the tide-wave.

Unfortunately we can get no direct knowledge as to the position of these lines in mid ocean; we can only determine a few points here and there on the coasts and on the islands, so that much is necessarily left to conjecture. Fig. 122 is a reduced copy of a cotidal map, borrowed by permission, with some modifications, from Guyot's *Physical Geography*.

Course of the daily tide-waves.

340. Course of Travel of the Tidal Wave. — In studying this map we find that the main or "parent" wave starts twice a day in the Pacific, off Callao, on the coast of South America. This is shown on the chart by a sort of oval "eye" in the cotidal lines, just as on a topographical chart the summit of a mountain is indicated by an eye in the contour lines. From this point the wave travels northwest through the deep water of the Pacific at the rate of about 850 miles an hour, reaching Kamchatka in ten hours. Through the shallower water to the west and southwest the velocity is only from 400 to 600 miles an hour, so that the wave arrives at New Zealand about twelve hours old. Passing on by Australia and combining with the small wave which the moon raises directly in the Indian Ocean, the resultant tide crest reaches the Cape of Good Hope in about twenty-nine hours and enters the Atlantic.

Here it combines with a smaller tide-wave, twelve hours younger, which has backed into the Atlantic around Cape Horn, and it is also modified by the *direct tide* produced by the moon's action upon the Atlantic. The tide resulting from the combination of these three then travels *northward* through the Atlantic at the rate of nearly 700 miles an hour. It is about *forty hours old*, reckoning from the birth of its principal component in the Pacific, when it first reaches the coast of the United States in Florida; and our coast is so situated that it arrives at all the principal ports within two or three hours of that time. It is forty-one or forty-two hours old when it reaches New York and Boston.

Age of the tide when it reaches various ports.

To reach London it has to travel around the northern end of Scotland and through the North Sea, and is nearly sixty hours old when it arrives at that port and at the ports of the German Ocean.

In the great oceans there are thus three or four tide crests traveling simultaneously, following each other nearly in the same track, but with continual minor changes. If we take into account the tides in rivers and

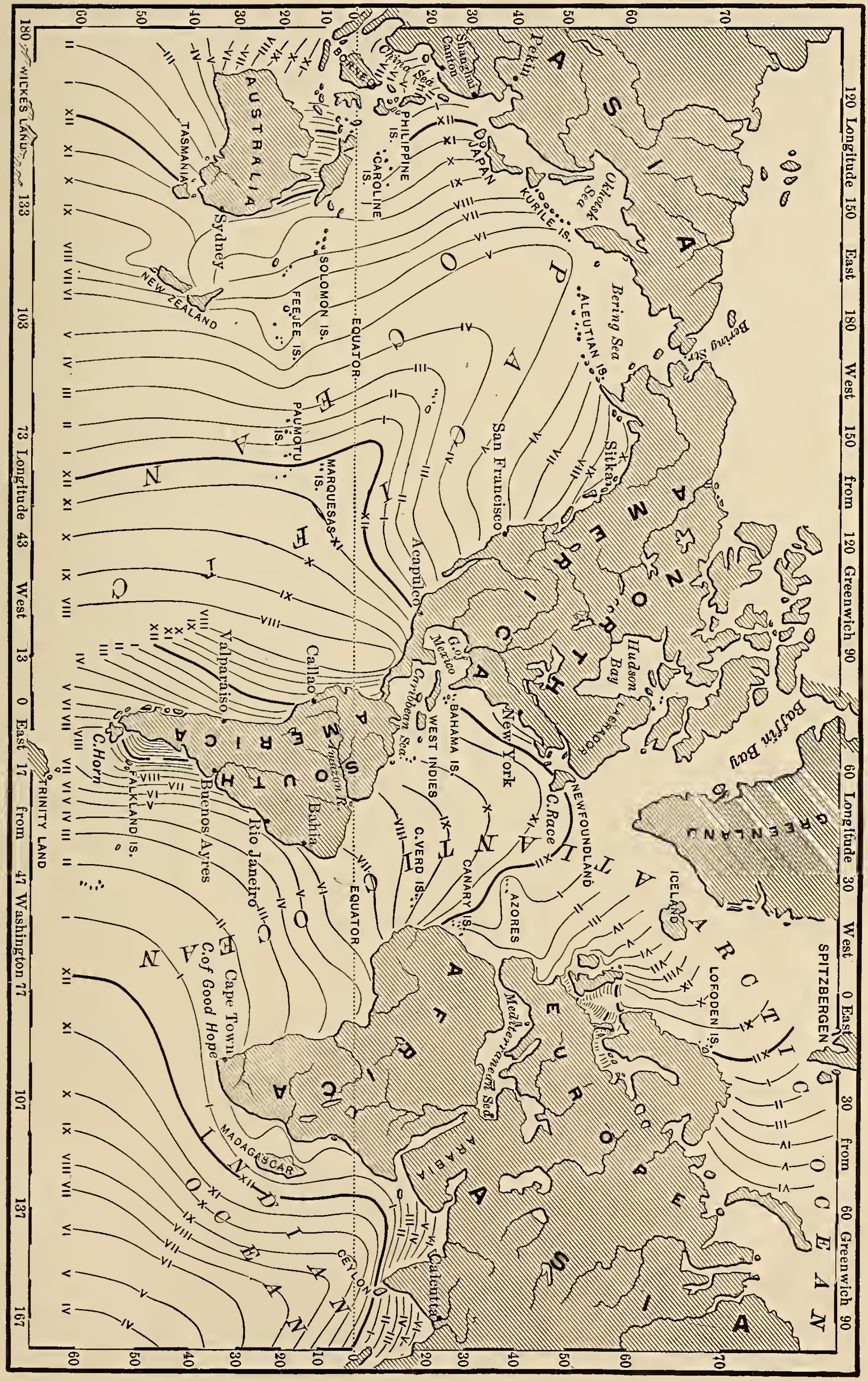
ASIA
AUSTRALIA
TASMANIA
Sydney
NEW ZEALAND
BORNEO
China Sea
Shanghai
Canton
Pekin
PHILIPPINE IS.
CAROLINE IS.
JAPAN
KURILE IS.
Okhotsk Sea
SOLOMON IS.
FEEJEE IS.
PAUMOTU IS.
MARQUESAS IS.
ALEUTIAN IS.
Bering Sea
Bering Str.
PACIFIC OCEAN
EQUATOR
San Francisco
Sitka
Acapulco
Callao
Valparaiso
NORTH AMERICA
Hudson Bay
LABRADOR
Baffin Bay
GREENLAND
ICELAND
G. of Mexico
Caribbean Sea
BAHAMA IS.
WEST INDIES
New York
C. Race
NEWFOUNDLAND
SOUTH AMERICA
Amazon R.
Bahia
Rio Janeiro
Buenos Ayres
C. Horn
FALKLAND IS.
TRINITY LAND
WICKES LAND
C. VERD IS.
CANARY IS.
AZORES
ATLANTIC OCEAN
ARCTIC OCEAN
SPITZBERGEN
LOFODEN IS.
EUROPE
Mediterranean Sea
AFRICA
ARABIA
Cape Town
C. of Good Hope
MADAGASCAR
CEYLON
Calcutta
INDIAN OCEAN
120 Longitude 150 East 180 West 150 from 120 Greenwich 90 60 Longitude 30 West 0 East 30 from 60 Greenwich 90
180 133 103 73 Longitude 43 West 13 0 East 17 from 47 Washington 77 107 137 167
70 60 50 40 30 20 10 0 10 20 30 40 50 60
I II III IV V VI VII VIII IX X XI XII

FIG. 122

sounds, the number of simultaneous tide crests must be at least six or seven; *i.e.*, the tidal wave at the extremity of its travel (up the Amazon River, for instance) must be at least three or four days old, reckoned from its birth in the Pacific.

Speed and limit of ascent of tides in rivers.

341. Tides in Rivers. — The tide-wave ascends a river at a rate which depends upon the depth of the water, the amount of friction, and the swiftness of the stream. It may, and generally does, ascend *until it comes to a rapid where the velocity of the current is greater than that of the wave.* In shallow streams, however, it dies out earlier. Contrary to what is usually supposed, *it often ascends to an elevation far above that of the highest crest of the tide-wave at the river's mouth.* In the La Plata and Amazon it goes up to an elevation of at least 100 feet above the sea-level. The velocity of the tide-wave in a river seldom exceeds 10 or 20 miles an hour, and is usually much less.

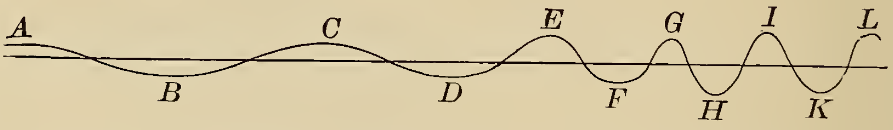

FIG. 123. — Increase in Height of Tide on approaching the Shore

Height of tides in mid ocean and near shore.

342. Height of Tides. — In mid ocean the difference between high and low water is usually between 2 and 3 feet, as observed on isolated islands in deep water; but on continental shores the height is ordinarily much greater. As soon as the tide-wave "touches bottom," so to speak, the velocity is diminished, the tide crests are crowded more closely together, and the height of the wave is increased somewhat as indicated in Fig. 123. Theoretically, *it varies inversely as the fourth root of the depth; i.e.*, where the water is 100 feet deep the tide-wave should be twice as high as at the depth of 1600 feet.

Maximum height of tides.

Where the configuration of the shore forces the tide into a corner it sometimes rises very high. In Minas Basin, near the head of the Bay of Fundy, tides of 70 feet are said to be not uncommon, and some of nearly 100 feet have been reported.

343. Effect of the Wind and Changes in Barometric Pressure. — When the wind blows into the mouth of a harbor, it drives in the water by its surface friction and may raise the level several feet. In such cases the time of high water, contrary to what might at first be supposed, is *delayed*, sometimes as much as fifteen or twenty minutes. This depends upon the fact that the water *runs into the harbor for a longer time* than it would do if the wind were not blowing.

Effect of wind and changes of barometer upon height of tide and time of high water.

When the wind blows out of the harbor, of course there is a corresponding effect in the opposite direction.

When the *barometer* at a given port is lower than usual, the level of the water is usually higher than it otherwise would be, at the rate of about 1 foot for every inch of difference between the average and actual heights of the barometer.

344. Tides in Lakes and Inland Seas. — These are small and difficult to detect. Theoretically, the range between high and low water in a land-locked sea should bear about the same ratio to the rise and fall of tide in mid ocean that the length of the sea does to the diameter of the earth. On the coasts of the Mediterranean the tide averages less than 18 inches, but it reaches the height of 3 or 4 feet at the head of some of the gulfs. In Lake Michigan, at Chicago, a tide of about $1\frac{3}{4}$ inches has been detected, the "establishment" (Sec. 331) of Chicago being about thirty minutes.

Tides in lakes.

345. Effects of the Tides on the Rotation of the Earth. — If the tidal motion consisted merely in the rising and falling of the particles of the ocean to the extent of some 2 feet twice daily, it would involve a very trifling expenditure of energy, and this is the case with the mid-ocean tide. But near the land this slight oscillatory motion is transformed into the bodily traveling of immense masses of water, which flow in upon the shallows and then out again to sea with a great amount of fluid friction, and this involves the expenditure of a very considerable amount of energy. From what source does this energy come?

Effect of tides upon length of day.

The answer is that it must be derived mainly from the earth's energy of rotation, and the necessary effect is to *lessen the speed* of rotation and to *lengthen the day*. Compared with the earth's whole stock of rotational energy, however, the loss by tidal

Counteracting causes.

friction even in a century is very small and the theoretical effect on the length of the day extremely slight. Moreover, while it is certain that the tidal friction, *by itself considered*, lengthens the day, it does not follow that the day grows longer. There are counteracting causes, — for instance, the earth's radiation of heat into space and the consequent shrinkage of her volume. At present we do not know *as a fact* whether the day is really longer or shorter than it was a thousand years ago. The change, if real, cannot well be as great as $\frac{1}{1000}$ of a second.

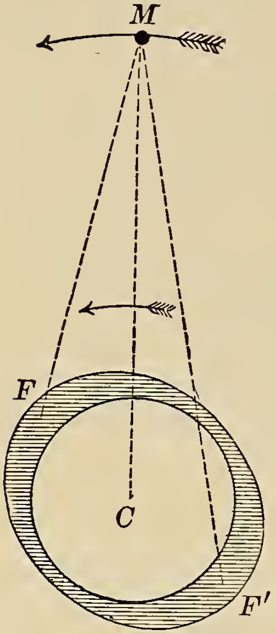

Fig. 124. — Effect of the Tide on the Moon's Motion

Effect of tide to cause the moon's distance to increase and to lengthen the month.

346. Effect of the Tide on the Moon's Motion. — Not only does the tide diminish the earth's energy of rotation directly by the tidal friction, but theoretically it also communicates a minute portion of that energy to the moon. It will be seen that a tidal wave, situated as in Fig. 124, would slightly accelerate the moon's motion, the attraction of the moon by the tidal protuberance, F, being slightly greater than that of the opposite wave at F. This difference would tend to draw it along in its orbit, thus slightly increasing its velocity, and so indirectly *increasing the major axis* of the moon's orbit as well as its *period.* The tendency is, therefore, to make the moon *recede* from the earth and to *lengthen* the month.

Tidal evolution.

Upon this interaction between the tides and the motions of the earth and moon Prof. George Darwin has founded his theory of *tidal evolution;* viz., that the satellites of a planet, having separated from it millions of years ago, have been made to recede to their present distances by just such an action.

An excellent popular statement of this theory will be found in the closing chapter of Sir Robert Ball's *Story of the Heavens*, and one more complete, but still popular, in Darwin's *Time and Tide.*

CHAPTER XII

THE PLANETS IN GENERAL

Bode's Law — The Apparent Motions of the Planets — The Elements of their Orbits — Determination of Periods and Distances — Perturbations, Stability of the System — Data referring to the Planets themselves — Determination of Diameter, Mass, Rotation, Surface Peculiarities, Atmosphere, etc. — Herschel's Illustration of the Scale of the System

Stars sensibly fixed on celestial sphere; planets move.

347. The *stars* preserve their relative configurations, however much they may alter their positions in the sky from hour to hour. The Dipper always remains a "dipper" in every part of the diurnal circuit.

But certain of the heavenly bodies, and the most conspicuous of them, behave differently. The sun and the moon move always steadily eastward through the constellations; and a few others, which look like brilliant stars, but are not stars at all, creep back and forth among the star groups in a less simple manner.

These moving bodies were called by the Greeks *Planets*, *i.e.*, "wanderers." They enumerated seven, — the Sun and Moon and, in addition, Mercury, Venus, Mars, Jupiter, and Saturn.

List of the planets.

348. List of Planets. — At present the sun and moon are not reckoned as planets; but the number of others known to the ancients has been increased by two new worlds, — Uranus and Neptune, of great magnitude, though inconspicuous on account of their distance, — besides a host of little asteroids.

The list of the principal planets in their order of distance from the Sun stands thus at present: Mercury, Venus, the Earth, Mars, Jupiter, Saturn, Uranus, and Neptune.

Asteroids and Eros.

Moreover, between Mars and Jupiter, where there is a wide gap in which another planet would naturally be looked for, there have already (January, 1902) been discovered more than five hundred little bodies called "asteroids," which probably represent a single planet, somehow "spoiled in the making," so to speak, or subsequently burst into fragments.

One of this family, Eros, discovered in 1898, crosses the inner boundary mentioned, — the orbit of Mars, — and at times comes nearer to the earth than any other heavenly body except the moon.

Planets non-luminous.

The planets are non-luminous bodies which shine only by reflected sunlight, — globes which, like the earth, revolve around the sun in orbits nearly circular, moving all of them in the same direction and (with numerous exceptions among the asteroids) nearly in the common plane of the ecliptic.

Satellites.

All but the inner two and the asteroids are attended by satellites. Of these the Earth has one (the moon), Mars two, Jupiter five, Saturn eight (and perhaps nine), Uranus four, and Neptune one.

Bode's Law.

349. Relative Distances of the Planets from the Sun; Bode's Law. — There is a curious approximate relation between the distances of the planets from the sun, usually known as *Bode's Law*.

It is this: Write a series of 4's. To the second 4 add 3; to the third add 3 × 2, or 6; to the fourth, 4 × 3, or 12; and so on, doubling the added number each time, as in the following scheme:

4	4	4	4	4	4	4	4	4
—	3	6	12	24	48	96	192	384
4	7	10	16	[28]	52	100	186	388
☿	♀	⊕	♂	①	♃	♄	♅	♆

No satisfactory explanation of the law yet reached.

The resulting numbers (divided by 10) are approximately equal to the true mean distances of the planets from the sun, expressed in radii of the earth's orbit (astronomical units) — excepting Neptune, however; in his case the law breaks down utterly. For the present, at least, it must therefore be regarded as a mere coincidence rather than a real "law," but it is not unlikely that its explanation may ultimately be found when the evolution of the solar system comes to be better understood.

It is known as Bode's Law because first brought prominently into notice by him in 1772, though it appears to have been discovered by Titius of Wittenberg some years earlier.

350. Table of Names, Distances, and Periods

Table of the planets: symbols, distances, and periods.

NAME	SYMBOL	DISTANCE	BODE	DIFF.	SID. PERIOD	SYN. PERIOD
Mercury . . .	☿	0.387	0.4	– 0.013	88^d or 3^m	116^d
Venus	♀	0.723	0.7	+ 0.023	$224^d.7$ or $7\frac{1}{2}^m$	584^d
Earth	⊕	1.000	1.0	0.000	$365\frac{1}{4}^d$ or 1^y	. . .
Mars	♂	1.523	1.6	– 0.077	687^d or $1^y\,10^m$	780^d
Mean asteroid		2.650	2.8	– 0.150	$3^y.1$ to $8^y.9$	various
Jupiter	♃	5.202	5.2	+ 0.002	$11^y.9$	399^d
Saturn	♄	9.539	10.0	– 0.461	$29^y.5$	378^d
Uranus	⛢ & ♅	19.183	19.6	– 0.417	$84^y.0$	370^d
Neptune . . .	♆	30.054	38.8	**– 8.746**	$164^y.8$	$367\frac{1}{2}^d$

The column headed "Bode" gives the distance according to Bode's Law; the column headed "Diff.," the difference between the true distance and that given by Bode's Law.

Definition of sidereal and synodic periods.

351. Periods. — The *sidereal period* of a planet is the time of its revolution around the sun, from a *star* to the same star again, as seen from the sun. The *synodic period* is the time between two successive conjunctions of the planet with the sun, *as seen from the earth.*

Equation expressing relation between them.

The sidereal and synodic periods are connected by the same relation as the sidereal and synodic months (Sec. 191), namely, $\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$, in which E, P, and S are, respectively, the periods of the earth and of the planet's synodic period; and the numerical difference between $\frac{1}{P}$ and $\frac{1}{E}$ is to be taken *without regard to* *sign; i.e.*, for an *inferior* planet, $\frac{1}{S}=\frac{1}{P}-\frac{1}{E}$; for a *superior* one, $\frac{1}{S}=\frac{1}{E}-\frac{1}{P}$.

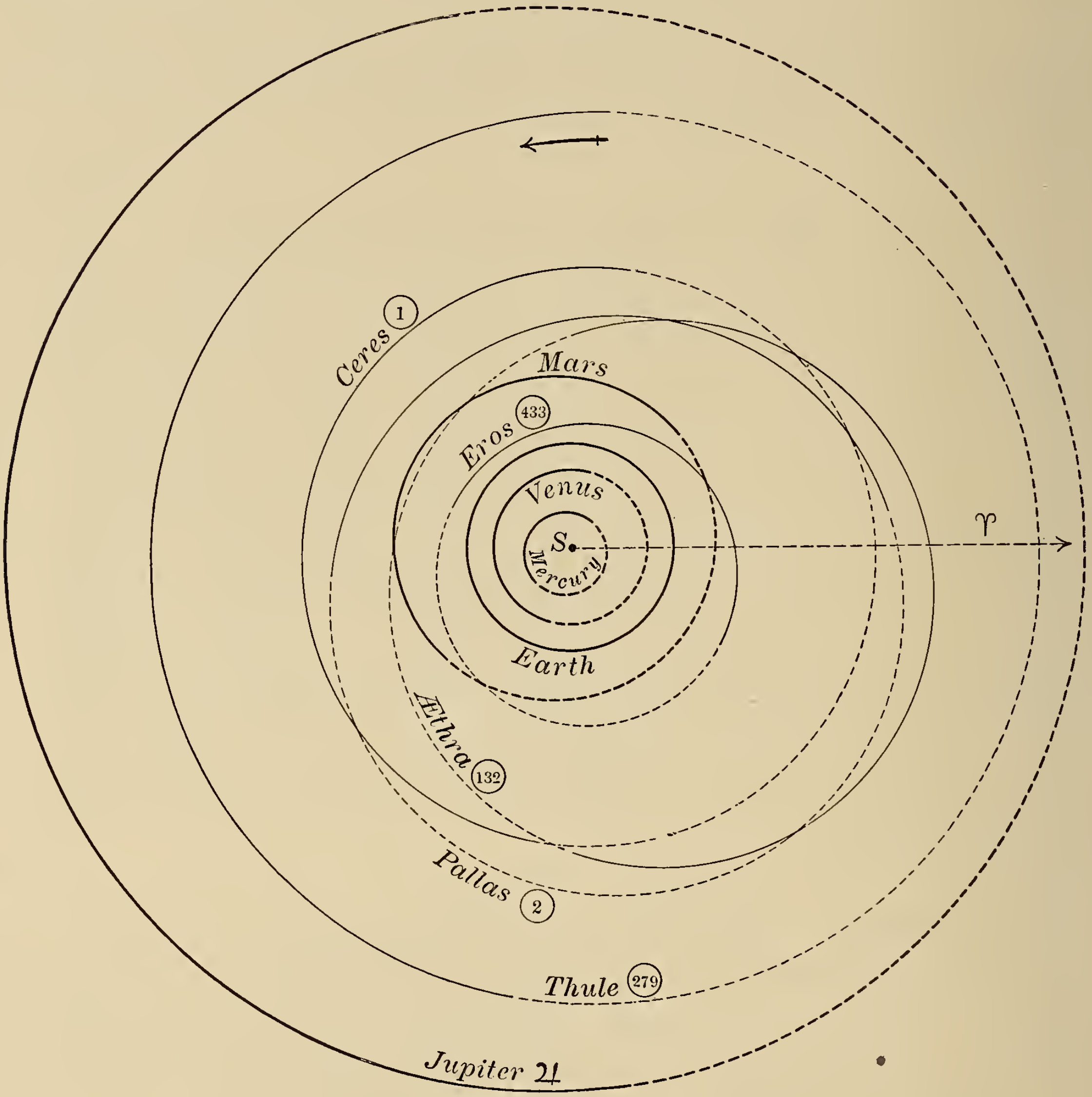
Ceres ①
Mars
Eros (433)
Venus
S
Mercury
Earth
♈
Æthra (132)
Pallas ②
Thule (279)
Jupiter ♃

Fig. 125. — The Planetary Orbits

The two last columns of the table of Sec. 350 give the approximate periods, both sidereal and synodic, for the different planets.

Map of the smaller orbits.

Fig. 125 shows the smaller orbits of the system (including the orbit of Jupiter), drawn to scale, the radius of the earth's orbit being taken as one centimeter.

On this scale the diameter of Saturn's orbit would be 19.08 centimeters, that of Uranus 38.36 centimeters, and that of Neptune 60.11 centimeters, or about 2 feet. The nearest fixed star, on the same scale, would be about a mile and a quarter away.

It will be seen that the orbits of Mercury, Mars, Jupiter, and several of the asteroids are quite distinctly eccentric.

Technical terms defined.

352. Explanation of Terms. — Fig. 126 illustrates the meaning of various terms used in describing the position of a planet with respect to the sun. *E* in the figure is the position of the earth, the inner circle is the orbit of an *inferior* planet (Mercury or Venus), and the outer circle is that of a *superior* planet, *Mars*, for instance.

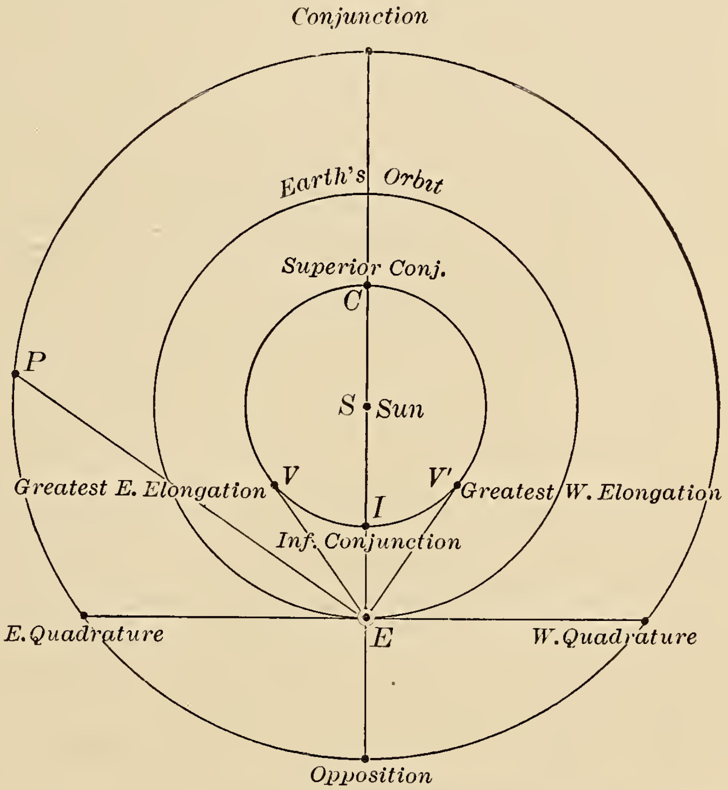
Conjunction
Earth's Orbit
Superior Conj.
C
P
S
Sun
Greatest E. Elongation
V
V′
Greatest W. Elongation
I
Inf. Conjunction
E. Quadrature
E
W. Quadrature
Opposition

Fig. 126. — Planetary Configurations and Aspects

Elongation.

The *Elongation* of a planet is the angle at the earth between lines drawn from the observer to the planet and to the sun, *i.e.*, the apparent angular distance of the planet from the sun; for a planet at *P* it is the angle *SEP*.

For a *superior* planet the elongation can have any value from 0° to 180°. For an *inferior* planet there is a certain maximum value, called the *greatest elongation*, which must be less than 90°. This greatest elongation is the angle between a line drawn from the earth to the sun and another line drawn tangent to the planet's orbit, — the angle *VES* in the figure.

Conjunction: superior and inferior.

Absolute *Conjunction* occurs when the elongation of the planet is zero; *superior* conjunction when the planet is beyond the sun; *inferior* when between earth and sun, — a position, of course, impossible for a superior planet. *Conjunction in longitude* occurs when the planet's longitude is the same as the sun's, and *in right ascension* when it has the same right ascension as the sun.

Opposition.

Opposition occurs when the elongation of a planet is 180° and the planet rises at sunset.

Quadrature.

Quadrature occurs when the planet has an elongation of 90°. An inferior planet cannot be in either opposition or quadrature.

The astrologers called these positions "aspects" and recognized several others, — for instance, "sextile," "trine," "octant," etc.

353. Apparent Motions of the Planets. — If we imagine ourselves looking down upon the orbits perpendicularly from their northern side, so as to see them *in plan*, they would appear as shown in Fig. 125, and the planets would travel regularly forward (contrary to the hands of a watch) with a steady, almost uniform, motion. Viewed from the earth, however, we see the orbits nearly edgewise, and their apparent motions are complicated, being made up of their own real motion around the sun, combined with a purely apparent motion due to the movement of the earth.

Apparent motion of planets complicated by earth's motion.

Their apparent motion as seen by us may be considered under three different aspects:

(1) The motion *in space relative to the earth.*

(2) The motion *on the celestial sphere relative to the constellations*, *i.e.*, change of right ascension and declination or of celestial latitude and longitude.

(3) With reference to *their apparent angular distance from the sun*, *i.e.*, motion *in elongation.*

354. Motion in Space Relative to the Earth. — The fundamental principle of relative motion is that if we look at a body at rest while we ourselves are moving, its *relative motion*, *i.e.*, the

Relative motion in space: a combination of the planet's real motion with that of the earth reversed.

change in its distance and direction from us, will be the same as if we were at rest and it possessed our motion *reversed.* If we look at a body while we move to the *south*, it appears to move towards the *north.* If we *approach* it, the effect is the same as if it *were coming towards us*, and so on.

If the body has a motion of its own, then the *total* apparent or relative motion will be the *resultant* of its real motion combined with our reversed motion, according to the law of composition of motions (*Physics*, p. 18).

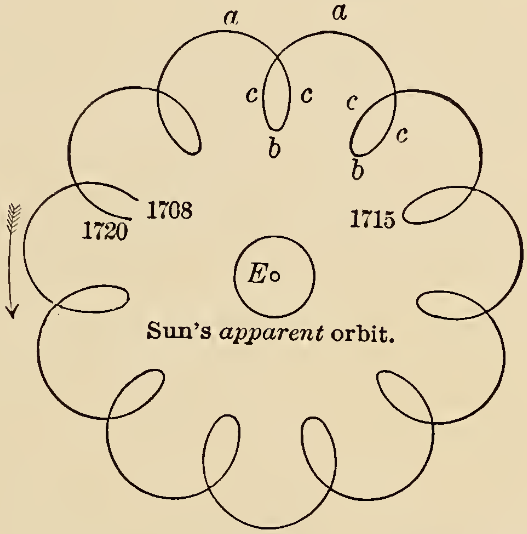

Fig. 127. — Geocentric Motion of Jupiter from 1708 to 1720

After Cassini

Result an epicycloidal motion relative to earth.

A planet at rest, therefore, would appear to move in an orbit precisely like that of the earth in form and size and in the same plane, always keeping its motion opposed to our own, though going around this apparent orbit *in the same direction* as the earth (just as any two opposite points on the circumference of a revolving wheel are always moving in opposite directions, though going the same way around the axis). And since the planets are really revolving around the sun, it follows that their apparent or *geocentric* motion is a combination of two motions, — that of a body moving once a year around the circumference of a circle[1] equal to the earth's orbit, while at the same time the center of that circle is carried around the sun in the real orbit of the planet, and in the same period with the planet. Jupiter, for instance, appears to move as in Fig. 127.

This is the orbit we should find if we were to attempt to map it out by the method used for determining the form of the orbit of the earth around the sun (Sec. 159), *i.e.*, by observing the *direction* of the planet from the earth, and at the same time measuring its *apparent diameter* in order to get its relative distances at different times. Practically, however, the method would not succeed very well, since the planet's apparent diameter is too small to permit the necessary precision in determining the variations of distance.

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[1] The "circles" spoken of here are strictly ellipses of small eccentricity.

Effect of the eccentricity of the real orbits.

A motion of the kind represented in the figure is loosely called "epicycloidal,"—not quite accurately, because the orbits concerned are not true circles, so that the loops are of varying size.

The Ptolemaic theory of the solar system was fundamentally an acceptance of this apparent motion of the planets relative to the earth as real, though his theory involved certain serious errors of arrangement and proportion.

Planet's motion on the celestial sphere: alternately direct and retrograde in right ascension and longitude.

355. Motion of a Planet on the Celestial Sphere, *i.e.*, in Right Ascension and Declination, or in Latitude and Longitude.—Looking at Fig. 127, we see that, viewed from the earth, the planet moves most of the time "direct," *i.e.*, *eastward* in the direction of the arrow, as at the points *aa*; but while rounding the loops at *bb*, where it comes nearest the earth, its apparent motion is reversed and "retrograde," and at certain points, *cc*, on each side of the loop the planet is "stationary" in the sky, its motion at the time being directly towards or from the earth.

Starting from the time of superior conjunction, when the planet is at *a*, it moves eastward, or "direct," among the stars, always increasing its right ascension or longitude, but at a rate continually slackening, until at last the planet becomes "stationary" at an eastern elongation from the sun, which depends upon the size of the orbit and its distance from the earth.

Stationary points.

From the stationary point it reverses its course and moves *westward* around the loop until it comes to the second stationary point at a western elongation, the same (if the real orbit is circular) as the eastern elongation of the former stationary point.

There it resumes its eastward motion and continues it until it reaches the next superior conjunction, at the end of a synodic period.

Planet retrogrades when nearest the earth.

The middle of this *arc of regression* is always very near the point where the planet comes nearest the earth, *i.e.*, at "opposition" for a superior planet, and at "inferior conjunction" for an inferior planet. In time, as well as in the number of degrees passed over, the direct motion always exceeds the retrograde in each synodic period of the planet.

As observed with a transit-instrument, all planets when moving eastward (direct) come *later* to the meridian each night *by the sidereal clock*, and *vice versa* when retrograding.

Motion in latitude: loops.

356. Motions in Latitude. — If the orbits of the planets all lay precisely in the same plane with the earth's orbit, their apparent orbits relative to the earth would do so also, and their apparent motions on the celestial sphere would be simply *forward and backward upon the ecliptic.*

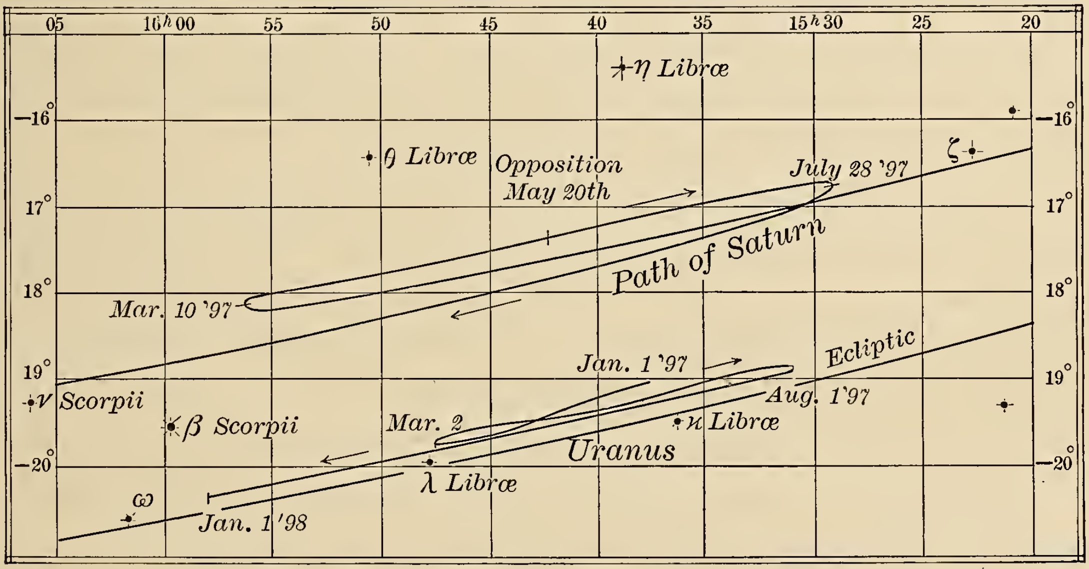

FIG. 128. — Motion of Saturn and Uranus in 1897

But while the orbits of the larger planets are only slightly inclined to the ecliptic, so that they never go very far from it, they do, in fact, deviate a few degrees one side and the other, so that their paths in the heavens form more or less complicated loops and kinks. Fig. 128 shows the loops made by Saturn and Uranus in 1897, when they happened to be very near each other in the sky.

Certain of the "asteroids" have orbits greatly inclined to the ecliptic and very eccentric, as, for instance, the little Eros. The description of apparent motions as given above would therefore require very serious modification in their case. Eros is sometimes found in circumpolar regions more than 40° north of the ecliptic; sometimes its nearest approach to the earth does not coincide with the time of its opposition within several weeks; and sometimes at the time of its opposition its motion is more nearly from *north to south* than from east to west.

Motion with respect to elongation from sun.

357. Motion of the Planets in Elongation, *i.e.,* with Respect to the Sun's Place in the Sky. — The visibility of a planet depends mainly on its *elongation*, because when near the sun the planet will be above the horizon only by day. As regards their motion, considered from this point of view, there is a marked difference between the inferior planets and the superior.

Superior planets come to meridian earlier every day by mean-time watch.

(1) *The Superior Planets drop always steadily westward with respect to the sun's place in the heavens*, continually increasing their western elongation or decreasing their eastern. As observed by an ordinary timepiece (keeping *solar time*), they therefore invariably rise earlier and come *earlier to the meridian* every successive night, never moving eastward among the stars as rapidly as the sun, even when their direct motion is most rapid. This relative motion westward with respect to the sun is not, however, uniform. It is slowest near superior conjunction, most rapid at opposition.

Beginning at conjunction the planet is then behind the sun, at its greatest distance from the earth, and invisible. It soon, however, reappears in the morning, rising before the sun as a "morning star," and passes on to western quadrature, when it rises at midnight. Thence it moves on to opposition, when it is nearest and brightest, and rises at sunset. Still dropping westward and receding, it by and by reaches eastern quadrature and is on the meridian at sunset. Thence it still crawls sluggishly westward as an "evening star," until it is lost in the twilight and completes its synodic period by again reaching conjunction.

Inferior planets oscillate from one side of sun to the other.

358. (2) The *Inferior Planets*, on the other hand, apparently *oscillate* across the sun, moving out equal, or nearly equal, distances on each side of it, but making the westward swing between us and the sun much more quickly than the eastward.

The westward swing much swifter than the eastward.

At superior conjunction an inferior planet is moving eastward *faster* than the sun. Accordingly, it creeps out into the twilight as an "evening star," and continues to increase its apparent distance from the sun until it reaches its *greatest eastern elongation* (47° for Venus; for Mercury, from 18° to 28°). Then the sun begins to gain upon it, and as the planet itself soon begins to retrograde, the elongation diminishes rapidly and the planet hurries back to *inferior conjunction*, passes it, and then as a "morning star" moves swiftly out to its western elongation. There it turns and climbs slowly back to superior conjunction again.

The system of Ptolemy. The Almagest.

359. The Ptolemaic System. — Assuming the fixity and central position of the earth and the actual revolution of the heavens, Ptolemy (who flourished at Alexandria about A.D. 140) worked out the system which bears his name.

The epicycle, fictitious planet, and deferent.

In his great work, the *Almagest* (Arabic for the Greek *The Greatest*), which for fourteen centuries was the authoritative "Scripture of astronomy," he showed that all the apparent motions of the planets, so far as then observed, could be accounted for by supposing each planet to move around the circumference of a circle called the "epicycle," while the center of this circle, sometimes called the "fictitious planet," itself moved *around the earth* on the circumference of another and larger circle, called the "deferent."

It was as if the real planet was carried on the end of a crank arm which turned around the "fictitious planet" as a center in such a way as to point towards or from the earth at times when the planet is in line with the sun.

Fig. 129 represents this Ptolemaic system, except that no attention is paid to dimensions, the deferents being spaced at equal distances.

It will be noticed that the epicycle radii, which carry at their extremities the planets Mars, Jupiter, and Saturn, are always parallel to the line which joins the earth and the sun.

Error of Ptolemy in respect to orbits of Mercury and Venus.

In the case of Venus and Mercury this was not so. Ptolemy supposed that for these planets the *deferent* circles lay *between* the earth and the sun, and that the fictitious planet in both cases revolved in its deferent once a year, always keeping exactly between the earth and the sun; the motion in the *epicycle* in this case was completed in the time of the planet's period. He did not recognize that for these two planets there should be only one deferent, viz., the orbit of the sun itself, as the ancient Egyptians are said to have understood.

To account for some of the irregularities of the planets' motions it was necessary to suppose that both the deferent and epicycle, though circular, are eccentric, the earth not being exactly in the center of the deferent, nor the "fictitious planet" in the exact center of the epicycle. In after times, when the knowledge of the planetary motions had become more accurate, the Arabian astronomers added epicycle upon epicycle until the system became very complicated.

Epicycles added by the Arabian astronomers. The Alphonsine tables.

King Alphonso of Spain is said to have remarked to the astronomers who presented to him the Alphonsine tables of the planetary motions, which had been computed under his orders, that "if he had been present at the creation he would have given some good advice."

System of Copernicus.

360. The Copernican System. — Copernicus (1473–1543) asserted the diurnal rotation of the earth on its axis, which was rejected by Ptolemy, and showed that it would fully account for the apparent diurnal revolution of the stars. He also showed that nearly all the known motions of the planets could be accounted for by supposing them to revolve around the sun, with the earth as one of them, in orbits *circular*, but slightly *out of center.* His system, as he left it, was nearly that which is accepted to-day, and Fig. 125 may be taken as representing it. He was, however, obliged to retain a few small epicycles to account for certain of the irregularities.

Discovery of the elliptical form of orbits by Kepler.

Up to this time no one dared to doubt the exact circularity of celestial orbits. It was considered metaphysically improper that heavenly bodies should move in any but *perfect* curves, and the circle was regarded as the only perfect one. It was left for Kepler, some sixty years later than Copernicus, to show that the planetary orbits are *elliptical* and to bring the system substantially into the form in which we know it now.

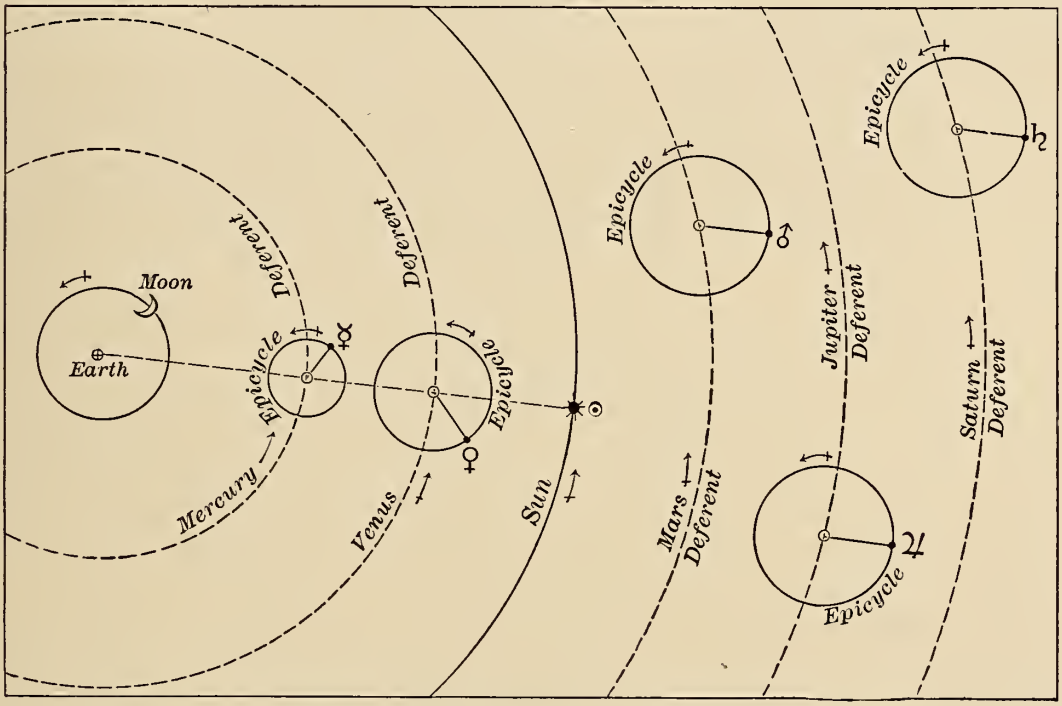
Moon
Earth
Deferent
Epicycle
☿
Mercury
Deferent
Venus
Epicycle
♀
Sun
☉
Epicycle
♂
Mars
Deferent
Jupiter
Deferent
Epicycle
♃
Epicycle
♄
Saturn
Deferent

FIG. 129. — The Ptolemaic System

It was nearly a century before the Copernican system, with the improvements of Kepler, finally replaced the Ptolemaic. In our oldest American universities, Harvard and Yale, the Ptolemaic was for a considerable time taught in connection with the Copernican.

System of Tycho. He could not detect any parallax of the stars and concluded that the earth must be at rest.

361. Tychonic System. — Tycho Brahe, who came between Copernicus and Kepler, found himself unable to accept the Copernican system for two reasons. One was that it was unfavorably regarded by the church, and he was a good churchman. The other was the really scientific objection that if the earth moved around the sun, the fixed stars all ought to appear to move in a corresponding manner, each star describing annually an oval in the heavens of the same apparent dimensions as the earth's orbit seen from the star. Technically speaking, they ought to have an *annual parallax.*

His instruments were by far the most accurate that had ever been made, and he could detect no such parallax (although it really existed and can now be observed); hence, he concluded, not illogically, but incorrectly, that the earth must be at rest.

He rejected the Copernican system, placed the earth at the center of the universe, according to the then received interpretation of Scripture, made the sun revolve around the earth once a year, and then (this was the peculiarity of his system) made the apparent orbit of the sun the *common deferent* for the epicycles of all the planets, making them to revolve around the sun.

This theory just as fully accounts for all the motions of the planets as the Copernican or Ptolemaic, but like the Ptolemaic breaks down absolutely when it encounters the *aberration of light* and the *annual parallax of the stars*, now observable with modern instruments, though not with Tycho's. The Tychonic system was never generally accepted, and the Copernican was soon firmly established by Kepler and Newton.

The seven elements of a planet's orbit.

362. Elements of a Planet's Orbit. — These are a set of numerical quantities, seven in number, which describe the orbit with precision and furnish the means of finding the planet's place in the orbit at any given time, whether past or future, so far as that place depends upon the attraction of the sun alone. They are as follows:

(1) The semi-major axis, a.
(2) The eccentricity, e.
(3) The inclination to the ecliptic, i.
(4) The longitude of the ascending node, ☊.
(5) The longitude of perihelion, π.
(6) The period, P, or else the daily motion, μ.
(7) The epoch, E.

Of these, the first five pertain to the orbit itself, regarded as an ellipse lying in space with one focus at the sun, while two are necessary to determine the planet's place in the orbit.

363. The *semi-major axis*, a (CA in Fig. 130), defines the *size* of the orbit and is usually expressed in astronomical units. (It will be remembered that the earth's mean distance from the sun is the "astronomical unit.")

Size defined by the semi-major axis.

The *eccentricity*, e, defines the orbit's *form*. It is a mere numerical quantity, being the fraction $\frac{c}{a}$, obtained by dividing the distance between the sun and the center of the orbit by the semi-major axis. In some computations it is convenient to use, instead of the decimal fraction itself, the angle ϕ, which has e for its sine, so that $e = \sin \phi$.

Form defined by the eccentricity

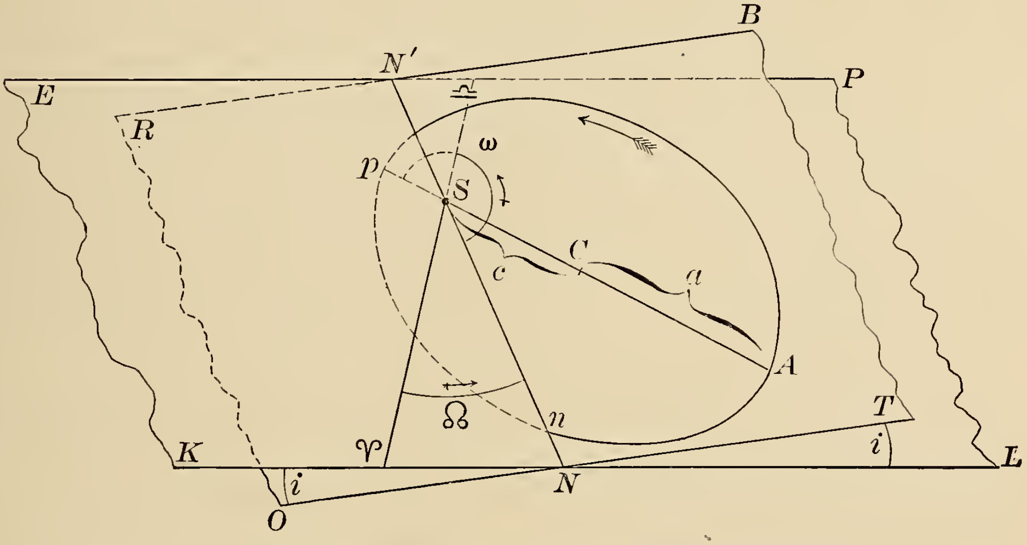

FIG. 130. — The Elements of a Planet's Orbit

The third element, i, the *inclination*, is the angle between the plane of the planet's orbit and that of the earth. In the figure it is the angle KNO, the plane of the ecliptic being lettered $EKLP$ and that of the orbit $ORBT$.

Inclination and longitude of node determine plane of the orbit.

The fourth element, ☊ (*the longitude of the ascending node*), defines what has been called the "aspect" of the orbit plane, *i.e.*, the direction in which it faces. The line of nodes is the line

NN′ in the figure (the intersection of the two planes of the orbit and ecliptic), and the angle ♈ *SN* is the longitude of the ascending node. The planet passes from the lower or southern side of the plane of the ecliptic to the northern at the point *n* in its orbit.

The fifth, and last, of the elements which belong strictly to the orbit itself is π, the so-called *longitude of the perihelion*, which defines the direction in which the major axis of the ellipse (the line *pA*) lies on the plane *ORBT*. Strictly, π is not a longitude, but equals the sum of the two angles ☊ and ω; *i.e.*, ♈ *SN* (in the plane of the ecliptic) plus *NSp* (in the plane of the orbit), both reckoned in the direction of the planet's motion. *NSp*, or ω, in the figure is about 210° and ♈ *Sp* is about 315°.

If we regard the orbit as an oval wire hoop suspended in space, these five elements completely define its *position*, *form*, and *size*. The *plane* of the orbit is fixed by the two elements numbered three and four, the *position* of the orbit in this plane by number five, the *form* of the orbit by number two, and finally its *magnitude* by number one.

To determine where the planet will be at any subsequent date we need two more elements:

The period and epoch furnish the means of computing the place of planet in its orbit.

Sixth, *the periodic time*. We must have the sidereal period, P, or else the mean daily motion, μ, which is simply 360° divided by the number of days in P.

Seventh, and finally, we must have a "starting-point," the *Epoch*, so called; *i.e.*, the longitude of the planet as seen from the sun at some given date, usually Jan. 1, 1850 or 1901, or else the precise date at which the planet passed the perihelion or the node.

364. If no force acted on the planets except the sun's attraction, these elements would never change, but on account of the interaction of the planets they do change; accordingly, it is usual to add in tables of the elements columns giving the amount by which each element changes in a century.

It is to be noted also that if Kepler's third law in its uncorrected form were strictly true, as it is not (Sec. 308), we should not need both a and P, for if a is expressed in *astronomical units*, P in years would be simply $\sqrt{a^3}$.

The method of determining the position of a planet in its orbit, *i.e.*, of computing an ephemeris, belongs to theoretical astronomy and will not be treated here. It is sufficient to say that it is possible from the elements of the planets to deduce, for any given time, their actual positions in their orbits and their distances and directions from the sun and from each other.

DETERMINATION OF THE PERIOD AND DISTANCE OF A PLANET

365. Since the planetary orbits are, for the most part, nearly circular and in the plane of the ecliptic, they are described with sufficient accuracy for ordinary purposes by simply giving the planet's period and distance from the sun. We proceed to show how these two elements may be determined, but note in passing that there is a *general method* by which all seven of the elements of a planet's orbit can ordinarily be deduced together from *three accurate observations* of the planet's position, separated by a few weeks' interval, though in certain special cases a *fourth* observation becomes necessary.

General method of Gauss for computing all the elements of a planet from three complete observations.

This general method involves long and complicated calculation,—treated in works on theoretical astronomy. It was invented in 1801 by Gauss, then a young man of twenty-three, in connection with the discovery of Ceres, the first of the asteroids, which, after its discovery by Piazzi, was lost to observation by passing into conjunction with the sun.

366. The observations upon which the calculation for the elements of a planet's orbit rest are determinations of the planet's right ascension and declination, usually made with the meridian-circle, but sometimes by the differential method (Secs. 116–117) with the equatorial telescope and micrometer, or often at present by photography.

These observations are, of course, made from the earth's *surface*, and before they can be utilized must be corrected for parallax, so as to give the *geocentric* place, *i.e.*, the place the planet would occupy if seen from the center of the earth. In many cases the geocentric right ascension and declination must, for convenient use, be further transformed into celestial latitude and longitude. (See Sec. 30 and Appendix, Sec. 702.)

Interpolation of observations to furnish a planet's place at moment when it could not be actually observed.

367. Interpolation of Observations. — It often happens that we want the place of a planet at some particular moment when it cannot be actually observed, as, for instance, when it is below the horizon. If we have a *series* of observations made about that time, say for several days before and after, the place at any moment included within the time covered by the observations can be determined by *interpolation*, and with an accuracy exceeding that of any single observation of the series.

The determination can be made *graphically* by simply plotting the observations on squared paper with a scale of times as abscissas, the observed data being plotted as ordinates, and then drawing a curve through the points determined by observation, as in so many operations of the physical laboratory. Whatever can be done *graphically* can, of course, be worked out still more accurately by *calculation*. The principle is of very extensive application.

Heliocentric place.

368. Heliocentric Place. — This is the place of a planet as it would be seen from the sun and is often wanted in calculations.

When we have once found the node of the planet's orbit and the inclination of the orbit, as well as the planet's distance from the sun, the *heliocentric* place and the distance of the planet from the earth can be immediately deduced from the *geocentric* by a simple calculation, which, though not difficult, is rather tedious and lies outside the scope of this work. (See Watson's *Theoretical Astronomy*, p. 86.)

369. Determination of the Sidereal Period of a Planet. — *First, by Observation of its Node Passage.* At the moment when a planet crosses its node its *latitude*, both geocentric and heliocentric, becomes zero, because the planet is then actually in the plane of the ecliptic. From a series of observations of its right ascension and declination made about that time and reduced to latitude and longitude, both the position of the node and the *time* when the planet crossed the node can be deduced.

Determination of a planet's period by observations of time when it passes the node.

The interval between two successive node passages thus determined is the planet's period, — exactly, if the node be stationary; very approximately in any case, for none of the nodes move rapidly.

The method is not very satisfactory, however, (1) because the planetary orbits cross the ecliptic at so small an angle that the latitude is almost zero for many hours, so that the precise second is difficult to determine; (2) then, also, the periods of the more distant planets are too long, — Uranus, 84 years; Neptune, 164 years, — too long to wait.

Period determined from observations of time of opposition.

370. *Second, by the Mean Synodic Period.* The sidereal period may also be determined by finding the *mean synodic* period of a planet from the dates of two *oppositions*, widely separated in time if possible.

The exact instant of opposition is found from a series of right ascensions and declinations observed about the proper date; by comparing the deduced longitudes of the planet with the corresponding longitudes of the sun we find easily the precise moment when the difference was 180°. When the synodic period is found the *sidereal* is at once given by the equations in Sec. 351, viz., $\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$ for an inferior planet, and $\frac{1}{S} = \frac{1}{E} - \frac{1}{P}$ for a superior. In the first case, $P = \frac{S \times E}{S + E}$; in the second, $P = \frac{S \times E}{S - E}$.

Necessary that the oppositions should be separated by a long interval.

It will not answer for this purpose to deduce the synodic period from two *successive* oppositions, because, on account of the eccentricity of the orbits, both of the planet and of the earth, the synodic periods are notably variable. The observations must be sufficiently separated in time to give a good determination of the *mean* synodic period.

In the case of all the older planets we have observations running back nearly two thousand years, so that no difficulty arises on this score. For the newly discovered planets the method would be seldom available.

Geometrical method of determining a planet's distance from the sun by two observations of its elongation separated by an interval of time exactly equal to the planet's period.

371. Geometrical Method of determining a Planet's Distance from the Sun in Astronomical Units. — When we know a planet's sidereal period it is easy to determine its distance from the sun by two observations of the planet's *elongation from the sun*, made at dates separated by an *interval of exactly one of its periods.*

The elongation, it will be remembered, is the difference between the longitude of the planet and that of the sun as seen from the earth, and is determined for any given date by a series of meridian-circle observations of the planet and sun covering that date.

To determine, for instance, the distance of Mars we must have two observations of the planet's elongation, *MAS* and *MCS* (Fig. 131), separated by an interval of 686.95 days; so that at the moment of the second observation from the earth at *C* the planet will occupy precisely the same point in its orbit as when observed from *A* nearly two years before.

In the figure the two angles at *A* and *C* are given directly by the observations. The angle at the sun, *ASC*, is determined *from the earth's motion during the elapsed time*, which is less than two *sidereal* years by 730.53 days *minus* 686.95 days, *i.e.*, by 43.58 days; this makes the angle *ASC* very nearly 43°.

The sides *AS* and *CS* are radii vectores of the earth's orbit, accurately known in terms of the mean distance of the earth from the sun, which is the astronomical unit.

In the quadrilateral *SAMC* we have, therefore, the three angles *A*, *S*, and *C* given, and the two sides *AS* and *CS*; we can, therefore, proceed just as in Sec. 196 in computing the line *SM*, finding both its *length* as compared with *AS*, the astronomical unit, and also the *planet's direction from the sun*, given by the angle ASM or CSM, both of which come out in the course of the calculation.

The student can follow out for himself the process by which from two elongations of Venus, SAV and SBV, observed at an interval of *225 days*, SV can be determined. A little modification is necessary from the fact that the point S falls within the triangle formed by the two positions of the earth and planet, instead of outside of it, as in the case of Mars.

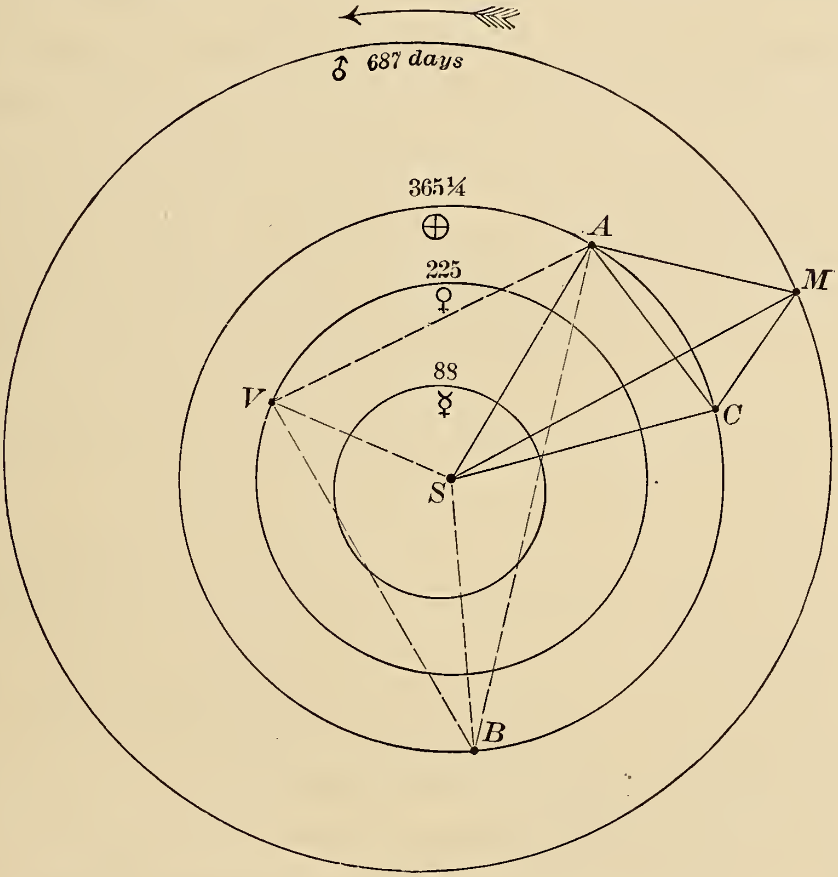

FIG. 131. — Determination of the Distance of a Planet from the Sun

372. From a sufficient number of such pairs of observations distributed around an orbit it is evidently possible to work out completely its magnitude and form; and it was precisely in this way that Kepler, utilizing the rich mine of data contained in Tycho's long series of observations, proved that the orbit of Mars is an *ellipse* (and later those of the other planets also) and

This method used by Kepler in proving the orbit of Mars to be an ellipse.

deduced their distances from the sun as compared with that of the earth. His Harmonic Law was then discovered by simply comparing the periods with the distances. Now that we have the Harmonic Law, a planet's *approximate* mean distance can, of course, after its period is known, be much more easily found by applying that law than by the geometrical method just explained.

The distance of an inferior planet determined by a single observation of its greatest elongation.

373. Simple Method of finding the Distance of an Inferior Planet. — In the case of Venus, which has an orbit almost perfectly circular, we can use the method indicated in Fig. 132. When the planet is at its greatest elongation the angle at V is sensibly a right angle, and if we then measure the elongation SEV, we have at once $SV = SE \times \sin SEV$.

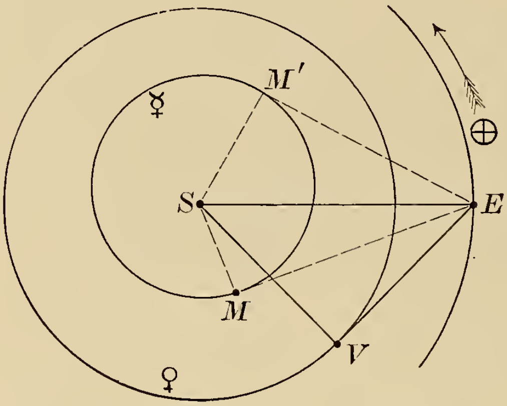

FIG. 132. — Distance of an Inferior Planet determined by Observations of its Greatest Elongation

Mercury's orbit is so eccentric that the method gives only a rough approximation, the angle at M not being a right angle; but, by taking many observations distributed all around the orbit, an accurate result may be obtained.

374. Planetary Perturbations. — The attractions of the planets for each other slightly disturb their otherwise elliptical motion around the sun, but their disturbing forces are, with few exceptions, extremely small, and the resulting perturbations are, as a rule, much less than in the case of the moon. The exception is in the case of some of the asteroids, which at times come near enough to the gigantic Jupiter to be displaced by as much as 8° or 10°. The interaction between Jupiter and Saturn also produces apparent displacements of these planets exceeding half a degree.

Planetary perturbations: periodic and secular.

The planetary perturbations are divided into two classes: (1) the *periodic* perturbations, which depend on the positions of the *planets in their orbits* and affect their orbital positions (these generally run through their cycle within a century); (2) the *secular* perturbations, which depend on the relative positions *of the orbits themselves* with reference to each other and produce changes *in the elements of the orbits affecting the positions of the planets only indirectly* (these have periods of thousands, and even millions, of years).

Amount of the periodic perturbations of the different planets.

375. Periodic Perturbations. — Those of Mercury never exceed 15″, as seen from the sun. Those of Venus may reach 30″; those of the earth about 1′ (say 30000 *miles*), and those of Mars a little exceed 2′. As already mentioned, the mutual disturbances of Jupiter and Saturn are much larger, reaching 28′ and 48′, respectively. Those of Uranus never reach 3′, as seen from the sun, and those of Neptune are smaller yet.

The great perturbation between Jupiter and Saturn is called a "long inequality," having a period of 913 years. It is due to the near commensurability of their periods, seventy-seven of Jupiter's periods being almost exactly equal to thirty-one of Saturn's. Between Uranus and Neptune there is a somewhat similar "long inequality" with a period exceeding 4000 years.

Secular perturbations of the orbits.

376. Secular Perturbations. — These, as already said, depend upon the relative positions of the *orbits*, but not of the planets themselves, and their effects are to change the orbits and only indirectly to alter the positions occupied by the planets. These secular perturbations are extremely slow in their development, running on, as the name implies, "from age to age."

Constancy of major axes and periods.

A most remarkable fact, first proved by Laplace and Lagrange about a century ago, is that the *major axes* and *periods* are never altered by these "secular perturbations." While subject to slight periodical changes, they remain absolutely constant in the long run, so far as planetary action goes.

Revolution of nodes and apsides.

The *nodes* and *perihelia*, on the other hand, move around continuously; all the nodes regress, and all the perihelia (that of Venus alone excepted) advance.

According to Leverrier, the shortest of these periods of revolution is 37000 years (the line of nodes of Uranus), and the longest is 540000 (that of the perihelion of Neptune). But these numbers must not be accepted too confidently, since the *rate of motion* is not constant, but itself is subject to secular variation.

Oscillation of inclinations and eccentricities.

The *inclinations* of the orbits to the ecliptic are all slightly changed in an irregular *oscillatory* manner, some increasing and some diminishing. As Laplace and Leverrier have proved, all these changes are for the principal planets *confined within narrow limits* of not more than a degree or two.

The *eccentricities* also change in the same irregular way, some increasing, some decreasing, but never changing greatly. These oscillations, both for inclinations and eccentricities, usually occupy from 10000 to 50000 years, but change continually; a long and extensive swing in one direction may or may not be followed by a short one reversing its effects.

The statements made with reference to the unimportant character of the planetary perturbations do not apply in the case of the asteroids, the orbits of which may possibly be subject to very material alterations.

Stability of system not affected by perturbations.

377. Stability of the Planetary System. — It was near the end of the last century that Laplace and Lagrange succeeded in showing that the orbits of the principal planets of the system would never be seriously changed by their mutual attractions. As has been set forth in the preceding section, changes occur, but none of such a character or extent as to be subversive.

Since the major axes and periods remain constant, and the revolution of the nodes and apsides is of little moment in affecting the conditions of the planets, the *eccentricities* and *inclinations* alone come into consideration; and with respect to these Laplace proved that the changes *due to perturbations* must be too small to be of any serious consequence.

It does not follow, however, that because the mutual attractions of the planets cannot seriously derange the system it is, therefore, of necessity securely stable. There are other conceivable actions which might end even in its ultimate destruction; such, for instance, as that of a *resisting medium*, or the intervention of large bodies coming from without.

Other causes might affect it.

378. The "Invariable Plane." — There is no reason why the ecliptic — the plane of the earth's orbit — should be made the fundamental plane of reference for the solar system, except that *we* terrestrials live on the earth. There is in the system, however, an "invariable plane," as discovered by Laplace in 1784, the position of which remains forever unchanged by any mutual action between the bodies of the system, just as does their common center of gravity.

The invariable plane defined.

This plane is defined by the following conditions: if from all the planets perpendiculars are drawn to it (technically, if the planets be "projected" on this plane, which passes through the center of gravity of the system), and if then we multiply the *mass* of each planet by the *area* which its projected radius vector describes upon this plane, in a unit of time, around the center of gravity, the *sum of the products will be a maximum.*

The determination of the exact position of this plane demands, however, an accurate knowledge of the masses and motions of all the planets, discovered and undiscovered, belonging to the system, and the data now in our possession hardly warrant a final assignment of its location. According to the computation of Stockwell, it is inclined to the present ecliptic at an angle of about 1° 55′, its *ascending node* on the ecliptic being in longitude 106° 56′. As might be expected, it lies between the planes of the orbits of Jupiter and Saturn, and very near to that of Jupiter.

Position of the invariable plane.

THE PLANETS THEMSELVES

In discussing the "personal peculiarities" of the planets we have to consider a variety of different data, mostly obtained by telescopic study and micrometric measurements, — such, for instance, as their *diameters;* their *masses* and *densities;* their *axial rotation;* their *surface markings;* their *atmospheric phenomena*, if any; their *albedo*, or light-reflecting power; and, finally, their *satellite systems.*

Determination of a planet's diameter by micrometric observations.

379. Determination of "Size," — Diameter, Surface, and Volume. — The size of a planet is found by measuring its *apparent* diameter in seconds of arc with some form of "micrometer" (Sec. 71) attached to a powerful telescope. Since from the elements of the orbit of a planet and of the earth we can find the distance of the planet from the earth at any time in astronomical units, we can at once deduce the real linear diameter from the apparent diameter D$''$ by an equation slightly modified from that given in Sec. 10, viz.,

$$\text{linear diameter} = \Delta \sin D'', \text{ or } \frac{\Delta \times D''}{206265},$$

Δ being the distance of the planet from the earth. This will give the linear diameter as a fraction of the astronomical unit and can be converted into miles by simply multiplying it by 93 000000, the number of miles in the unit.

The planet's relative radius, ρ.

For many purposes it is convenient to express the planet's radius in terms of the earth's radius by dividing half the diameter in *miles* by 3959 (the number of miles in the mean radius of the earth), designating this *relative radius* by ρ.

Surface area equals ρ^2, volume equals ρ^3.

The *surface area* of the planet in terms of the earth's surface is then ρ^2, and the *volume* or *bulk* of the planet is ρ^3 in terms of the earth's volume; *i.e.*, if, as is nearly true in the case of Jupiter, $\rho = 11$, then the surface of the planet is 121 times that of the earth, and its bulk 1331 times that of the earth.

Greater effect of observational error in case of remote planets.

The nearer the planet, other things being equal, the more accurately ρ and the quantities derived from it can be determined. An error of 0$''$.1 in measuring the apparent diameter of Venus when nearest counts for less than thirteen miles, but in the case of Neptune it would correspond to more than 1300.

Planet's mass determined by means of satellite.

380. Mass, Density, and Surface Gravity. — If the planet has a satellite, its *mass* compared with the sun is very easily and accurately found from the proportion

$$S + p : p + s : : \frac{A^3}{T^2} : \frac{a^3}{t^2},$$

in which S is the mass of the sun, p that of the planet, A the mean distance of the planet from the sun, and T the planet's sidereal period; while s is the mass of the satellite, a its mean distance from the planet, and t its sidereal period.

In almost all cases p in the first term may be neglected as compared with S, and in the second term s as compared with p, which makes the proportion read

$$S : p :: \frac{A^3}{T^2} : \frac{a^3}{t^2}; \text{ whence, } p = S \times \frac{a^3}{t^2} \times \frac{T^2}{A^3}.$$

Mass of planet compared with the earth.

If we want the *mass as compared with the earth*, the first proportion becomes

$$\begin{aligned}&(\text{earth} + \text{moon}) : (\text{planet} + \text{satellite})\\ &:: \left(\frac{\text{cube of moon's distance}}{\text{square of moon's sidereal period}}\right)\\ &: \left(\frac{\text{cube of satellite's distance}}{\text{square of satellite's period}}\right).\end{aligned}$$

The mass of the moon being $\frac{1}{81}$ of that of the earth, it cannot be neglected in comparison with the earth's mass. (No other satellite has a mass more than $\frac{1}{3000}$ of its planet.)

It is to be noted also that instead of the *actual* sidereal period of the moon we must use a period about an hour shorter, in order to allow for the action of the sun (Sec. 327, (1)).

Data for mass: the satellite's distance and period.

The observations upon which this method of determining a planet's mass depend are those of the satellite's *greatest elongation*, the measures of *distance* being especially important, since the distance enters into the formula by its cube.

Mass of planet determined by the perturbations it causes.

When a planet has no satellite, as is the case with Mercury and Venus, its mass can be determined only by means of the *perturbations it produces* in the motions of other planets, or of comets that happen to come near it.

In the case of Mercury the mass is still very uncertain. Venus, however, disturbs the earth sufficiently to give a very good determination of her mass.

Demonstration of the formula for planet's mass.

331. The proportions given in the preceding section are easily derived for circular orbits from the equation for the general equation of the motion of a small body revolving around a larger, viz.,

$$(M+m) = \frac{4\pi^2}{G} \times \frac{r^3}{t^2}. \qquad (1)$$

This equation is obtained by combining the equation for the gravitational attraction between two spheres *expressed as an acceleration* (Sec. 146), viz.,

$$f = G\frac{M+m}{D^2},$$

with the expression for the central force in circular motion (Sec. 306), viz.,

$$f = \frac{4\,\pi^2 r}{t^2}.$$

Replacing D in the first equation by r, and equating the two values of f, we have

$$G\frac{M+m}{r^2} = 4\pi^2\frac{r}{t^2},$$

from which equation (1) follows at once.

In forming the proportion the constant factor drops out, and we have

$$M_1 + m_1 : M_2 + m_2 : : \frac{r_1{}^3}{t_1{}^2} : \frac{r_2{}^3}{t_2{}^2}.$$

As Newton proved, this is accurately true for elliptical orbits also if for r we put a, the semi-major axis of the orbit; but the demonstration lies beyond our scope.

Surface gravity equals mass ÷ ρ^2.

382. Surface Gravity and Density. — When the mass has been determined the surface gravity and density follow at once. Putting γ for surface gravity as compared with the earth, we have

$$\gamma = \frac{m}{\rho^2},$$

Density equals mass ÷ ρ^3.

m being the planet's mass, and ρ its radius as compared with the earth's. The *density*, compared with the earth, is simply $\frac{m}{\rho^3}$; if we want the *specific gravity*, *i.e.*, density as compared with water, we must multiply the result by 5.53, the density of the earth. Any error in the measured diameter of course affects very seriously the computed density and gravity.

Rotation period and position of planet's equator determined by observation of spots upon its surface.

383. Rotation Period and Data connected with it. — The length of the planet's "day," when it can be determined at all, is usually ascertained by observing some well-marked spot on its disk and noting the times of its successive returns. An approximate value of the rotation period is obtained from the observation of such returns during a few days or weeks, and this is afterwards corrected by data furnished from observations separated by the longest interval obtainable,—a century or more if possible.

Mars, however, is the only planet of which the rotation period is known with great accuracy; the others either show no well-defined markings, or only such markings as seem to be more or less movable on the planet's surface, like spots on the sun.

In reducing the observations account has to be taken of the continual change in the direction of the planet from the earth and also of the variations of its distance, which alter the time taken by light to reach us.

Rotation period of Eros determined from variations of its brightness.

In the case of the little planet Eros, a large and regular variation in its brightness, observed for some months early in 1901 in a certain portion of its orbit, was probably due to its axial rotation; if so, the photometrically measured period of variation of brightness gives a determination of the length of its day. (See Sec. 428.) The planet is far too small to show a disk in the telescope, and of course no observations of spots are possible.

The *inclination* of the planet's equator to the plane of its orbit and the positions of its poles and equinoxes are deduced from the observations of the *paths* of the spots as they cross the disk. Such data, however, are available only in the cases of Mars, Jupiter, and Saturn.

It may be added that the disappearance of the variations of the brightness of Eros in May, 1901, after persisting over two months, is naturally explained by Professor Pickering as due to the fact that in May its *pole was turned towards us;* and, if so, this gives us the position of the planet's axis and equator.

Oblateness determined from observations of planet's diameters; also from perturbations of its satellites.

The *oblateness*, or polar compression, of the planet, due to its rotation, is found simply by measuring the difference between the polar and equatorial diameters; but the difference is always very small, so that the percentage of its probable error is rather large.

In some cases also the oblateness can be determined from observation of the motion of the nodes of the planet's satellites.

384. Data relating to the Light of a Planet. — The *brightness* of the planet and the reflecting power of its surface, or *albedo*, are determined by observations with the photometer, which is sometimes used direct, and sometimes attached to a telescope; we have just pointed out how, in one case at least, such observations may also be available for determining the rotation of a planet.

Planet's albedo determined by photometric observations.

Spectroscopic peculiarities.

The *spectroscopic peculiarities* of the planet's light are of course studied with a spectroscope, and usually by spectroscopic photography. A planet always shows, so far as its brightness permits, the lines of the solar spectrum and, in some cases, additional lines or bands of its own, which give information as to the constitution of its atmosphere.

Surface markings and topography.

385. The Planet's Surface Markings and Topography. — These are studied with the telescope by making careful notes and drawings of the appearances and markings seen at different times. If the planet has any well-defined and characteristic features by which its rotation can be determined, it is soon possible to identify such as are permanent and to chart them more or less perfectly.

At present, however, Mars is the only planet of which we have been able to obtain what may be called a real map, though some preliminary chartings have been attempted for Venus and Mercury. The surface markings, which are often very distinct and beautiful upon Jupiter, are all of a more or less transient character.

Thus far *photography* has given but little help in the study of planetary surfaces. The images formed even by the largest telescope are too small compared with the "grain" of the sensitive film; and the light of the planet is so feeble that long exposure is required, during which the atmospheric disturbances usually confuse the image. It is perhaps not impossible that in the future these difficulties may be overcome by the invention of still more sensitive plates with a finer grain and by the use of telescopes of extremely long focal length placed in a fixed position and "fed" by a siderostat, — a method suggested, and now being tried, by Professor Pickering of Harvard.

Satellite systems: the elements determined by micrometric observations.

386. The Satellite Systems. — The principal data to be determined in respect to these systems are the distances and periods of the satellites. These are got by *micrometric* measures of the apparent distance and direction of each satellite from the planet; or from other satellites, as is now quite the usual method, since the distance and direction between two satellites (which are mere points of light) can be measured much more precisely than between a satellite and the center of the large disk of a planet. The reduction of the observations in this latter case is, however, very complicated.

Diameters.

Masses.

In a few cases the satellites present *disks* large enough to be measured and show spots upon them, so that questions of their rotation and surface markings admit of discussion. Also, where there are a number of satellites attending a planet, their mutual perturbations furnish a very interesting subject of study and make it possible to determine their *masses* relative to that of the planet.

Near satellites move nearly in plane of planet's equator, and have orbits nearly circular.

With the exception of our moon and Iapĕtus (the outer satellite of Saturn), all the satellites move very nearly in the plane of their planet's equator, — so far at least as known, since the position of the equators of Uranus and Neptune has never yet been ascertained. Moreover, all the satellites except the moon and Hyperion, the seventh satellite of Saturn, move in orbits of very small eccentricity, in fact almost perfect circles. Laplace and Tisserand have shown that the "equatorial

Remote satellites move nearly in plane of planet's orbit.

protuberance" of a planet, due to its axial rotation, compels a near satellite to move nearly in the *equatorial plane*. The more distant satellites, like the moon and Iapetus, on the other hand, move nearly in *the orbital plane* of the planet.

The circularity of the satellite orbits is not yet accounted for.

Humboldt's classification of the planets.

387. Classification of Planets. — Humboldt has classified the planets in two groups, — the "terrestrial planets," as he calls them, and the "major planets." The terrestrial group contains the four planets nearest the sun, — Mercury, Venus, the Earth, and Mars. They are all bodies of similar magnitude, ranging from 3000 to 8000 miles in diameter; not very different in density and probably roughly alike in physical constitution, though probably also differing very much in the extent, density, and character of their atmospheres.

The four major planets — Jupiter, Saturn, Uranus, and Neptune — are much larger bodies, ranging from 32000 to 90000 miles in diameter; are much less dense; and, so far as we can make out, present only cloud-covered surfaces to our inspection. There are strong reasons for supposing that they are at a high temperature, and that Jupiter especially is a sort of "semi-sun"; but this is not certain.

As to the *asteroids*, the probability is that they represent a fifth planet of the terrestrial group, which, as has been already intimated, failed somehow in its evolution, or else has been broken to pieces.

Fig. 133 gives an idea of the relative sizes of the planets. The sun on the scale of the figure would be about a foot in diameter.

Relative accuracy of different planetary data.

388. Tables of Planetary Data. — In the Appendix we present tables of the different numerical data of the solar system, derived from the best authorities and calculated for a solar parallax of 8''.80, the sun's mean distance being therefore taken as 92 897000 miles. These tabulated numbers, however, differ widely in accuracy. The *periods* of the planets and their *distances in astronomical units* are very precisely known; probably the last decimal place in the table may be trusted. Next in certainty come the *masses* of such planets as have satellites, expressed in terms of the *sun's mass.* The masses of Venus and, especially, of Mercury are much more uncertain. The distances of the planets *in miles*, their masses in *terms of the earth's mass*, and their diameters *in miles*, all involve the solar parallax and are affected by the slight uncertainty in its amount. For the remoter planets, moreover, *diameters*, *volumes*, and *densities* are subject to a very considerable percentage of error, as explained above (Sec. 379). The student need not be surprised, therefore, at finding serious discrepancies between the values given in these tables and those given by other authorities, amounting in some cases to ten per cent or twenty per cent, or even more. Such differences merely indicate the actual uncertainties of our knowledge.

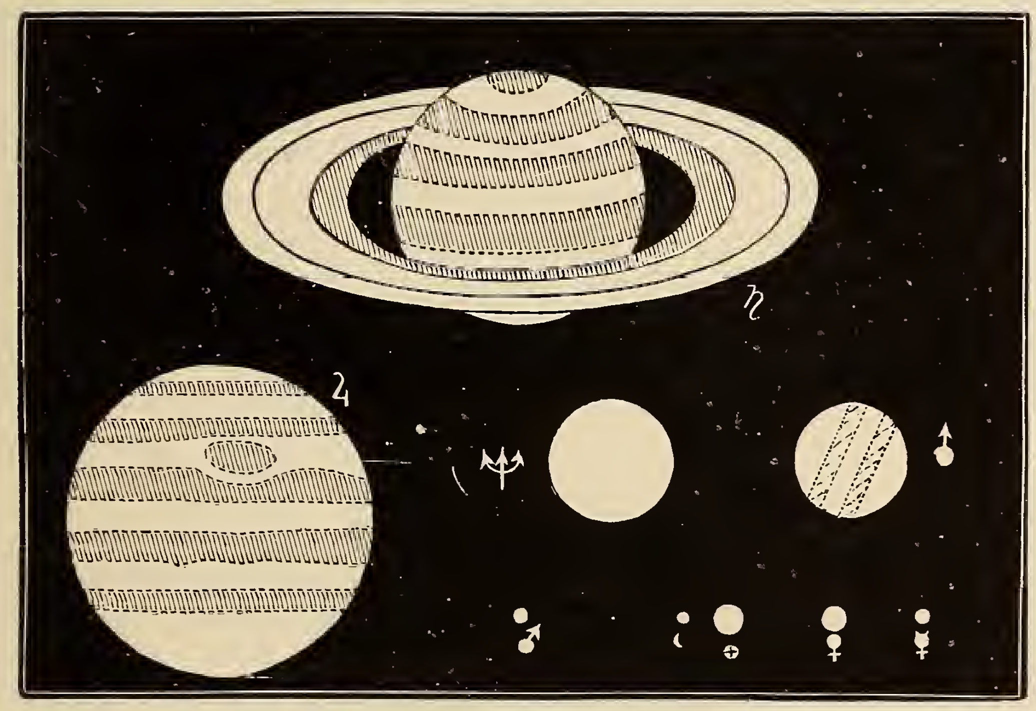

FIG. 133. — Relative Sizes of the Planets

Sir John Herschel's illustration of the scale of the solar system.

389. Sir John Herschel's Illustration of the Dimensions of the Solar System. — In his *Outlines of Astronomy* Herschel gives the following illustration of the relative magnitudes and distances of the members of our system:

> Choose any well-levelled field. On it place a globe two feet in diameter. This will represent the sun. *Mercury* will be represented by a grain of mustard seed on the circumference of a circle 164 feet in diameter for its orbit; *Venus*, a pea, on a circle of 284 feet in diameter; the *Earth*, also a pea, on a circle of 430; *Mars*, a rather large pin's head, on a circle of 654 feet; the *asteroids*, grains of sand, on orbits having a diameter of 1000 to 1200 feet; *Jupiter*, a moderate-sized orange, on a circle nearly half a mile across; *Saturn*, a small orange, on a circle of four-fifths of a mile; *Uranus*, a full-sized cherry or small plum, upon a circumference of a circle more than a mile in diameter; and, finally, *Neptune*, a good-sized plum, on a circle about $2\frac{1}{2}$ miles in diameter.

We may add that on this scale the nearest star would be on the opposite side of the earth, 8000 miles away.

EXERCISES

1. What is the mean daily gain of the earth on Mars as seen from the sun, *i.e.*, the synodic motion of Mars, assuming their sidereal periods as 365.25 days for the earth, and 687 days for Mars?

2. Find the synodic period of Venus, her sidereal period being 225 days.

3. Given the synodic period of a planet as 3 years, what is its sidereal period?

Ans. $\left\{\begin{array}{l}\frac{3}{4}\text{ of a year, or}\\ 1\frac{1}{2}\text{ years.}\end{array}\right.$

4. Given a synodic period of 4 years, find the sidereal period.

5. What would be the sidereal period of a planet which had its synodic period equal to the sidereal? *Ans.* 2 years.

6. Within what limits of distance from the sun must lie all planets having synodic periods longer than 2 years? (Apply Kepler's third law after finding the sidereal periods that would give a synodic period of 2 years.)

Ans. $\left\{\begin{array}{l}0.763\text{ astron. units, or }70\,950000\text{ miles, and}\\ 1.588\text{ astron. units, or }147\,500000\text{ miles.}\end{array}\right.$

7. A brilliant starlike object was seen about 7 P.M. on May 1 exactly at the east point of the horizon. Could it have been one of the planets? If not, why not?

8. Mercury was at inferior conjunction on Feb. 8, 1896, at 1 P.M. On May 6, at 15 minutes after noon (exactly one sidereal period later), its elongation from the sun was observed to be 18° 50′ E. Find the distance of the planet from the sun in astronomical units, the earth's orbit being regarded as circular. (See Sec. 371.)

(The fact that the first observation was made at conjunction greatly simplifies the calculation.)

Ans. { Distance from the sun, 0.335 astron. units. (The planet was near perihelion.)

9. At a time when Jupiter's distance from the earth was 4.6 astronomical units its apparent equatorial diameter was observed to be 43″.3. Find the diameter in miles as determined by this observation.

Ans. 89700 miles.

New Physical Observatory, Greenwich

CHAPTER XIII

THE TERRESTRIAL AND MINOR PLANETS

Mercury, Venus, and Mars — The Asteroids — Intramercurial Planets — Zodiacal Light

MERCURY

390. Mercury has been known from remote antiquity, and we have recorded *observations* running back to 264 B.C. At first astronomers failed to recognize it as the same body on the eastern and western side of the sun, and among the Greeks it had for a time two names, — Apollo when morning star, and Mercury when evening star. It is so near the sun that it is comparatively seldom seen with the naked eye (Copernicus is said never to have seen it), but when near its greatest elongation it is easily enough visible as a brilliant star of the first magnitude, though always low down in the twilight. It is best seen in the evening at such *eastern* elongations as occur in March and April. As a morning star it is best seen at western elongations in September and October.

Two names for Mercury.

Best times for seeing Mercury.

It is an exceptional planet in various ways. It is the *nearest* to the sun, *receives the most light and heat*, is the *swiftest in its movement*, and (excepting some of the asteroids) *has the most eccentric orbit*, with the *greatest inclination to the ecliptic*. It is also the *smallest in diameter* (again excepting the asteroids) and has the *least mass*.

391. Its Orbit. — Its mean distance from the sun is about 36 000000 miles, but the eccentricity of its orbit is so great (0.205) that the sun is 7 500000 miles out of the center, and the distance of the planet from the sun ranges all the way from

Peculiarities of Mercury's orbit. Exceptional in many respects.

28 500000 to 43 500000, while the velocity in its orbit varies from 36 miles a second at perihelion to only 23 at aphelion. Its distance from the earth ranges from about 50 000000 miles at the most favorable inferior conjunction to about 136 000000 at the remotest superior conjunction.

Seasons due to the eccentricity of its orbit.

A given area upon its surface receives on the average nearly seven times as much light and heat as the same area on the earth; and the heat received at perihelion is greater than that at aphelion in the ratio of 9 : 4. For this reason, even if the planet's equator should be found to be parallel to the plane of its orbit, there must be two seasons in each Mercurian year, due to the changing distance; and if the planet's equator is inclined nearly at the same angle as ours, the seasons must be extremely complicated.

The *sidereal* period is 88 days, and the *synodic* period (from conjunction to conjunction) 116 days. The *greatest elongation* ranges from 18° to 28°, on account of the eccentricity of its orbit, and occurs about 22 days before and after inferior conjunction. The *inclination* of the orbit to the ecliptic is about 7°.

Diameter, mass, etc., of Mercury.

392. The Planet's Magnitude, Mass, etc. — The apparent diameter of Mercury ranges from $5''$ to about $13''$, according to its distance from us, and the real *diameter* is very nearly 3000 miles. Its *surface* is about *one seventh* that of the earth, and its *volume*, or *bulk*, *one eighteenth.*

The planet's *mass* is not accurately known; it is very difficult to determine, since it has no satellite, and it is so near the sun that its disturbing effect upon the other planets is extremely small, so that the values calculated from perturbations produced by it are very discordant. Different computers give results ranging all the way from $\frac{1}{9}$ of the earth's mass to $\frac{1}{30}$. It probably lies somewhere between $\frac{1}{20}$ and $\frac{1}{52}$. Its mass is, however, unquestionably smaller than that of any other planet, asteroids excepted.

Small surface gravity.

Our uncertainty as to its mass prevents us from assigning any certain values to its *density* or *surface gravity;* probably it is not quite so dense as the earth. Assuming Newcomb's mass of $\frac{1}{21}$ that of the earth, the density comes out about 0.85, and its surface gravity a little less than $\frac{1}{3}$.

Telescopic appearance and phases.

393. Telescopic Appearances, Phases, etc. — In the telescope the planet looks like a little moon, showing phases precisely similar to those of the moon. At inferior conjunction the dark side is towards us, at superior conjunction the illuminated surface. At greatest elongation the planet appears as a half-moon.

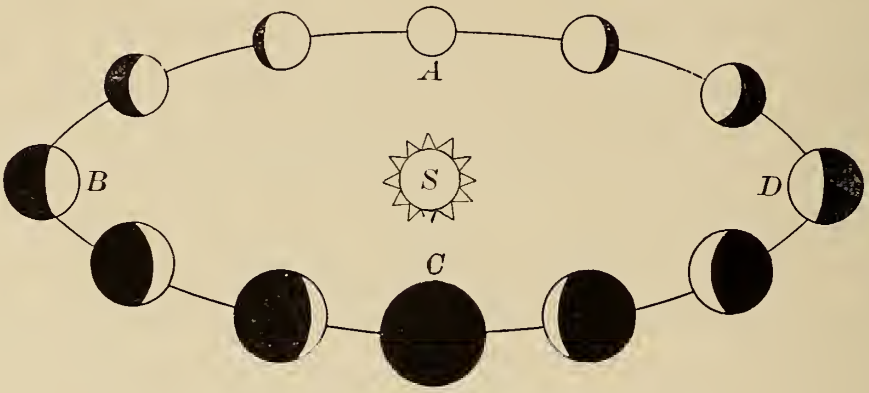

Fig. 134. — Phases of Mercury and Venus

It is gibbous between superior conjunction and greatest elongation, while between inferior conjunction and elongation it shows the crescent phase.

Fig. 134 illustrates the phases of Mercury (and of Venus also).

Atmosphere of Mercury.

The *atmosphere* of the planet cannot be as dense as that of Venus, because at a transit across the sun it shows no encircling ring of light, as Venus does (Sec. 401). Both Huggins and Vogel, however, report spectroscopic observations which imply the presence of water vapor; *i.e.*, the planet's spectrum, in addition to the ordinary dark lines belonging to the spectrum of reflected sunlight, shows other bands known to be due to water vapor, but it is not yet quite certain whether the vapor is in the planet's atmosphere or in our own. On the whole, it is probable that the atmospheric conditions are much like those upon the moon, since under the powerful action of the solar heat a planet of so small a mass would probably lose most of its atmosphere, if it ever possessed any.

Planet best observed with the telescope in daytime.

Generally the planet is so near the sun that it can be observed only by day, but when proper precautions are taken to screen the object-glass from direct sunlight, its observation is not specially difficult. The surface presents very little of interest to an ordinary telescope. Like the moon, it is brighter at the edge than at the center, but until recently no markings have been observed upon its disk well enough defined to give us any trustworthy information as to its geography or even its rotation.

No satisfactory map.

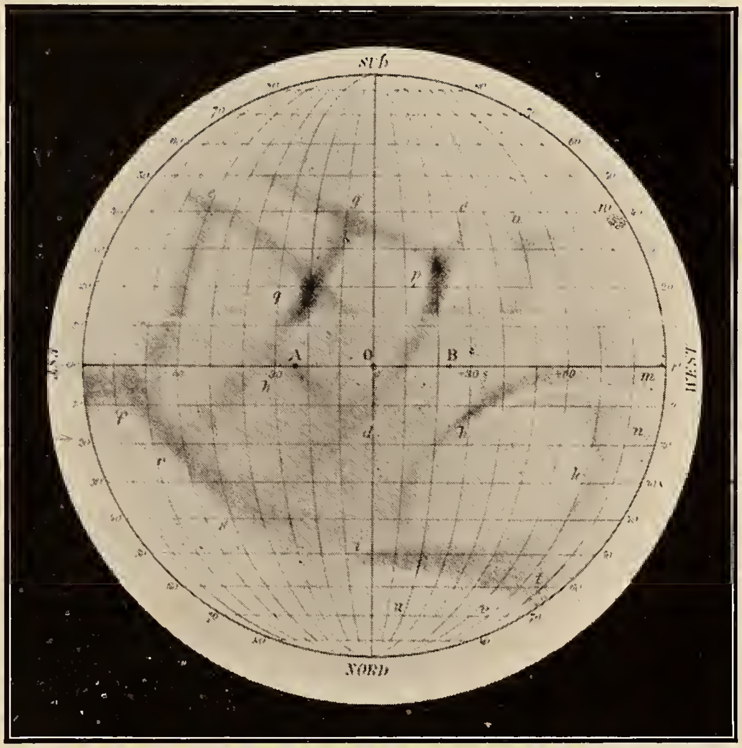

FIG. 135. — Mercury
After Schiaparelli

The planet in its rotation keeps the same face always turned towards the sun.

394. The Planet's Rotation.—Schröter, a German astronomer and a contemporary of Sir William Herschel, and, to speak mildly, an imaginative man, early in the last century reported certain observations which he considered to indicate high mountains on the planet and deduced a rotation period of $24^h 5^m$, — a result that stood uncontradicted until about 1890 and still appears in many text-books, though unconfirmed by other observers with instruments certainly much better than his.

In 1889 the Italian astronomer, Schiaparelli, announced the discovery upon the planet of certain dark permanent markings, of which he presented a map (Fig. 135). He found also that these markings did not change their positions upon the planet's disk *even in the course of several hours* (a fact obviously inconsistent with rotation in twenty-four hours), but remained always nearly fixed in their position with respect to the "terminator," — the boundary between the illuminated and unilluminated hemispheres of the planets. Granting this permanency, it follows that *the planet rotates on its axis only once during its orbital period of eighty-eight days;* i.e., *it keeps the same face always towards the sun as the moon does towards the earth.* Slight changes in the positions of the spots show, however, a comparatively large *libration in longitude* (Sec. 203, (2)), as there ought to be, considering the great eccentricity of the planet's orbit. This libration amounts to about 23½°; *i.e.*, the sun, seen from a favorable position on the planet, instead of rising and setting as with us, must seem to oscillate east and west in the sky to the extent of 47° in a period of eighty-eight days.

Large libration in longitude.

Schiaparelli's reported discovery excited great interest, but the observations are extremely difficult even under the Italian atmosphere, and confirmation was tardy. In 1896, however, Mr. Lowell reported its complete corroboration as the result of observations at his Flagstaff Observatory, though it is rather difficult to reconcile his drawings of the surface markings with those of Schiaparelli. Partial confirmations have also been received from other quarters.

If this rotation period is correct, as it probably is, one face of the planet is always sunless and probably intensely cold, while the opposite is always exposed to a sevenfold African blaze of sunbeams. Between these regions is a space in which, as a consequence of librations, the sun alternately rises above the horizon and drops back again.

Albedo extremely low.

395. Albedo. — The reflecting power of the planet's surface is very low, — according to Zöllner, 0.13, a little less than that of the moon and much below that of any other planet, hardly higher than that of a darkish granite.

In the proportion of light given out at its different phases it behaves like the moon, flashing out strongly near the "full," *i.e.*, near superior conjunction, — a fact which probably indicates a rough surface with very little atmospheric absorption of light.

Transits of Mercury in May and November. May transits rare.

396. Transits of Mercury. — At the time of inferior conjunction the planet usually passes north or south of the sun, the inclination of its orbit being 7°; but if the conjunction occurs when the planet is very near its *node*, it crosses the disk of the sun as a small black spot, — not, however, large enough to be seen without a telescope. Since the earth passes the planet's node on May 7 and November 9, transits can occur only near those dates.

If the planet's orbit were truly circular, the transit limit (corresponding to the ecliptic limit, Secs. 286 and 293) would be 2° 10′, and the conditions of transit would be the same at each node; but at the May transits the planet is near its aphelion and exceptionally near the earth, so that the May transits are only about half as numerous as the other.

Intervals between transits.

For the November transits the interval is sometimes only 7 years, but is usually 13 or 46 years. For the May transits the 7-year interval is impossible. Twenty-two synodic periods of Mercury are pretty nearly equal to 7 years; 41 much more nearly equal to 13 years, and 145 are almost exactly equal to 46 years. Hence, 46 years after a given transit another one at the same node is almost certain.

The last transit was in November, 1894, and was completely visible in the United States. During the first half of the present century transits will occur as follows:

Nov. 12, 1907,	May 7, 1924,	May 10, 1937,
Nov. 6, 1914,	Nov. 8, 1927,	Nov. 12, 1940.

Only the two first of these will be visible in the United States, and not the entire transit in either case. The first transits of which the whole will be visible here occur on Nov. 13, 1953, and Nov. 6, 1960.

Transits of Mercury a test of uniformity of earth's rotation.

Transits of Mercury are of no special astronomical importance, except as furnishing accurate determinations of the planet's place.

Newcomb has made a thorough examination of all the recorded transits in order to test the uniformity of the earth's rotation. They appear to indicate certain small irregularities in it, but hardly establish the fact as absolutely certain.

VENUS

The next planet in order from the sun is Venus, by far the brightest and most conspicuous of all, — the earth's twin sister in magnitude, density, and general constitution, if not in other physical conditions. Like Mercury, it had two names among the Greeks, — Phosphorus as morning star, and Hesperus as evening star.

Brilliance of Venus.

It is so brilliant that it is easily seen by the naked eye in the daytime for several weeks when near its greatest elongation; occasionally it is bright enough to catch the eye at once, but usually is seen by daylight only when one knows precisely where to look for it.

397. Distance, Period, and Inclination of Orbit. — Its *mean distance* from the sun is 67 200000 miles.

Peculiarities of orbit of Venus. Its eccentricity smaller than that of any other planet.

The *eccentricity* of the orbit is the smallest in the planetary system (only 0.007), so that the whole variation of its distance from the sun is less than a million miles.

Its *orbital velocity* is 22 miles per second.

The *heat and light* received from the sun are most exactly double the amount received by the earth.

Its *sidereal period* is 225 days, or nearly seven and one-half months, and its synodic period 584 days, — a year and seven months. From superior conjunction to elongation on either side is 220 days, while from inferior to elongation it is only 72 days, — less than one third as long.

The *greatest elongation* is 47° or 48°.

The *inclination of its orbit* is about 3½°.

398. Magnitude, Mass, Density, etc. — The apparent diameter of the planet ranges from 67″ at the time of inferior conjunction to only 11″ at superior conjunction, the great difference depending upon the enormous variation in the distance of the planet from the earth, which is only 26 000000 miles at inferior conjunction and 160 000000 at superior. The *real diameter of the planet* is about 7600 miles, according to the recent measures of See at Washington.

Diameter, mass, density, surface gravity, etc.

According to this, its *surface*, compared with that of the earth, is 0.91; its volume, 0.87. (These numbers differ somewhat from those given in the tables in the Appendix, which are allowed to stand unchanged, as illustrating the discrepancies between good authorities in such cases.)

By means of the perturbations she produces upon the earth, the *mass* of Venus is found to be a little more than four fifths (0.82) of the earth's; hence, her *density* is about ninety-four per cent and her superficial gravity ninety per cent of the earth's. A man who weighs 160 pounds here would weigh about 140 pounds on Venus.

399. Phases. — The telescopic appearance of the planet is striking on account of her great brilliance. When midway between greatest elongation and inferior conjunction she has an apparent diameter of 40″, so that, with a magnifying power of only 45, she looks exactly like the moon four days old, and of precisely the same apparent size, though very few persons would think so on first viewing the planet through a telescope. The novice always underrates the apparent size of a telescopic object, because he instinctively adjusts his focus as if looking at a picture or a page only a few inches away, instead of projecting the object visually into the sky.

Phases of Venus.

According to the theory of Ptolemy, Venus could never show us more than half her illuminated surface, since, according

Phases irreconcilable with Ptolemaic system.

to his hypothesis, she was *always between us and the supposed orbit of the sun.* Accordingly, when in 1610 Galileo discovered with his newly invented telescope that she exhibited the *gibbous* phase as well as the crescent, it was a strong argument for the Copernican theory.

Galileo's discovery of the phases.

Galileo announced his discovery in a curious way, by publishing the anagram, —

Haec immatura a me iam frustra leguntur; o. y.

Some months later he furnished a translation, which is found by merely transposing the letters of the anagram and reads, "Cynthiæ figuras æmulatur Mater Amorum," meaning "The Mother of the Loves (Venus) imitates the phases of Cynthia," *i.e.*, of the moon.

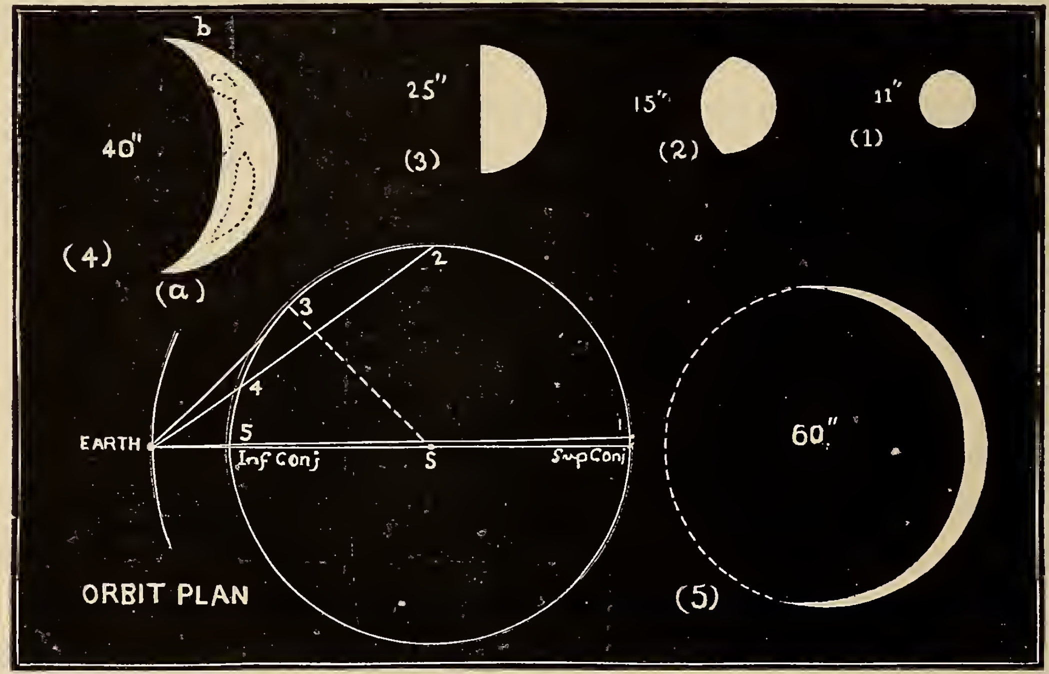

Fig. 136. — Telescopic Appearances of Venus

Fig. 136 represents the disk of the planet as seen at five points in its orbit. 1, 3, and 5 are taken respectively at superior conjunction, greatest elongation, and near inferior conjunction, while 2 and 4 are at intermediate points. Number 2 is badly engraved, however; the sharp corners are impossible since the terminator is always a semi-ellipse (Sec. 205).

The planet attains its *maximum brightness* thirty-six days before and after inferior conjunction, at a distance of about 38° or 39° from the sun, when its phase is like that of the moon about five days old. It then casts a strong shadow and, as already said, is easily visible by day with the naked eye.

High albedo of Venus.

400. Albedo. — According to Zöllner, the albedo of the planet is 0.50, which is about three times that of the moon and almost four times that of Mercury. It is, however, exceeded by the reflecting power of the surface of Jupiter and Uranus, while that of Saturn appears to be about the same.

This high reflecting power probably indicates that the surface is mostly covered with cloud, as few rocks or soils could match it in brightness.

Lowell, however, denies the existence of anything like a continuous cloud veil such as has been generally supposed.

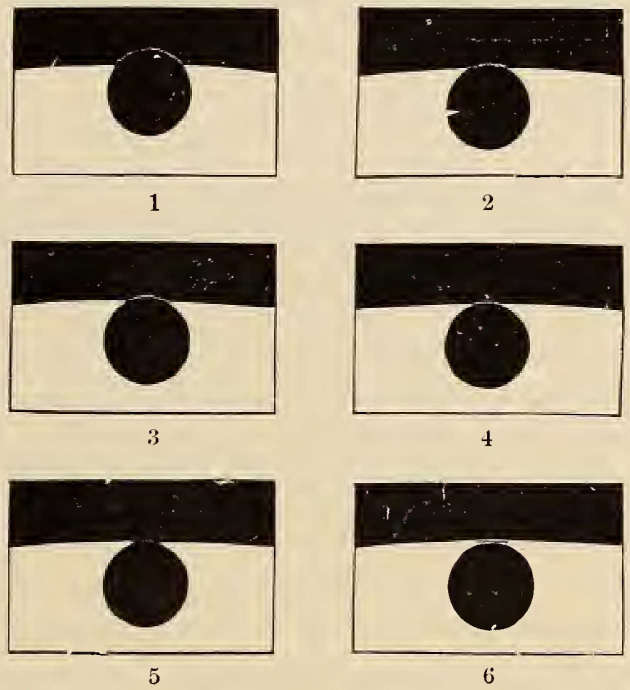

Fig. 137. — Atmosphere of Venus as seen during the Transit of 1882
Vogel

Atmosphere of Venus.

401. Atmosphere of the Planet. — There is no question that this planet has an atmosphere of considerable density.

When the planet is entering upon the sun's disk, or leaves it at a "transit," the portion of the disk outside the sun is encircled by a beautiful ring of light, due to the refraction, reflection, and dispersion of light by the planet's atmosphere (Fig. 137). If it were due *solely* to refraction, it would indicate that this atmosphere, according to the computations of Watson and others, must have an elevation of some 55 miles and be considerably denser than our own; but this conclusion is very doubtful.

Bright ring surrounding planet's disk due to reflection and diffusion of light rather than to refraction.

When the planet is near the sun, about the time of inferior conjunction, the horns of its crescent extend notably beyond the diameter, and when *very* near the sun they can be seen, by carefully screening the object-glass of the telescope from the sunlight, to form a complete ring around the disk, as observed by Professor Lyman of New Haven and others in 1860 and 1874. This phenomenon, which is unquestionable, has usually been ascribed to *refraction;* but the observations of Russell at Princeton in 1898 showed that it must be due *mainly* to diffuse *reflection* of light by the planet's atmosphere, like that which causes our twilight, and that refraction proper plays only a very secondary part.

If the ring were due to refraction, as by a lens, the widest and brightest part of it should be on the side of the planet *most distant* from the sun, where the rays would be bent *towards* the observer, and not on the side next the sun, as is actually and conspicuously the case. On this side refracted rays are bent away from the observer and would not reach his eye, while *reflected* rays are thrown towards it.

Density of the atmosphere of Venus.

The same observations also cast doubt on the hitherto accepted conclusion as to the great density of the atmosphere, making it probable that it is somewhat rarer than our own, rather than much denser; and this might be expected, considering the planet's smaller mass and presumably higher temperature.

Question as to presence of water vapor.

The presence of water vapor in the planet's atmosphere has been announced by several of the earlier spectroscopic observers. The evidence, however, is hardly conclusive.

Unexplained light on planet's surface.

Another curious phenomenon, not very satisfactorily explained as yet, is the occasional appearance of light on the unilluminated part of the planet's surface, making the whole disk visible, like the new moon in the old moon's arms. This light cannot be accounted for by any effect of *sunlight*, but must originate on the planet's surface or in her atmosphere. It recalls the aurora borealis of the earth and other electrical manifestations.

402. Surface Markings. — The surface of the planet is so brilliant as seen in the telescope that it is very difficult to make out any markings upon it; indeed, it is generally best in studying the surface to use a light shade glass. The disk is brightest at the limb, but the light fades off rapidly at the terminator, and over the surface there have been made out indistinct patches of less or greater brightness, as shown in Fig. 138, from drawings by Mascari made at the observatory on Mt. Etna in 1895, — an excellent representation of the planet's usual appearance. The darkest shadings may possibly be continents and oceans, dimly visible, though their comparative permanence with respect to the terminator makes this questionable; more probably they are purely atmospheric effects. But observations are as yet hardly decisive.

Surface markings.

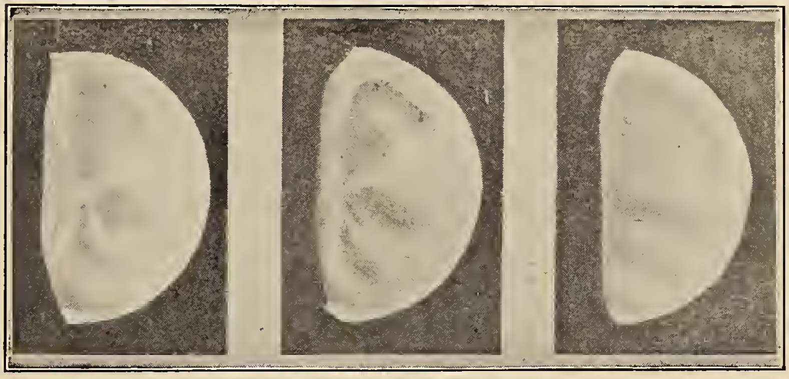

Fig. 138. — Venus
After Mascari

Occasionally very bright spots appear at the ends of the terminator, which may possibly be *polar caps* like those of Mars, and, if so, show that the planet's axis must be nearly perpendicular to its orbit. On the terminator roughnesses and irregularities are sometimes seen which may perhaps be due to mountains, to some of which Schröter assigned extravagant elevations exceeding 20 miles.

Polar caps.

Lowell, in 1898, in opposition to all previous observers, reports the discovery of permanent markings consisting of rather narrow, nearly straight, dark streaks, radiating like spokes from a sort of central hub. He describes them as fairly definite in outline, but *dim*, as if seen through a luminous but unclouded atmosphere of considerable depth; and he goes so far as to give a map of the planet, with names attached to some of the leading

Surface markings according to Lowell.

features. As yet, however, his observations want confirmation. Fig. 139 is from one of his drawings.

Rotation: probably planet keeps same face towards sun, like Mercury.

403. Rotation, Position of Axis, etc. — The earlier observers, from the first Cassini in 1666 down to De Vico in 1841, assigned to the planet a rotation period of about 23^h21^m, — uncontradicted, it is true, but regarded with a good deal of distrust, because the observations were of spots extremely vague and indistinct and were not very accordant.

Some highly respected authorities still accept this period, but the general opinion of astronomers now concurs with the conclusion of Schiaparelli, who considers that his observations make it certain that the rotation must be very *slow*, and render it highly probable that Venus follows the example of Mercury in keeping the same face always towards the sun, having, therefore, a diurnal period of 225 days. This is confirmed by Perrotin at Nice, and by Lowell at the Flagstaff Observatory, and by several other observers.

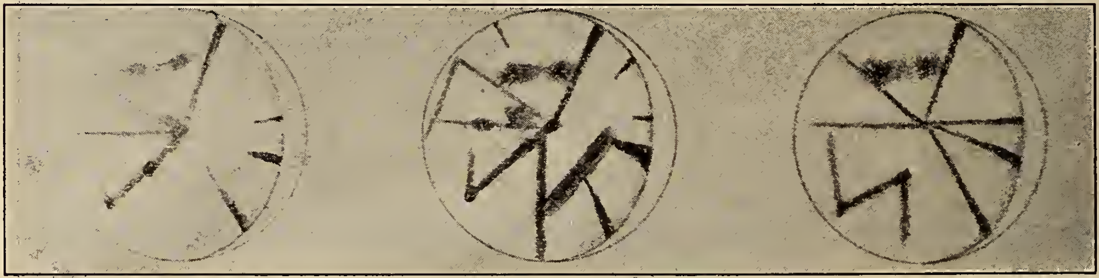

FIG. 139. — Venus
From drawings of P. Lowell

Possible decision by the spectroscope.

It is probable that the spectroscope will ultimately settle the question (though the observation will be very difficult) by showing, according to the Doppler principle (Sec. 254), how fast the eastern and western edges of the planet's disk respectively advance and recede; the observation has already been attempted by Belopolsky at Pulkowa, and it is a little disconcerting that the result, though by no means decisive, rather favors the twenty-three-hour period.

On the other hand, the planet shows no sensible oblateness, as it should if it had a day of the same length as the earth's; if that were the case, there should be a difference of nearly $\frac{1}{4}''$ between the equatorial and polar diameters, which has never been observed.

No measurable oblateness.

The *inclination* of the planet's equator cannot be exactly determined, but it is almost certain that it must nearly coincide with the plane of its orbit. The old determination of De Vico, still found in many text-books, making the inclination 37°, is certainly erroneous.

Inclination of equator to orbit probably small.

404. Question of Satellite. — No satellite has yet been discovered, and it is certain that the planet has none of any considerable size. It is not impossible, however, that it may have some pygmy attendant, like those of Mars, since the great brilliance of the planet and its nearness to the sun would make the discovery of such a body extremely difficult. There have been in the past several announcements of a satellite; but not one has been verified, and most of them were mistakes, since explained, either as observations of stars, or by reflections in the eyepiece of the observer's telescope.

No satellite.

405. Transits. — Occasionally Venus passes between the earth and the sun at inferior conjunction and "transits," or crosses, the disk of the sun from east to west as a round black spot, easily seen by the naked eye through a suitable shade glass. When the transit is central it occupies about eight hours, but when the track is near the edge of the disk it is correspondingly shortened. Since the transit can occur only when the sun is within about 4° of the node, the phenomenon is rare and can happen only within a day or two of the dates when the earth passes the nodes, viz., June 5 and December 7.

Transits.

Transit months June and December.

The special interest of the transits lies in their availablity for the purpose of finding the parallax and distance of the sun, as first pointed out by Halley in 1679.

Importance of determining the solar parallax.

The earliest observed transit in 1639 was seen by two persons only (Horrocks and Crabtree, in England), but the four which have since occurred, in June, 1761 and 1769, and in December, 1874 and 1882, were extensively observed by scientific expeditions sent out by the different governments to all parts of the world where they were visible. The transits of 1769 and 1882 were visible in this country.

It is, however, hardly likely that so much trouble and expense will be hereafter expended upon observations of transits. Other methods of determining the solar parallax have been found to be more trustworthy.

Recurrence and dates of transits.

406. Recurrence and Dates of Transits. — Five synodic revolutions of Venus are very nearly equal to eight years, the difference being little more than one day; and still more nearly, — in fact, almost exactly, — 243 years are equal to 152 synodic revolutions. If, then, we have a transit at any time, another *may* occur at the same node eight years earlier or later. Sixteen years before or after it will be impossible, and no other transit can then occur *at the same node* until after the lapse of 235 or 243 years, though a transit or pair of transits may, and usually will, occur *at the other node* in about half that time: thus, the next pair of transits of Venus will occur on June 8, 2004, and June 6, 2012.

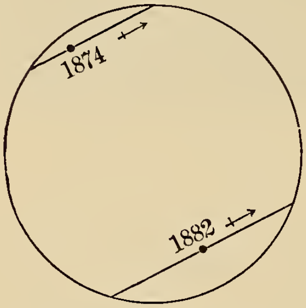

Fig. 140. — Transit of Venus Tracks

Transits at present come in pairs. Solitary transits possible.

If the planet crosses the sun nearly centrally, the transit will be "solitary," *i.e.*, not accompanied by another eight years before or after. If, however, the track is more than 12′ from the sun's center, it will be accompanied by another at eight years interval. At present transits come thus in pairs and have been doing so for several centuries; after a time this will cease to be the case, and they will become solitary for another long period.

Fig. 140 shows the tracks of Venus across the sun's disk in 1874 and 1882.

MARS

Mars: data relating to its orbit.

407. This planet, like Mercury and Venus, is prehistoric as to its discovery. It is so conspicuous in color and brightness and in the extent and apparent capriciousness of its movement among the stars, that it could not have escaped the notice of the very earliest observers.

Its *mean distance* from the sun is a little more than one and a half times that of the earth (141 500000 miles), and the *eccentricity* of its orbit is so considerable (0.093) that its radius vector varies more than 26 000000 miles.

At opposition the planet's *average* distance from the earth is 48 600000 miles. When opposition occurs near the planet's perihelion this distance is reduced to 35 500000 miles, while near aphelion it is over 61 000000. At superior conjunction the average distance from the earth is 234 000000.

Enormous variation of distance and brightness.

The apparent diameter and brilliancy of the planet vary enormously with those great changes of distance. At a *favorable* opposition (when the distance is at its minimum) the planet is more than fifty times as bright as at superior conjunction and fairly rivals Jupiter; when most remote it is hardly as bright as the pole-star.

Favorable oppositions.

The favorable oppositions occur always in the latter part of August (at which time the earth as seen from the sun passes the perihelion of the planet) and at intervals of fifteen or seventeen years. The last such opposition was in 1892; the next will be in 1907.

The *inclination* of the orbit is small,—1° 51′.

The planet's *sidereal period* is 687 days, or one year and ten and one-half months; its *synodic period* is much the longest in the planetary system, being 780 days, or nearly two years and two months. During 710 of the 780 days it moves eastward, and during 70 retrogrades through an arc of 18°.

408. Magnitude, Mass, etc.—The apparent diameter of the planet ranges from 3″.6 at conjunction to 24″.5 at a favorable

Diameter, etc.

opposition. Its *real* diameter is very near 4200 miles. This makes its *surface* about two sevenths, and its *volume* one seventh, of the earth's.

Mass, density, and gravity.

Its *mass* is a little less than one ninth of the earth's mass and is accurately determined by means of its satellites. Its *density* is 0.73, as compared with the earth's 0.73 and *superficial gravity* 0.38; a body which here weighs 100 pounds would have a weight of only 38 pounds on the surface of Mars.

General telescopic aspect.

409. General Telescopic Aspect, Phases, Albedo, Atmosphere, etc. — When the planet is nearest the earth it is more favorably situated[1] for telescopic observation than any other heavenly body, the moon alone excepted. It then shows a ruddy disk which, with a power of 75, is as large as the moon. Since its orbit is outside the earth's, it never exhibits the *crescent* phases like Mercury and Venus; but at quadrature it appears distinctly *gibbous*, about like the moon three days from the full.

Like Mercury, Venus, and the moon, its disk is brighter at the *limb* (*i.e.*, at the circular edge) than at the center; but at the *terminator*, or boundary between day and night on the planet's surface, there is a shading which, taken in connection with certain other phenomena, indicates the presence of an *atmosphere*.

Atmosphere probably not dense.

This atmosphere, however, contrary to opinions formerly held, is probably much less dense than that of the earth, the low density being indicated by the infrequency of clouds and of other atmospheric phenomena familiar to us upon the earth, to say nothing of the fact that, since the planet's superficial gravity is less than two fifths of the force of gravity on the earth, a dense atmosphere would be impossible.

Question as to presence of water vapor.

More than twenty years ago Huggins, Janssen, and Vogel all reported the lines of water vapor in the spectrum of the planet's atmosphere; but the observations of Campbell, at the

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[1] Venus at times comes nearer, but when nearest she is visible only by daylight, and shows only a very thin crescent.

Lick Observatory in 1894, throw great doubt on their result and show that the water vapor, if present at all, is too small in amount to give decisive evidence of its presence.

Albedo low.

Zöllner gives the *albedo* of Mars as 0.26, — just double that of Mercury, and much higher than that of the moon, but only about half that of Venus and the major planets. Near opposition the brightness of the planet suddenly increases in the same way as that of the moon near the full (Sec. 210).

Rotation period very accurately known.

410. Rotation, etc. — The spots upon the planet's disk enable us to determine its period of rotation with great precision. Its *sidereal day* is found to be $24^h 37^m 22^s.67$, with a probable error not to exceed one fiftieth of a second. This very exact determination is effected by comparing drawings of the planet made by Huyghens and Hooke more than two hundred years ago with others made recently.

Position of axis.

The *inclination* of the planet's equator to the plane of its orbit is very nearly 24° 50′ (26° 21′ *to the ecliptic*). So far, therefore, as depends upon that circumstance, Mars should have *seasons* substantially the same as our own, and certain phenomena of the planet's surface, soon to be described, make it evident that such is the case.

Oblateness $\frac{1}{200}$.

The planet's rotation causes a slight but sensible flattening at the poles, — about $\frac{1}{200}$, according to the latest determinations.

Much larger values, now certainly known to be erroneous, are found in the older text-books.

Surface markings.

411. Surface and Topography. — With even a small telescope, not more than 3 or 4 inches in diameter, the planet is a very beautiful object, showing a surface diversified with markings dark and light, which, for the most part, are found to be permanent objects. Occasionally, however, for a few hours at a time, we see others of a temporary character, supposed to be clouds, since they for the time obliterate the permanent ones; but these are surprisingly rare as compared with clouds upon the earth.

The permanent markings on the planet are broadly divisible into three classes.

Polar caps: question as to their nature.

First, *the white patches*, two of which are specially conspicuous near the planet's poles and are called the "polar caps." They are by many supposed to be masses of snow or ice, since they behave just as would be expected if such were the case. The northern one dwindles away during the northern summer, when the north pole is turned towards the sun, while the southern one grows rapidly larger; and *vice versa* during the southern summer. But the probable low temperature of the planet (Sec. 415) makes it at least doubtful whether the apparent "snow and ice" is really congealed *water*, or some quite different substance.

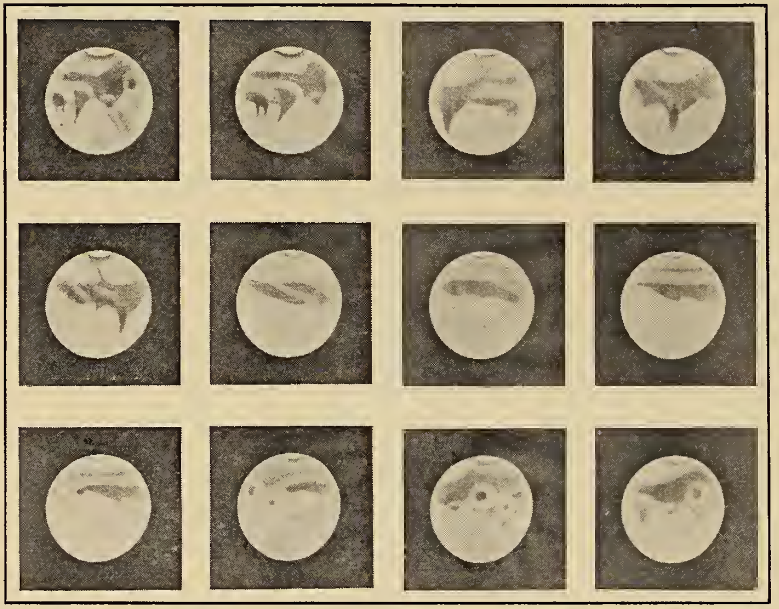

Fig. 141. — Mars

Keeler, 1892

Darkish spots called seas, but with doubtful propriety.

Second, *patches of a bluish gray or greenish shade*, covering about three eighths of the planet's surface, until recently generally supposed to be bodies of water, and therefore called "seas" and "oceans." But more recent observations, if they can be depended on, show a great variety of details within these areas, and such changes of appearance following the seasons of the planet, that this theory is no longer tenable, and they seem more likely to be regions covered with something like vegetation.

Third, *extensive regions of various shades of orange and yellow*, covering nearly five eighths of the surface, and interpreted as land. These markings are, of course, best seen when near the center of the planet's disk; near the limb they are lost in the brilliant light which there prevails, and at the terminator they fade out in the shade.

Continents.

Fig. 141, from drawings by Green of Madeira, and Fig. 142, from drawings by Keeler of the Lick Observatory, give an excellent idea of the planet's appearance as seen by most observers under good conditions.

412. Recent Discoveries; the Canals and their Gemination.— In addition to these three classes of markings the Italian astronomer Schiaparelli, in 1877 and 1879, reported the discovery of a great number of fine straight lines, or "canals," as he called them, crossing the ruddy portions of the planet's disk in all directions, and in 1881 he announced that some of them *become double* at times.

The canals.

These new markings are faint and very difficult to see, and for several years there was a strong suspicion that he was misled by some illusion,—in respect to their "gemination," at least,—which is still ascribed, by some very high authorities, to astigmatism in the eye of the observer or bad focusing of his telescope. Still, the weight of evidence at present favors the reality of the phenomena which Schiaparelli describes. Many observers, both in Europe and the United States, have confirmed his results, and they are now generally accepted, although some of the best, armed with very powerful telescopes, still fail to see the canals as anything but the merest shadings,—not at all as shown in the drawings of Schiaparelli and Lowell. It appears that in the observation of these objects the power of the telescope is less important than steadiness of the air and keenness of the

Their gemination.

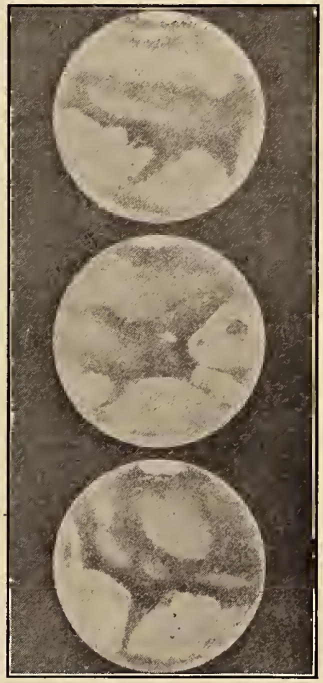

FIG. 142
Green, 1878

Canals difficult to observe.
Question as to possible illusions.

observer's vision. Nor are they usually best seen when Mars is nearest, but their visibility depends largely upon the *season* of the planet; and this is especially the case with their "gemination." Fig. 143, from one of Mr. Lowell's drawings made in 1894, gives an idea of the extent and complexity of the canal system; but the reader must not suppose that in the telescope it stands out with any such conspicuousness. The figure shows also how some of the canals cross the so-called "seas" and disprove the propriety of the name.

413. As to the real nature and office of the "canals" there is a wide difference of opinion, and it is very doubtful if their true explanation has yet been reached. Indeed, it is still quite possible that some of the peculiar phenomena reported are illusions, based on what the observers think they ought to see; it is easy to be deceived in attempting to interpret intelligibly what is barely visible.

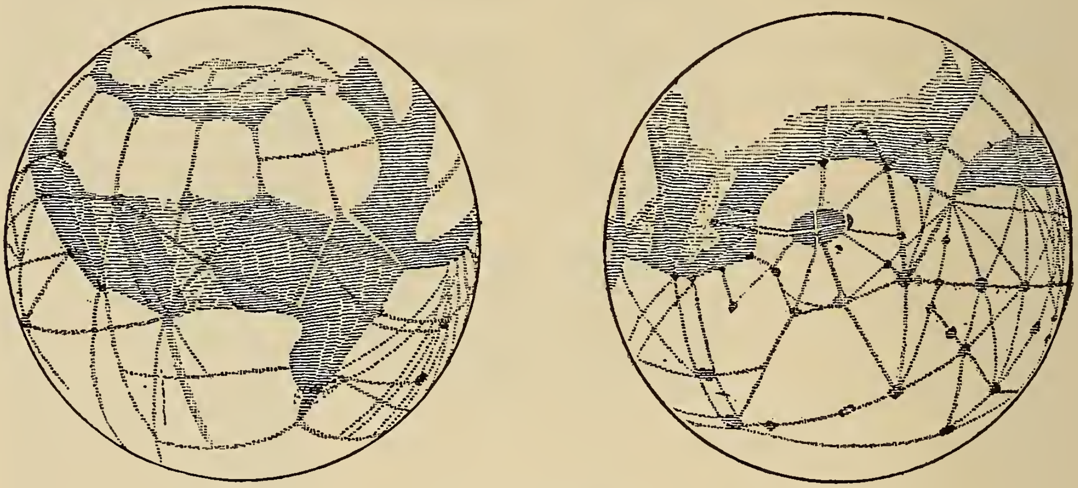

FIG. 143. — Mars
After Lowell

Views of Flammarion and Lowell.

According to Flammarion, Lowell, and other zealous observers of the planet, the polar caps are really *snow sheets*, which melt in the (Martian) spring and send the water towards the planet's equator over its nearly level plains (for no high mountains have yet been discovered there), obscuring for several weeks the well-known markings which are visible at other times.

In Lowell's view the dark regions on the planet's surface are areas covered with some sort of vegetation, while the ruddy portions are barren deserts, intersected by the canals, which he believes to be really irrigating watercourses; and on account of their straightness, and some other characteristics, he is disposed to regard them as *artificial.*

Office of the canals.

When the water reaches these canals vegetation springs up along their banks, and these belts of verdure are what we see with our telescopes, — not the narrow water channels themselves.

Where the canals cross each other and the water supply is more abundant there are dark round "lakes," as they have been called, which he interprets as *oases.*

Vegetation and oases.

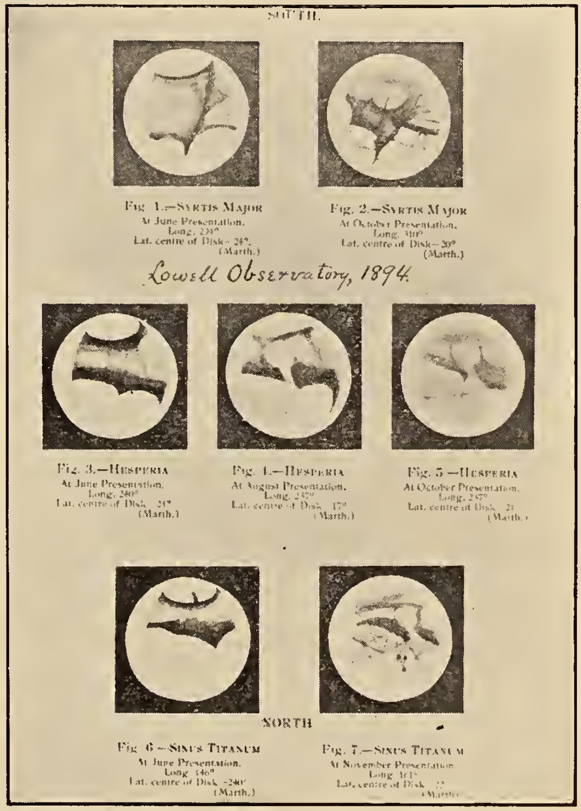
SOUTH.
Fig. 1.—Syrtis Major
At June Presentation.
Long. 23?°
Lat. centre of Disk—24°.
(Marth.)
Fig. 2.—Syrtis Major
At October Presentation.
Long. 310°
Lat. centre of Disk—20°
(Marth.)
Lowell Observatory, 1894.
Fig. 3.—Hesperia
At June Presentation.
Long. 280°
Lat. centre of Disk—24°
(Marth.)
Fig. 4.—Hesperia
At August Presentation.
Long. 257°
Lat. centre of Disk—17°
(Marth.)
Fig. 5.—Hesperia
At October Presentation.
Long. 257°
Lat. centre of Disk—21
(Marth.)
NORTH
Fig. 6.—Sinus Titanum
At June Presentation.
Long. 146°
Lat. centre of Disk—24°
(Marth.)
Fig. 7.—Sinus Titanum
At November Presentation
Long. 161°
Lat. centre of Disk
(Marth.)

Fig. 144. — Seasonal Changes on Mars
Lowell

All of this theoretical explanation rests, however, upon the *assumption* that the planet's temperature is high enough to permit the existence of water in the liquid state, to say nothing of other difficulties. But whatever may be the explanation, there is no longer much doubt as to the existence of the canals, nor that they and other features of the surface undergo real changes with the progress of the planet's seasons.

Their "gemination," however, still remains a mystery, nor is it entirely certain that it may not be a purely optical effect, as already intimated; experiments made at Harvard College Observatory in 1896, and later in France, point strongly in this direction.

Certain changes on the surface of the planet are clearly connected with its seasons. This is, of course, the case with the alternate growth and shrinkage of the polar caps, and Flammarion and Lowell have reported others. Fig. 144, from the observations of Lowell in 1894, shows their nature and amount.

Seasonal changes.

414. Maps of the Planet. — A number of maps of Mars have been constructed by different observers since the first was made by Maedler in 1830. Fig. 145 is reduced from one published in 1888 by Schiaparelli and shows most of his "canals" and

Maps of Mars.

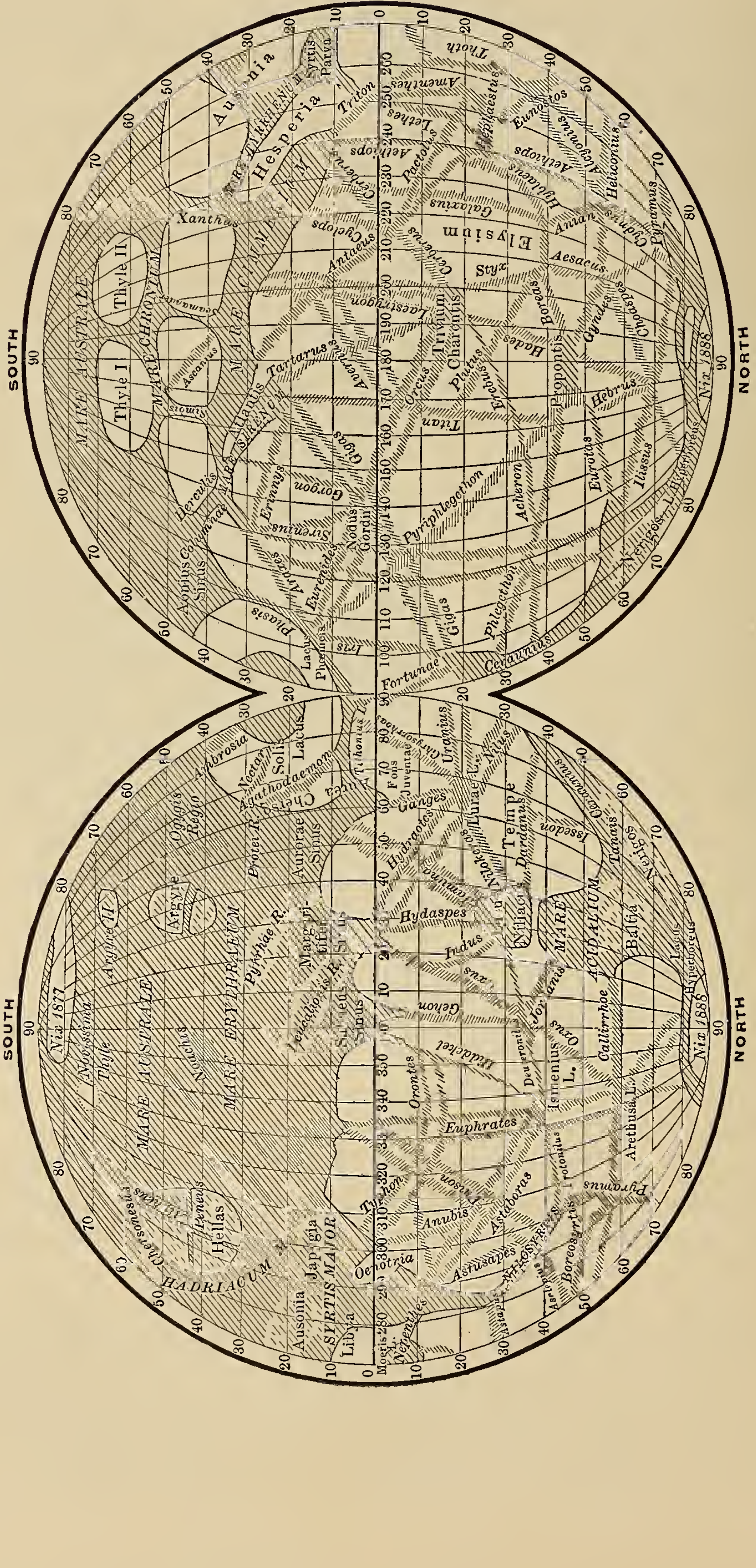
SOUTH
NORTH
SOUTH
NORTH
MARE AUSTRALE
MARE CHRONIUM
Thyle I
Thyle II
Xanthus
Ausonia
Hesperia
Syrtis Parva
MARE TYRRHENUM
MARE CIMMERIUM
MARE SIRENUM
Triton
Thoth
Amenthes
Lethes
Aethiops
Cerberus
Elysium
Styx
Galaxius
Eunostos
Hephaestus
Heliconius
Aesacus
Propontis
Hades
Trivium Charontis
Laestrygon
Tartarus
Avernus
Titan
Orcus
Gigas
Gorgon
Erinnys
Sirenius
Pyriphlegethon
Acheron
Eurotas
Hebrus
Nix 1888
Phlegethon
Ceraunius
Fortunae
Iris
Phasis
Lacus Phoenicis
Eumenides
Nodus Gordii
Atlantis
Icarius
Ascanius
Hercules
Aonius Sinus
Columnae
MARE ERYTHRAEUM
Argyre
Ambrosia
Nectar
Agathodaemon
Lacus Solis
Aurorae Sinus
Chrysorrhoas
Ganges
Uranius
Nilus
Tempe
Hydaspes
Indus
Margaritifer
Pyrrhae R.
Protei R.
Ogygis Regio
Noachis
Nix 1877
Thyle
Gehon
Hiddekel
Oxus
Jordanis
Ismenius L.
Deuteronilus
Euphrates
Orontes
Callirrhoe
Arethusa L.
MARE ACIDALIUM
Baltia
Lacus Hyperboreus
Tanais
Isseden
Sabaeus Sinus
Typhon
Anubis
Astaboras
Astusapes
NILOSYRTIS
Boreosyrtis
Pyramus
Protonilus
Oenotria
Nepenthes
Moeris L.
Libya
SYRTIS MAJOR
Japygia
Ausonia
HADRIACUM M.
Hellas
Peneus
Chersonesus
MARE AUSTRALE

FIG. 145.—Chart of Mars as observed from 1877 to 1888

Schiaparelli

their "gemination." While there may be some doubt as to the accuracy of the minor details, it is probable that the main features of the planet's surface are substantially correct.

The nomenclature, however, is in a very unsettled condition. Schiaparelli has taken his names mostly from ancient geography, while the English areographers,[1] following the analogy of the lunar maps, have mainly used the names of astronomers who have contributed to our knowledge of the planet's surface.

Temperature of the planet, *a priori*, supposably low.

415. Temperature.—As to the temperature of Mars we have no certain knowledge at present. Unless the planet has some unexplained sources of heat it *ought to be* very cold.

Its distance from the sun reduces the intensity of solar radiation upon its surface to less than half its value upon the earth, and its atmosphere cannot well be as dense as at the tops of our loftiest mountains.

On the other hand, things look very much as if the poles were really *snow*-capped, and as if liquid water and vegetable life were present in other regions.

Facts that suggest unknown sources of heat.

If so, we must suppose that the planet has sources of heat, external or internal, which are not yet explained; otherwise the polar "snow" must be something else than frozen *water*, as is perhaps not impossible. It is earnestly to be hoped, and may be expected, that before long we shall obtain some heat-measuring apparatus sufficiently delicate to decide whether the planet's surface is really intensely cold or reasonably warm, — for of course there are various conceivable hypotheses which might explain a high temperature at the surface of Mars.

Satellites: their discovery and names.

416. Satellites. — The planet has two satellites, discovered by Hall, at Washington, in 1877. They are extremely small and observable only with very large telescopes. The outer one, Deimos, is at a distance of 14600 miles from the planet's center and has a sidereal period of $30^h 18^m$; while the inner one, Phobos, is at a distance of only 5800 miles and has a period of $7^h 39^m$, — less than one third of the planet's day. (This is the only case known of a satellite with a period shorter than the revolution of its primary.) Owing to this fact, it *rises in the west*, as seen from the planet's surface, and *sets in the east*, completing its strange backward diurnal revolution in about eleven hours. Deimos, on the other hand, rises in the east, but takes nearly 132 hours in its diurnal circuit, which is more than four of its months. Both the orbits are sensibly circular and lie very closely in the plane of the planet's equator.

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[1] The Greek name of Mars is *Ares;* hence, *areography* is the description of the surface of Mars.

Peculiar behavior of Phobos.

Their diameters.

Micrometric measures of the diameter of such small objects are impossible, but, from *photometric* observations, Prof. E. C. Pickering, assuming that they have the same reflecting power as that of Mars itself, has estimated the diameter of Phobos as about 7 miles and that of Deimos as 5 or 6. Mr. Lowell, however, from his observations of 1894, deduces considerably larger values, viz., 10 miles for Deimos and 36 for Phobos. If this is correct, Phobos, seen in the zenith from the point on the planet's surface directly beneath him, would appear somewhat larger than the moon, but only about half as bright. Deimos would be no brighter than Venus.

Question of habitability.

417. Habitability of Mars. — As to this question we can only say that, different as must be the conditions on Mars from those prevailing on the earth, they differ less from ours than those on any other heavenly body observable with our present telescopes; and if life, such as we know it upon the earth, can exist on any of the planets, Mars is the one. If we could waive the question of temperature and assume, with Flammarion and others, that the polar caps really consist of frozen *water*, then it would become extremely probable that the growth of vegetation is the explanation of many of the phenomena actually observed.

Suggestion of intelligence in the system of canals.

Mr. Lowell goes further and argues the presence of intelligent beings, possessed of high engineering skill, from the apparent "accuracy" with which the "canals" seem to be laid out in a well-planned system of irrigation. But at present, and until the temperature problem is solved, such speculations appear rather premature; and as to the establishment of communication with the hypothetical inhabitants, the idea, in the present state of human arts at least, is simply chimerical.

THE ASTEROIDS

418. The "asteroids," or minor planets, are a host of small bodies circulating around the sun between the orbits of Mars and Jupiter. The name "asteroid," *i.e.*, "starlike," was suggested by Sir William Herschel early in the century, as indicating that, though really planets, they appear like stars.

The asteroids taking the place of a single planet in region indicated by Bode's Law.

Kepler had noticed the wide gap between Mars and Jupiter and had tried to account for it, though unsuccessfully, and when Bode's Law (Sec. 349) was published in 1772 the impression became very strong that there must be a missing planet in the vacant space, — an impression greatly strengthened by the discovery of Uranus in 1781, at a distance almost precisely corresponding to that law. An association of twenty-four astronomers, mostly German, was formed to look for the missing planet, but failed to find one after a dozen years of search, and the first discovery was made by the Sicilian astronomer, Piazzi, who was then engaged in forming his extensive catalogue of stars.

Discovery of Ceres by Piazzi.

On the first night of the nineteenth century (Jan. 1, 1801) he observed a small star where there had been no star a few days earlier, and the next day it had obviously moved, and it continued to move. He named the new planet *Ceres*, after the tutelary divinity of the island, and observed it carefully for several weeks, until he was taken ill; but before he recovered the planet was lost in the evening twilight. It was rediscovered at the close of the next year by means of Gauss' calculations. (See Sec. 365.)

Discovery of Pallas, Juno, and Vesta.

In 1802, while searching for Ceres, Pallas was discovered by Olbers. Juno was found by Harding in 1804, and in 1807 Olbers, who had broached the theory that these new bodies were fragments of an exploded planet, discovered Vesta, the only one ever visible to the naked eye. The search was kept up for several years longer without success, because those engaged in it did not look for small enough objects.

Discovery of Astræa.

The *fifth* asteroid, *Astræa*, was discovered in 1845 by Hencke, an amateur, who had resumed the search afresh by studying the smaller stars and after fifteen years of fruitless labor was rewarded by the new discovery. In 1847 three more were found, and not a year has passed since then without the discovery of from one to thirty.

Over five hundred known.

At present (January, 1902) the number known exceeds five hundred, and for the past ten years has been increasing with great rapidity.

Designation by numbers and names.

They are all designated by numbers; *i.e.*, each one receives a number after having been observed a sufficient number of times to determine its orbit, which usually happens soon after its discovery. Most of them also have names, usually mythological, and feminine for all but Eros; but it has already become a pretty serious matter to find names for all the new discoveries, and the custom may be given up before long.

Method of search with telescope.

419. Method of Search. — Formerly the asteroid hunter conducted his operations by making special telescopic star charts of regions near the ecliptic, and from time to time comparing the chart with the heavens. If an interloper appeared on the chart, a few hours' watching would decide whether it moved or not, *i.e.*, whether it was a planet or merely a variable star. The work, especially that of chart making, was very laborious.

Photographic method.

In 1891 a new method was introduced by Dr. Max Wolf of Heidelberg. A camera with a wide-angle lens of several inches aperture is mounted equatorially and moved by clockwork; with this photographs are made of portions of the sky from 5°

to 10° in diameter. On the negative the *stars*, if the clockwork runs correctly, show as small black dots, but a *planet*, if present, will move among the stars during the two or three hours of exposure, and its image will be a *streak* instead of a dot, and so recognizable at once.

Fig. 146 is a direct reproduction of the plate on which Dr. Wolf discovered planet 1892, V (Gudrun, (328)), the "trail" of which, due to about two hours motion, is shown exactly in the center of the cut. The first planet discovered by this method, in December, 1891, Wolf has named "Brucia," (323), in honor of the late Miss Catherine W. Bruce of New York, who provided the funds for his camera and its mounting.

Fig. 146. — Wolf's Discovery of (328) "Gudrun," 1892

It has happened several times that more than one planet is found on the negative; in one instance as many as *five*, three of which were new, and in another (on a plate made at Harvard) no less than *seven*. Already, during the past ten years, nearly two hundred have been thus discovered, almost all by Wolf of Heidelberg and Charlois of Nice, though a few others have contributed.

Great care is necessary to be sure that the objects discovered are really *new*. There are a number of the older ones which, not having been observed for many years, are now adrift and practically lost, and are likely to be rediscovered at any time. Several of them indeed have been already picked up by the new method.

Provisional designation before assignment of number.

Since 1892 the newly discovered bodies, while awaiting the final numbers (and perhaps a name), are provisionally designated by letters, as AM, DQ, etc.; they are now (1902) far along in the H's.

A full list, brought down to date as nearly as possible, is published every year in the *Annuaire du Bureau des Longitudes*, Paris, giving their number, name, date of discovery, and the elements of their orbits.

THEIR ORBITS

Distance and period: Adalberta nearest, Thule remotest. Periods three to nine years.

420. Mean Distance and Period. — The *mean distances* of the different asteroids from the sun differ widely, and their *periods* correspond. Excepting Eros, the nearest to the sun so far as yet determined is Adalberta, (330), its mean distance being about 2.09 (194 000000 miles) and its period three years and three days. Thule, (279), is the most remote, with a distance of 4.30, or 400 000000 miles, and a period only a month less than nine years.

Average mean distance of group about 2.65.

The mean distances are not distributed at all uniformly through their range, but there are several marked gaps, doubtless due to the action of Jupiter, since they come just where the period of the asteroid would be exactly commensurable with that of the great planet, *i.e.*, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$, or $\frac{2}{3}$ of Jupiter's period. The distances are grouped most densely about 2.8, which Bode's Law would indicate as that of the "missing" planet; but the *average* mean distance comes out somewhat smaller, about 2.65 (246 500000 miles), corresponding to a period of about four and one-third years.

Large inclinations and eccentricities.

421. Inclinations and Eccentricities. — These average much greater than for the principal planets. The mean *inclination* of the asteroid orbits to the ecliptic is about 8°. The orbit of Pallas, (2), is inclined 35°, and seven or eight others exceed 25°.

The *eccentricity* is also very large in some cases. For Æthra,[1] (132), and Andromache, (175), it is fairly cometary, exceeding 0.35, and there are a dozen others above 0.30.

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[1] A planet very recently discovered (October, 1901) but not yet numbered proves to have an eccentricity greater than even that of Æthra, exceeding 0.38.

The orbits so cross and interlink that if they were material hoops or rings the lifting of one would take all the others with it, and that of Mars also, caught up by that of Eros.

422. Diameter, Surface, etc. — These bodies are so small that micrometrical measurements, even of the largest, are extremely difficult, and of the smaller ones impossible. Since 1890, however, Barnard, with the Lick and Yerkes telescopes, has obtained measures of the disks of the four brightest and presumably largest, with the following rather surprising results, viz.: Ceres, 488 miles; Pallas, 304; Vesta, 248; Juno, 118. The surprise consists in the fact that Vesta, which is fully twice as bright as Ceres, should have a diameter only half as great, showing a wide difference of *albedo*. Müller of Potsdam, accepting Barnard's diameters, finds for Ceres from his photometric observations an *albedo* about the same as that of Mercury (0.13), while that of Vesta is put at 0.72, — higher than that of any other planet, and nearly equal to that of writing-paper.

Diameters for most part too small for measurement.

Barnard's results for Ceres, Pallas, Vesta, and Juno.

As to the other asteroids, probably no one of them has a diameter as great as 100 miles, and the smaller ones, such as those which are now being discovered, are mostly of the thirteenth and fourteenth magnitude, so small that they cannot be seen (though easily photographed) with a telescope of much less than 12 inches aperture and cannot be more than 10 or 15 miles in diameter, — mere "mountains broke loose," with a surface area no more extensive than some western farms.

423. Mass and Density. — On these points we have no absolute knowledge; but if we assume that the density is about the same as that of the planet Mars (seventy-three per cent of the density of the earth), which is probably an overestimate, Ceres would have a mass of about $\frac{1}{5800}$ that of the earth, and the force of gravity at her surface would be about $\frac{1}{22}$ of gravity here.

Mass of Ceres perhaps $\frac{1}{5800}$ of earth's.

A stone would descend only about 8½ inches in the first second of its fall, and the "parabolic velocity" at the planet's surface would be about 1900 feet a second (Sec. 319), — a rifle bullet shot from the planet would

Force of gravity on asteroids.

never return. For a planet 10 miles in diameter of the same density, the critical velocity would be only 38 feet a second, so that if the hypothetical dweller on one of these "planetules," as Miss Clerke calls them, should throw away a stone, it would never come back, but would become an independent planet.

Total mass of group certainly less than $\frac{1}{4}$ of earth's, perhaps less than $\frac{1}{100}$.

It is, however, possible from the perturbations which the asteroids produce (or rather do *not* produce) on Mars to estimate, for the aggregate mass of the flock, *a limit* which it cannot exceed, — including the presumably undiscovered multitude as well as the five hundred now known. Leverrier found long ago that the total mass could not be as great as one quarter the mass of the earth; and a much more recent computation by Ravené in 1896 puts it below one per cent. The united mass of all thus far discovered would make but a small fraction of this one per cent, — certainly not over $\frac{1}{1000}$ of the mass of the earth.

Total number probably thousands.

424. The number not yet discovered is probably enormous, though it is practically certain that nearly all that exceed 40 or 50 miles in diameter are already in our catalogue. How long it will be considered worth while to search for new ones is doubtful, as it is quite certain that the computers will not continue to follow by calculation the motions of any except such as possess peculiar interest for their size or some other reason.

An asteroid is much more difficult to observe than a large planet, and immensely more troublesome to follow by calculation, because of the great perturbations to which it is exposed from Jupiter's attraction. One little family of these bodies, twenty-two in number, which were discovered by Professor Watson of Ann Arbor, is, however, "endowed" with a fund which he left in his will to pay for the calculations necessary to keep them from getting lost.

Theories as to origin.

425. Origin. — As to this we can only speculate. It is hardly possible to doubt that this swarm of little rocks in some way represents a single planet of the terrestrial group.

A generally accepted view is that the material, which, according to the nebular hypothesis, once formed a ring or rings like those of Saturn, either continuous or of separate pieces, — matter which ought to have collected to make a single planet, — has failed to be so concentrated; and the failure is ascribed to the perturbations produced by its neighbor, the giant Jupiter, whose powerful attraction is supposed to have disintegrated the ring, or at least prevented the union of the separate parts, and thus stopped the development of a normal planet.

A ring disrupted by attraction of Jupiter.

Another view is that the asteroids may be fragments of an exploded planet. If so, there must have been not one but many explosions; first of the original, and then of the separate pieces in different portions of their orbits. It is demonstrable that *no single explosion* could account for the present tangle of orbits.

An exploded planet.

426. The Planet Eros. — This little planet, insignificant in size but of great astronomical interest, and already several times referred to, should probably be regarded as a member of the asteroid family. It has, however, an orbit so much smaller than any other asteroid that the discoverer claims for it a status of its own.

Eros a doubtful member of the asteroid family.

It was discovered in August, 1898, photographically, by Witt of Berlin, and at once attracted notice by the rapidity of its motion. After a preliminary calculation of its orbit had been made by Dr. Chandler, so that its place could be approximately computed for dates in the past, its trail was found on a considerable number of photographic plates made at Harvard College Observatory during several years preceding (1893, 1894, and 1896); and this rendered it possible at once to compute a very accurate orbit.

Its discovery.

427. Orbit of Eros. — Its *mean distance* from the sun is only 1.46 (135 500000 miles). Its *sidereal period* is 643 days, its *synodic* 845 days. The *eccentricity* of its orbit is 0.22, which makes the aphelion distance 165 500000 miles (well outside the orbit of Mars and well within the asteroid region), while its

Its orbital peculiarities.

perihelion distance (105 300000 miles) is only a little more than 12 000000 miles greater than the mean distance of the earth from the sun. Its orbit is shown in Fig. 125.

Its occasional close approach to the earth. Important as a means of determining solar parallax.

The *inclination* of its orbit is 11°, and this, combined with the fact that the perihelion of the planet's orbit nearly faces that of the earth, makes the *least possible distance between the earth and Eros about 13 500000 miles.* This is only a little more than half the least distance of Venus, and it gives the planet immense importance from an astronomical point of view, since observations made at such a time of close approach will furnish a far more precise determination of the solar parallax and astronomical unit than any other method known.

Next opportunity in 1931.

Unfortunately, these close oppositions are rare; one occurred in 1894, and another such opportunity will not occur again until 1931. In the winter of 1900–01 the conditions were better than they will be until then, the planet having come within about 30 000000 miles of the earth. An extensive series of observations, both visual and photographic, was made, participated in by all the leading observatories of the world. The mass of material accumulated is such that it will probably be two or three years at least before the results can be fully worked out.

Enormous range of brightness.

428. Eros itself.— The planet is small, probably not more than 15 or 20 miles in diameter, though this is merely an estimate. On account of the enormous variation in its distance from the earth (from 13 500000 miles to 260 000000), its brightness when nearest us is nearly four hundred times as great as when remotest. Near aphelion it is observed, if at all, only with the very largest telescopes; when nearest, in 1931, it will for a few days, perhaps, become visible even to the naked eye.

Periodic variations of brightness.

A very remarkable thing is the apparent *periodic variation in its brightness* observed during the early winter and spring of 1901,—shown also in some of the Harvard photographs of 1894 and 1896. At certain times the variation was very striking, the planet in February and March, 1901, being at the maximum fully three times as bright as at the minimum, only two and one-half hours later. At other times the variation disappeared entirely, as in May, 1901. The period of variation, which is 5^h16^m, gives some evidence of two unequal half periods, one of 2^h25^m and the other of 2^h51^m, but this is not yet certain.

Explained by rotation of a spotted sphere or by the revolution of two bodies of egg-shaped form around each other.

The most natural explanation of the variation, as already mentioned (Sec. 383), is that it is caused by the axial rotation of the planet, which is supposed to have light and dark markings on its surface. If these are arranged something like the continents on the earth (continents light, oceans dark), the variation of light would be about as observed when we are in the plane of the planet's equator, and would cease when the planet's pole is directed towards us.

Another explanation, preferred by the French astronomer André, is that the planet is *double*, "a pair of twins," consisting of two bodies revolving around each other almost in contact, in an oval orbit, and with a period of 5^h16^m. When we are in the plane of the orbit occultations occur twice in every revolution, one of the bodies eclipsing the other; but on account of the eccentricity of the orbit these eclipses are not at equal intervals.

To account for the greatness of the light change André further supposes that the bodies are *egg-shaped*, on account of their mutual tidal action, so that when seen sidewise they present three times as much surface as when seen "end on," one behind the other. When we are very much above or below the orbit-plane, so that the bodies pass each other without eclipse, the variation of light would cease. It remains to be seen what future observations may show as to the rival theories.

Possible intramercurial planets.

429. Intramercurial Planets. — It is not impossible that there is a considerable quantity of matter circulating around the sun inside the orbit of Mercury. This has been believed to be indicated by the otherwise unexplained advance of the perihelion of Mercury's orbit, but the investigations of Newcomb render very doubtful the validity of such an explanation, since the *nodes* of the planet's orbit are not affected as they would be on that hypothesis. It has been somewhat persistently supposed that this intramercurial matter is concentrated into one, or possibly two, planets of considerable size, and such a planet has several times been reported as discovered, notably in 1857 (when it was even named "Vulcan"), and again in 1878. We can only say that the supposed discoveries have never been confirmed, and the careful observations during total solar eclipses during the past twenty years make it practically certain that there is no "Vulcan," *i.e.*, no single considerable planet. Perhaps, however, there may be an intramercurial family of *asteroids.* If so, they must be very small or some of them would certainly have been found during the eclipses; and, if as large as 100 or 200 miles in diameter, some of them would probably have been caught crossing the sun's disk.

Reported discovery of Vulcan never verified.

Attempts to find intramercurial planets by photography during solar eclipses.

An attempt was made to detect any existing body of this kind during the eclipses of 1900 and 1901 by means of photography, with a photographic lens of long focus arranged to throw the image of an extensive region of the sky on a collection of photographic plates arranged upon a suitable frame behind the lens. The attempt failed on both occasions.

430. The Zodiacal Light. — This is a faint pyramid of light, for the most part less luminous than the Milky Way, extending from the sun both east and west along the ecliptic. In northern latitudes it is best seen in the evening during the months of February and March; in the morning, in October and November. Its summit is sometimes as far as 90° from the sun, and in the tropics it is said to be sometimes visible at midnight as a complete belt extending clear across the heavens.

The zodiacal light: its appearance and when best seen.

Opposite to the sun there is a slightly brighter patch 10° or 20° in diameter, called the "Gegenschein," or "counter glow." This, and indeed the whole phenomenon, is so faint that it can be well seen only when the observer is where there is no interference from artificial lights. Even the presence of one of the brighter planets greatly embarrasses the observation. The region near the sun is fairly bright, it is true, but is always more or less immersed in the twilight.

The Gegenschein.

The spectrum is a simple continuous one, *without perceptible lines or marking of any kind.* We emphasize this because it has often been erroneously reported that it shows the bright yellow line which characterizes the spectrum of the aurora borealis.

The spectrum of the zodiacal light.

The most probable explanation of the zodiacal light is that it is due to reflection of sunlight from myriads of small particles revolving around the sun in a comparatively thin, flat sheet or ring (something like Saturn's ring), which extends far beyond the orbit of the earth, and perhaps even to that of Mars.

Probable explanation of the zodiacal light as a meteoric ring.

Near the sun the particles are supposed to be more numerous than elsewhere, as well as more brilliantly illuminated, so that although less than half the sunlit surface of each is visible to us, yet on the whole the total sum of brightness is greater than elsewhere.

As for the Gegenschein, this may be accounted for by supposing that the particles nearly opposite the sun "flash out" in the same way that the moon does at the full.

It has been attempted to explain the zodiacal light as due to a ring of meteoric particles revolving *around the earth;* but in that case the Gegenschein would be replaced by a *dark spot* caused by the shadow of the earth, unless indeed the ring had a very improbable diameter of many million miles. As to the size of the particles, we have no direct evidence, though from the analogy of shooting-stars it is likely that they are very small, perhaps not larger than pinheads.

But it cannot be said that the problem is completely solved. There are serious difficulties with every theory yet proposed.

CHAPTER XIV

THE MAJOR PLANETS

Jupiter: its Satellite System; the Equation of Light, and the Distance of the Sun — Saturn: its Rings and Satellites — Uranus: its Discovery, Peculiarities, and Satellites — Neptune: its Discovery, Peculiarities, and Satellite

JUPITER

Jupiter: its conspicuousness.

JUPITER, the nearest of the major planets, stands next to Venus in the order of brilliance among the heavenly bodies, being five or six times as bright as Sirius, the most brilliant of the stars, and decidedly superior to Mars, even when Mars is nearest. It is not, like Venus, confined to the twilight sky, but at the time of opposition dominates the heavens all night long.

Orbital data.

431. Its orbit presents no marked peculiarities. The *mean distance* of the planet from the sun is a little more than five astronomical units (483 000000 miles), and the *eccentricity* of the orbit is not quite $\frac{1}{20}$, so that the distance from the sun varies about 42 000000 miles between perihelion and aphelion.

At an average opposition the planet's distance from the earth is about 390 000000 miles, while at conjunction it is about 580 000000; but it may come as near to us as 370 000000 and may recede to a distance of nearly 600 000000.

The *sidereal* period is 11.86 years, and the *synodic* period is 399 days (a figure easily remembered), a little more than a year and a month.

Diameter, oblateness, surface, and bulk.

432. Diameter, Mass, Density, etc. — The planet's apparent diameter ranges from 50″ to 32″, according to its distance from the earth. The disk, however, is distinctly oval, the *equatorial* diameter being nearly 90000 miles, while the *polar* diameter is 84200. The *mean* diameter $\left(\frac{2a+b}{3}\right)$ (see Sec. 139) is 88000 miles, or a little over eleven times that of the earth.

These values are from the recent measures of Barnard and See, and are notably larger than those determined by earlier observers with a double-image micrometer and given in the table in the Appendix. Very likely the truth may lie intermediate.

The *oblateness* is $\frac{1}{16}$, — very much greater than that of any other planet, Saturn excepted.

Its *surface* is 122, and its *volume* or bulk 1355, times that of the earth. It is by far the largest of all planets, — larger, in fact, than all the rest united.

Its mass 317 times that of the earth.

Its *mass* is very accurately known, both by means of its satellites and by the perturbations which it produces upon certain asteroids. It is $\frac{1}{1048.35}$ of the sun's mass, or about 317 times that of the earth.

Its density and surface gravity.

Comparing this with its volume, we find its mean *density* to be 0.23, *i.e.*, less than one fourth the density of the earth and a little less than that of the sun. Its *surface gravity* is about two and two-thirds times that of the earth, but varies nearly twenty per cent between the equator and poles of the planet.

General aspect.

433. General Telescopic Aspect, Albedo, etc. — In even a small telescope the planet is a fine object, since a magnifying power of only 60 makes its apparent diameter, even when remotest, equal to that of the moon. With a large instrument and magnifying power of 300 or 400 the disk is covered with an infinite variety of detail, interesting in outline, rich in color, — mostly reds and brown, with here and there an olive-green, — and these details change continually as the planet turns on its axis.

The belts.

For the most part, the markings are arranged in "belts" parallel to the planet's equator, as shown in Fig. 147. The left-hand one of the two larger figures is from a drawing by Trouvelot (1870), and the other from one by Vogel (1880). The smaller figure below represents the planet's ordinary appearance in a 3-inch telescope. Fig. 148 is from a beautiful drawing by Keeler, made in 1889, which still continues to be an excellent representation of the planet's aspect. Near the limb of the planet the light is less brilliant than in the center of the disk, and the belts there fade out.

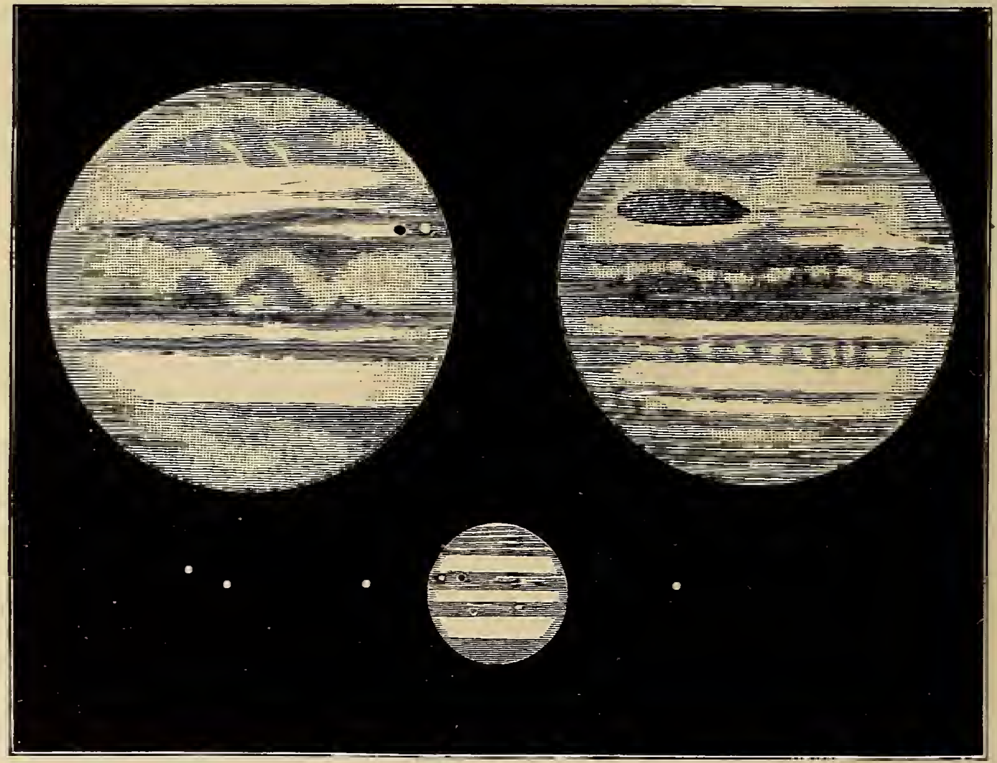


Fig. 147. — Telescopic Views of Jupiter

The planet shows no perceptible *phases*, but at quadrature the edge which is turned away from the sun is sensibly darker than the other.

High albedo, — 0.62.

According to Zöllner, the mean *albedo* of the planet is 0.62, which is very high, that of white paper being 0.78. The question has been raised whether Jupiter is not to some extent self-luminous, but there is no proof, and little probability, that such is the case.

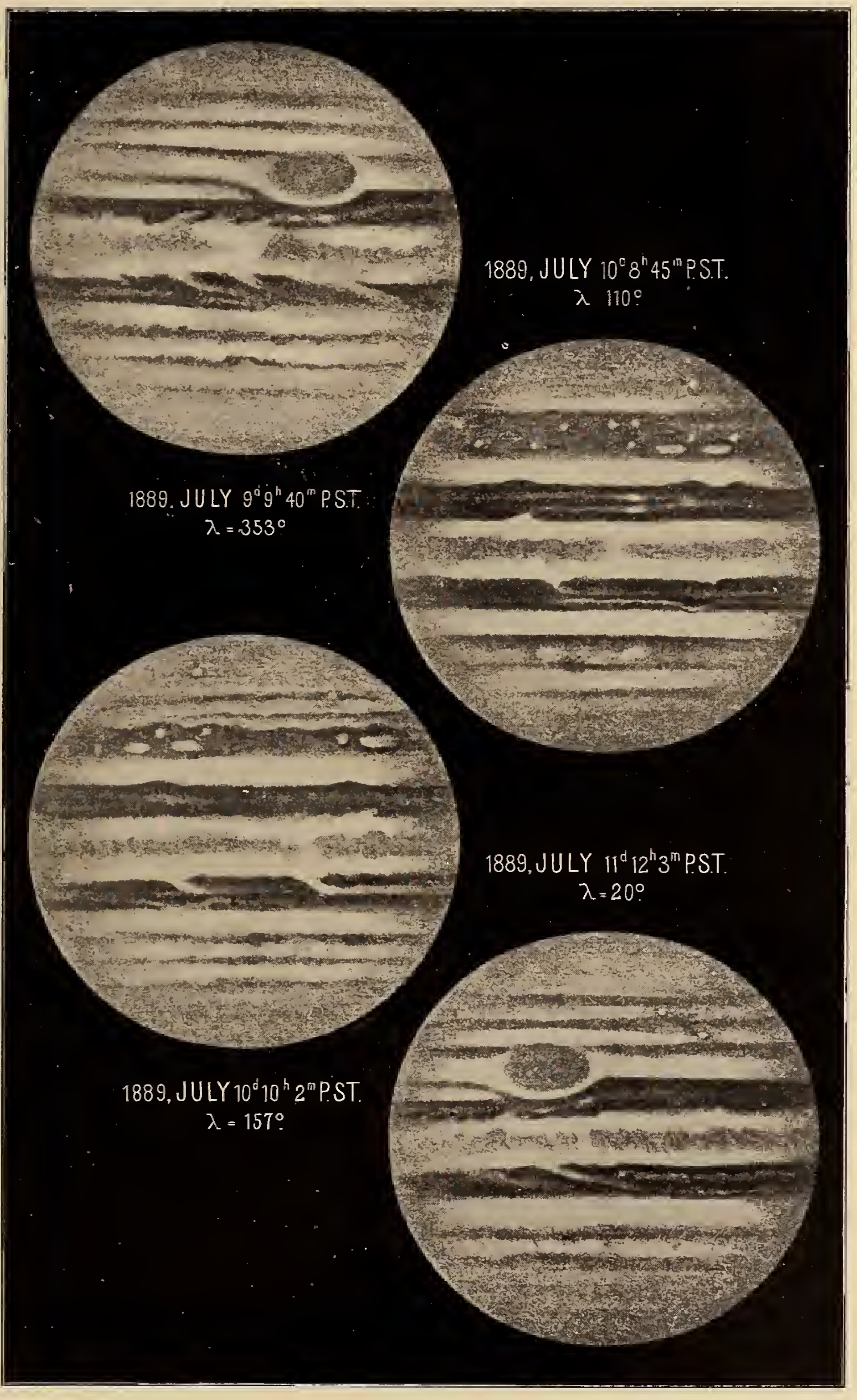

1889, JULY $10^c 8^h 45^m$ P.S.T.
λ 110°
1889, JULY $9^d 9^h 40^m$ P.S.T.
λ = 353°
1889, JULY $11^d 12^h 3^m$ P.S.T.
λ = 20°
1889, JULY $10^d 10^h 2^m$ P.S.T.
λ = 157°

Fig. 148. — Jupiter

After drawings by Keeler, at Lick Observatory

The planet's atmosphere.

434. Atmosphere and Spectrum. — The planet's atmosphere must be very extensive. The forms visible with the telescope are nearly all evidently "atmospheric," — *i.e.*, like clouds, — as is obvious from their rapid changes, though Professor Hough considers that we see the pasty, semi-liquid surface of the globe itself at times. The low mean density of the planet makes it, however, very doubtful whether there is anything solid about it anywhere, — whether it is anything more than a ball of fluid, overlaid by cloud and vapor.

Spectrum of the planet. Shadings in the red and orange.

The *spectrum* of the planet differs less from that of mere reflected sunlight than might have been expected, showing that the light is not obliged to penetrate the atmosphere to any great depth before it is reflected towards us from the clouds. There are, however, faint shadings in the red and orange parts of the spectrum that are probably due to some unidentified constituent of the planet's atmosphere; they seem to be identical in position with certain bands which are intense in the spectra of Uranus and Neptune.

Rotation period about 9^h55^m. Different for different classes of markings.

435. Rotation. — Jupiter rotates on its axis more swiftly than any other planet, — in about 9^h55^m. The time can be given only approximately; not because it is difficult to find, and to observe with accuracy, well-defined objects on the disk, but because different results are obtained from different spots, ranging all the way from 9^h50^m for certain small bright spots to $9^h56\frac{1}{2}^m$ for others of a different character. Well-marked features near each other on the planet's surface often drift by each other, sometimes at the rate of from 200 to 400 miles an hour.

On the whole, spots near the equator usually show a shorter period than those in higher latitudes, but there are numerous exceptions. There is no such regular difference as on the sun, but there apparently are a number of different zones, each with its own rate of rotation, and one or two of the swiftest are not near the equator; neither are the two hemispheres, the northern and southern, alike in their behavior.

The plane of rotation nearly coincides with that of the orbit, the inclination being only 3°, so that there can be no well-marked seasons on the planet due to causes such as produce our own seasons.

Plane of rotation nearly in plane of orbit.

436. Physical Condition; the "Great Red Spot." — The condition of the planet is obviously very different from that of the earth or Mars. No permanent markings are found upon the disk, though there are some which may be called at least *sub-permanent*, persisting for years with only slight apparent change.

No permanent surface markings.

The most remarkable instance of such a marking is *the great red spot*, shown in Figs. 147 and 148. It was first noted in 1878, was extremely conspicuous for several years, and then gradually faded away, slightly changing its form and becoming rounder; even yet (1901), while hardly visible itself, the place which it occupies is clearly marked by the "bed" it has hollowed out in the great southern belt. In its prime it was about 30000 miles long by about 7000 wide.

The great red spot.

Were it not that during the first six or seven years of its visibility it lengthened its rotation period by about six seconds (from $9^h55^m35^s$ to $9^h55^m41^s$), we might suppose it permanently attached to a solid nucleus below; but this change of rotation means that, *relative to its position in 1878*, the spot must have traveled completely around the nucleus of the planet in the six years, unless the nucleus itself changed its own period to the same extent, and that without affecting the motions of the other spots and markings.

No really satisfactory explanation of the spot and its strange behavior has yet been found.

437. Temperature. — Many things about the planet indicate a *probable high temperature*, as, for instance, the abundance of clouds and the rapidity of their motions and transformations, which almost certainly indicate a rapid exchange of matter and a vigorous vertical circulation between the surface and the underlying nucleus, if there is one. To maintain such an

Temperature probably high. The planet a semi-sun.

ebullition requires a continuous supply of heat, and since on Jupiter the solar light and heat are only $\frac{1}{27}$ as intense as here, we are forced to conclude that it gets very little of its heat from the sun, but is probably *hot on its own account*, and for the same reason that the sun is hot, *i.e.*, *as the result of a process of condensation.* In short, it appears very probable, as has been intimated before, that the planet is a sort of "semi-sun," — hot, though not so hot as to be sensibly self-luminous.

The five satellites of Jupiter; their discovery.

438. Satellites. — Jupiter has five satellites, four of them so large as to be seen easily with a common opera-glass. These were in a sense the first heavenly bodies ever "discovered," having been found by Galileo in January, 1610, with his newly invented telescope. The fifth satellite, discovered by Barnard at the Lick Observatory in 1892, is, on the other hand, extremely small and visible only in the most powerful instruments.

The fifth satellite.

It is nearest to the planet, its distance from the center of Jupiter being only 112500 miles and its sidereal period $11^h 57^m.4$. Its diameter probably does not exceed 100 miles.

Data relating to the Galilean satellites.

The old satellites, though more remote, are still usually known as the first, second, etc., in the order of their distance from the planet. Their distances range from 262000 to 1 169000 miles and their sidereal periods from forty-two hours to sixteen and two-thirds days. Their orbits are almost perfectly circular and lie very nearly in the plane of the planet's equator. The third satellite is much the largest, having a diameter of about 3600 miles, while the others are between 2000 and 3000, — all of them larger than our moon, though much less massive.

Third satellite the largest.

Peculiarities of the fourth satellite: very dark surface.

For some reason, the fourth satellite is a very dark-complexioned body, so that when it crosses the planet's disk it looks like a black spot, hardly distinguishable from its own shadow; the others under similar circumstances appear bright, dark, or are invisible, according to the brightness of the part of the planet which happens to form the background. With very powerful instruments spots are sometimes visible on their surfaces, and there are variations in their brightness; W. H. Pickering, Douglass, and some other observers have also reported periodic irregularities in their forms, as if they were cloudlike in constitution.

In the case of the fourth satellite the regularity in the changes of brightness indicates that it follows the example of our moon in always keeping the same face towards the planet, and the observations of Douglass at Flagstaff, in 1897, of spots upon the surfaces of the third and fourth satellites also indicate a rotation agreeing with their orbital periods far within the limits of error to be expected in such observations. It may be considered practically certain that both these satellites behave like our moon.

Keeps same face to planet during its rotation.

The four satellites of Galileo have names also: viz., Io, Europa, Ganymede, and Callisto, — Io being the nearest to the planet. But these names are seldom used.

Names of Galilean satellites.

439. Eclipses and Transits. — The orbits of the satellites are so nearly in the plane of the planet's orbit that with the exception of the fourth, which at certain times escapes, they are eclipsed at every revolution, and also cross the planet's disk at every conjunction.

Eclipses and transits of the satellites.

When the planet is either at opposition or conjunction the shadow, of course, is directly behind it, and we cannot see the eclipse at all. At other times we ordinarily see only the beginning or the end; but when the planet is at or near quadrature the shadow projects so far to one side that the whole eclipse of every satellite, except the first, takes place clear of the disk.

An eclipse is a *gradual* phenomenon, the satellite disappearing by becoming slowly fainter and fainter as it plunges into the shadow, and reappearing in the same leisurely way.

The phenomena gradual.

Two important uses have been made of these eclipses: they have been employed for the determination of longitude, and they *furnish the means of ascertaining the time required by light to traverse the space between the earth and the sun.*

Their use in astronomy.

The equation of light: its constant the time occupied by light in traveling from sun to earth.

440. The Equation of Light.— When we observe a celestial body we see it, not as it *is* at the moment of observation, but as it *was* at the moment when the light which we see left it. If we know its distance in astronomical units, and know how long light takes to traverse that unit, we can at once correct our observation by simply *dating it back* to the time when the light started from the object.

The necessary correction is called the *Equation of Light*, and *the time required by light to traverse the astronomical unit of distance is* ***the Constant of the light-equation*** (not quite five hundred seconds, as we shall see).

Roemer's discovery of the progressive motion of light.

It was in 1675 that Roemer, the Danish astronomer (the inventor of the transit-instrument, meridian-circle, and prime vertical instrument,—a man almost a century in advance of his day), found that the eclipses of Jupiter's satellites show a peculiar variation in their times of occurrence, which he explained as due to the *time taken by light to pass through space.* His bold and original suggestion was neglected for more than fifty years, until long after his death, when Bradley's discovery of *aberration* proved the correctness of his views.

Apparent alternate retardation and acceleration of the satellite eclipses as the distance of the earth from the planet varies.

441. Eclipses of the satellites recur at intervals which are really almost exactly equal (the perturbations being very slight), and the interval can easily be determined and the times tabulated. But if we thus predict the times of the eclipses during a whole synodic period of the planet, then, beginning at the time of opposition, it is found that as the planet recedes from the earth the eclipses, *as observed*, fall constantly *more and more behindhand*, and by precisely the same amount for all four satellites. The difference between the predicted and observed time continues to increase until the planet is near conjunction, when the eclipses are almost seventeen minutes later than the prediction. After the conjunction they quicken their pace and make up the loss, so that when opposition is reached once more they are again on time.

It is easy to see from Fig. 149 that at opposition the planet is nearer the earth than at conjunction by just two astronomical units, *i.e.*, $JB - JA = 2\,SA$. Light coming from J to the earth when it is at A will, therefore, make the journey quicker than when it is at B, by *twice* the time it takes light to pass from S to A, provided it moves through space at a uniform rate, as there is every reason to believe.

How this determines the constant of the light-equation.

The whole apparent retardation of eclipses between opposition and conjunction must, therefore, be exactly *twice the time required for light to come from the sun to the earth.* In this way the "light-equation constant" is found to be very nearly 499 seconds, or 8^m19^s, with a probable error of perhaps two seconds.

The constant, $499^s \pm 2^s$.

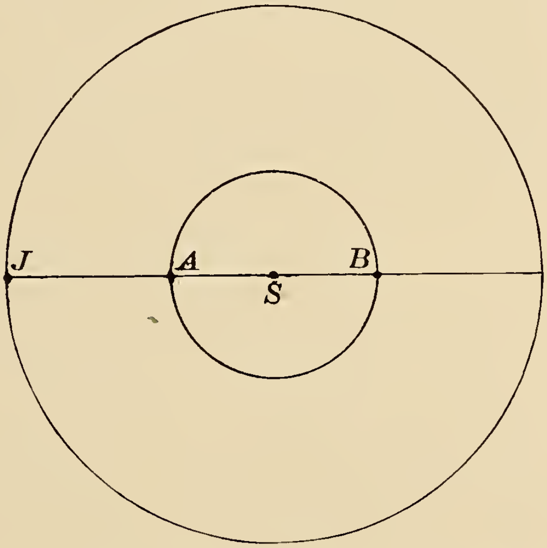


FIG. 149. — Determination of the Equation of Light

Attention is specially directed to the point that the observations of the eclipses of Jupiter's satellites give *directly* neither the *velocity of light* nor *the distance of the sun;* they give only the *time* required by light to make the journey from the sun. Many elementary text-books, especially the older ones, state the case carelessly.

Since these eclipses are *gradual* phenomena, the determination of the exact moment of a satellite's disappearance or reappearance is very difficult, and this renders the result somewhat uncertain. Prof. E. C. Pickering of Cambridge has proposed to utilize *photometric* observations for the purpose of making the determination more precise, and two series of observations of this sort and for this purpose are now completed, and are being reduced, one in Cambridge, and the other in Paris under the direction of Cornu, who devised a similar plan. Pickering has also applied *photography* to the observation of these eclipses with encouraging success.

Photometric method of observing the eclipses.

442. The Distance of the Sun determined by the "Light-Equation." — Until 1849 our only knowledge of the *velocity of*

Distance of sun obtained by multiplying the constant of the light-equation by the velocity of light.

light was obtained from such observations of Jupiter's satellites. By assuming as known *the earth's distance from the sun*, the velocity of light can be obtained when we know the *time* occupied by light in coming from the sun. At present, however, the case is reversed. We can determine the velocity of light by two independent *experimental* methods, and with a surprising degree of accuracy. Then, knowing this velocity and the "light-equation constant," *we can deduce the distance of the sun.* According to the latest determinations, the velocity of light is 186330 miles per second. Multiplying this by 499, we get 92 979000 miles for the sun's distance. (Compare Sec. 173.)

SATURN

Saturn: its brightness and variations of its light.

443. Saturn is the most remote of the planets known to the ancients. In brilliance it is inferior to Venus and Jupiter, or even Mars when nearest; still, it is a conspicuous object of the first magnitude, outshining all the stars (except Sirius) with a steady, yellowish radiance, not varying much in appearance from month to month, though in the course of fifteen years it alternately gains and loses nearly fifty per cent of its brightness with the changing phases of its rings; for it is unique among the heavenly bodies, a great globe attended by eight, perhaps nine, satellites, and surrounded by a system of rings which has no counterpart elsewhere in the universe, so far as known at present.

Orbital peculiarities.

444. Orbit. — Its mean distance from the sun is about nine and one-half astronomical units, or 886 000000 miles; but the distance varies nearly 100 000000, on account of the considerable *eccentricity* of the orbit (0.056). Its nearest opposition approach to the earth is about 774 000000 miles, while at the remotest conjunction it is 1028 000000 miles away.

The *sidereal period* of the planet is about twenty-nine and one-half years, the *synodic* being 378 days. The inclination of the orbit to the ecliptic is about 2½°.

445. Dimensions, Mass, etc. — The apparent mean diameter of the planet varies, according to the distance, from 14″ to 20″. The *equatorial diameter* is about 75000 miles, the *polar diameter* only 67400; the mean diameter, therefore, is about 72500, — a little more than *nine* times the diameter of the earth. The *oblateness* of Saturn (the flattening at the poles) is nearly $\frac{1}{10}$, being greater than that of any other planet.

Diameter, oblateness, surface, and volume.

The *surface* is about eighty-four times that of the earth, and its *volume* 768.

Its *mass* is found by means of its satellites to be ninety-five times that of the earth, so that its mean *density* comes out only *one eighth* that of the earth, — only two thirds that of water! It is by far the least dense of all the planetary family. Its mean *superficial gravity* is about 1.2 times gravity upon the earth, varying, however, nearly twenty-five per cent between the equator and the pole.

Mass, density, and surface gravity. Saturn less dense than water.

The *rotation period* is about 10^h14^m, as determined by Professor Hall in 1876 from a white spot that appeared on the planet's surface and continued visible for several weeks. Later observations of Stanley Williams in 1893, while confirming this result, indicate that there are vigorous surface currents, as on Jupiter, so that different spots give rotation periods differing by several minutes.

Rotation period 10^h14^m.

The equator of the planet is inclined about 27° to the plane of the orbit.

Its equator inclined 27° to plane of its orbit.

446. Surface, Albedo, Spectrum. — The disk of the planet, like that of Jupiter and the sun, is darker at the edge, and, like that of Jupiter, it shows a number of belts arranged parallel to the equator. The equatorial belt is very much brighter than the rest of the surface (not quite so much so, however, as represented in Fig. 150), and is often of a delicate pinkish tinge. The belts in higher latitudes are comparatively faint and narrow, while just at the pole there is a dark cap, sometimes distinctly olive-green in color. Compared with Jupiter, however, there is very little detail observable on the surface; the edges of the belts are usually smooth, with only occasional irregularities, and the spots, when they appear, are as a rule ill-defined and very faintly contrasted with the background, so that they are difficult to observe. Like the markings on

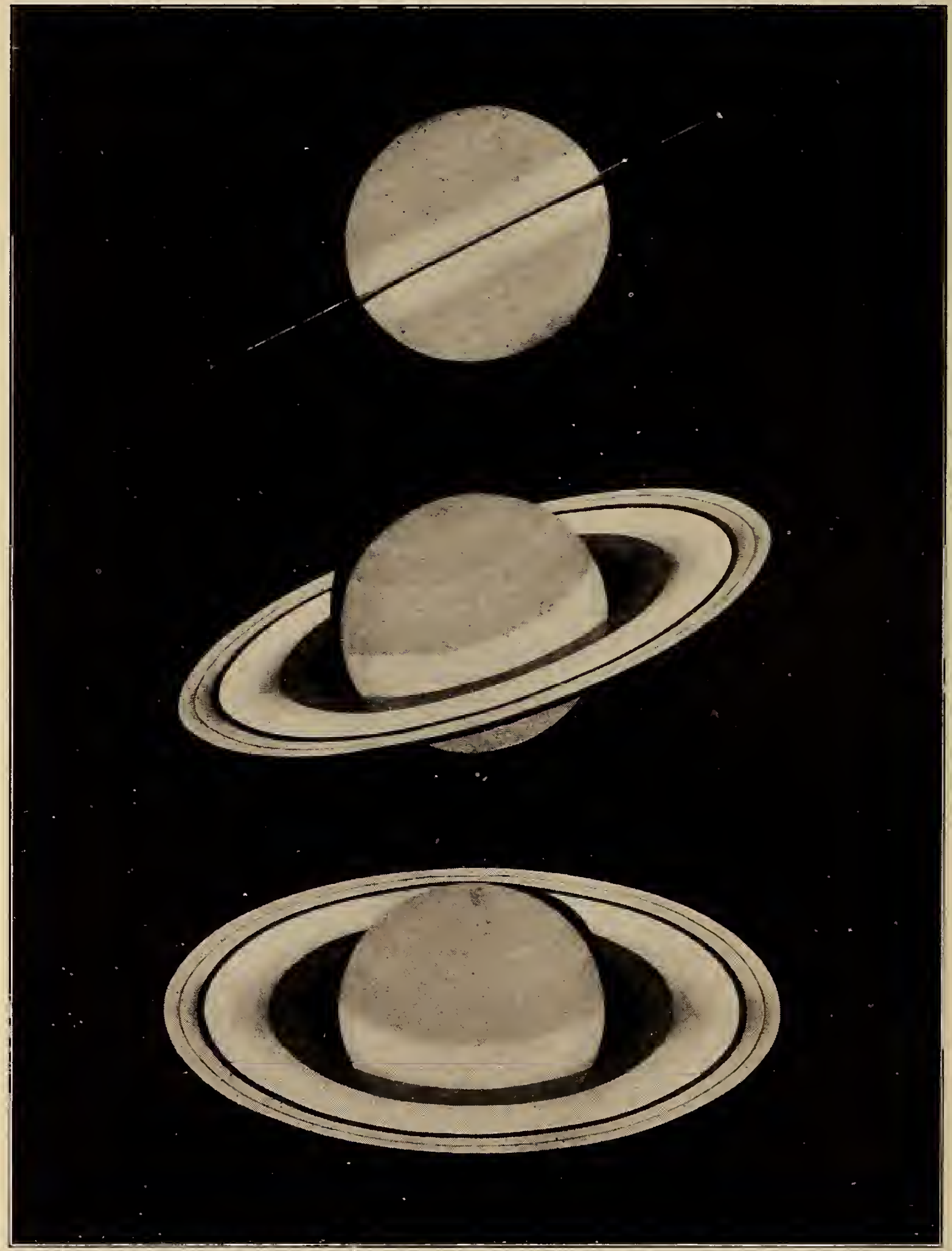


FIG. 150. — Saturn
After Proctor

Jupiter, they are almost certainly atmospheric, *i.e.*, *clouds* of no great density.

The mean *albedo* of the planet is 0.52, according to Zöllner, — very nearly the same as that of Venus.

Albedo 0.52.

The *spectrum* of Saturn is substantially like that of Jupiter, but the dark bands in the lower part of the spectrum are more pronounced. These bands, which are doubtless due to some unidentified constituent of the planet's atmosphere, *do not appear, however, in the spectrum of the rings*, which presumably have very little atmosphere upon them.

Spectrum of the planet. Bands more pronounced than in case of Jupiter.

As to the physical condition and constitution of the planet, it is probably essentially like that of Jupiter, though still farther from solidity; it does not, however, seem to boil quite so vigorously at the surface. Its supply of solar heat and light is less than $\frac{1}{90}$ of that which we receive on the earth.

Probably warm from internal sources.

447. The Rings. — The most remarkable peculiarity of the planet is its *ring system*. The globe is surrounded by three thin, flat, concentric rings in the plane of Saturn's equator, like circular disks of paper perforated through the center. They are generally referred to as *A*, *B*, and *C*, *A* being the exterior ring.

Its ring system.

Galileo *half* discovered them in 1610; that is, he saw with his little telescope two appendages on each side of the planet, but he could make nothing of them, and after a while he lost them, to regain them again some years later, greatly to his perplexity.

Half discovered by Galileo.

The problem remained unsolved for nearly fifty years, until Huyghens explained the mystery in 1655. Twenty years later D. Cassini discovered that the ring is *double*, *i.e.*, composed of two concentric rings, with a dark line of separation between them, and in 1850, Bond of Cambridge, U.S., discovered the third "dusky" or "gauze" ring between the principal ring and the planet. (It was discovered a fortnight later, and independently, by Dawes in England.)

Discovery completed by Huyghens and D. Cassini.

Gauze ring discovered by Bond in 1850.

Dimensions of the rings.

The outer ring, *A*, has an exterior diameter of about 173000 miles and a width of not quite 12000. Cassini's division between this and *B* is about 1800 miles wide; the ring *B*, much the broadest and brightest of the three, has a breadth of about 17000 miles. The semi-transparent ring, *C*, has a width of about 11000 miles, leaving a clear space of from 7000 to 8000 miles in width between the planet's equator and its inner edge. Their thickness is exceedingly small,—probably less than 50 miles. (These dimensions are from the recent measurements of Professor See and differ slightly from those given in the *General Astronomy*.) There is some reason to suspect that the rings may have changed their dimensions at different times, but as yet the proof is insufficient.

Rings extremely thin.

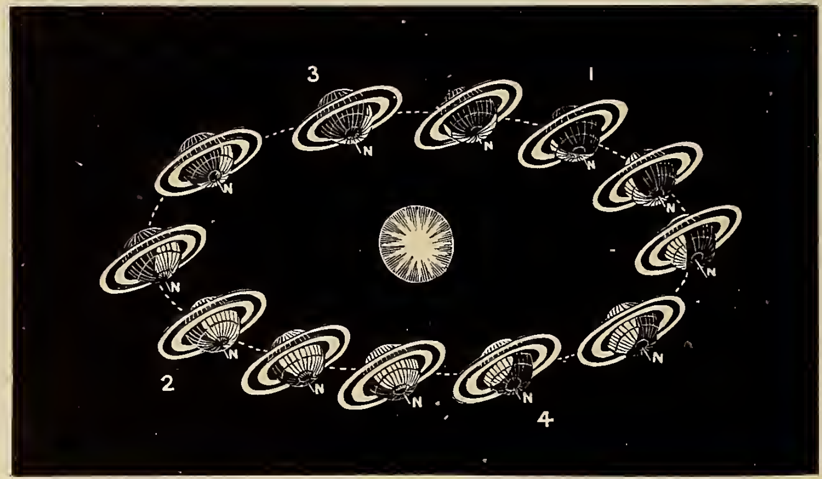


FIG. 151.—The Phases of Saturn's Rings

Their phases. Disappear once in fifteen years, when planet passes the nodes of its orbit.

448. Phases of the Rings.—The rings are inclined about 28° to the ecliptic (27° to the planet's *orbit*), having their nodes in longitude 168° and 348°, and of course maintain their plane parallel to itself at all times. Twice in a revolution of the planet (once in about fifteen years) this plane sweeps across the orbit of the earth (too small to be shown in Fig. 151), occupying not quite a year in so doing; and whenever the plane passes between the earth and the sun the dark side of the ring is towards us and the edge alone is visible. The plane of the ring traverses the orbit of the earth in about 359.6 days, and during this time the earth herself passes the plane either *once* or *three* times, according to circumstances, — usually three times, thus causing two periods of disappearance during the critical year. When the earth is crossing the plane of the ring, so that its edge is exactly towards us, the ring becomes absolutely invisible to all existing telescopes for several days; and in the longer periods, while the dark side of the ring is presented to us, — sometimes for several weeks, — only the most powerful instruments can see it, like a fine needle of light piercing the planet's ball, and with satellites strung like beads upon it, as shown in the upper view of Fig. 150.

Structure of the rings: a swarm of independent moonlets.

449. The Structure of the Rings. — It is now universally admitted that the rings are not continuous sheets, either solid or liquid, but a *flock or swarm of separate particles*, little "moonlets," each pursuing its own independent circular orbit around the planet, though all moving nearly in the same plane.

Origin and development of the idea.

The idea was first suggested by J. Cassini in 1715, and later by Wright in 1750, but was quite lost sight of until brought forward again by G. P. Bond in connection with his father's discovery of the dusky ring. Peirce soon demonstrated that the rings *could not be solid*, though he was disposed to think they might be liquid. Clerk Maxwell in 1857 went further, by showing mathematically that while they could be neither solid nor liquid, they must, in order to be permanent, be constituted as explained above.

Photometric observations of Seeliger and Barnard.

There are also observational facts that confirm the theory. Seeliger has shown that the variations in the naked-eye brightness of the planet, due to the phases of the rings, can be explained only on the hypothesis that they are like clouds of dust. Again, in 1892 Barnard observed the satellite Iapetus during one of its eclipses (a very rare event) and found that the shadow of the ring is not opaque; the satellite did not disappear when immersed in it, but vanished as soon as it entered the shadow of the ball.

Keeler's spectroscopic proof that the outer edge of the ring moves more slowly than the inner.

450. Keeler's Demonstration of the Meteoric Theory of Saturn's Rings. — In 1895 Keeler, then at Allegheny, obtained *spectroscopic proof* that the *outer* edge of the ring revolves *more slowly than the inner*, as the theory requires, but as would not be the case if the ring were a continuous sheet. Photographs were made of the spectrum of the planet with the slit of the spectroscope crossing the planet and its rings, as shown in Fig. 152, which is a much magnified drawing of the actual image.

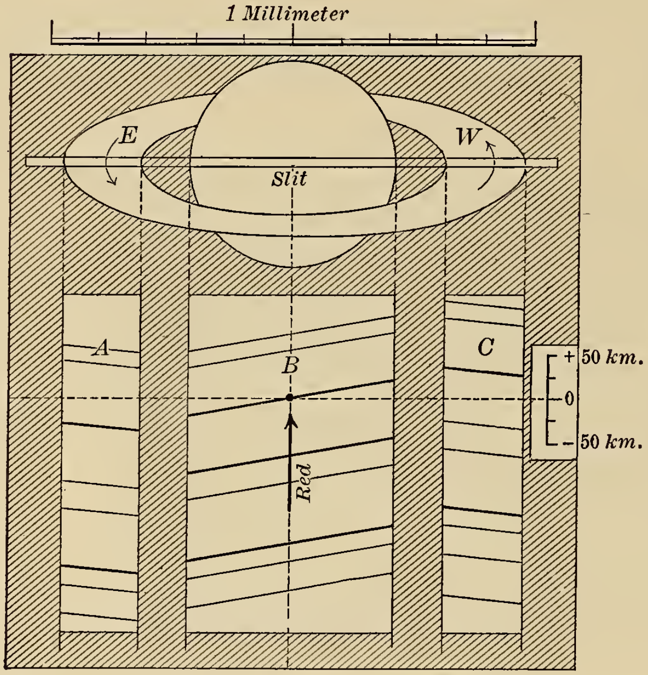


Fig. 152. — Spectroscopic Observation of Saturn's Ring
Keeler

Effect of reflection on shift of spectrum lines.

At the western limb of the planet and the western extremity of the ring the motion of rotation carries the particles *from* us, and the displacement of the spectrum lines should be towards the red, according to Doppler's principle; moreover, since the particles shine by *reflected* sunlight, the displacement is practically *doubled* at the time of the planet's opposition, — *twice as great as if the particles were self-luminous.* On the eastern side there is an equal shift towards the violet. Now, on looking at the diagram of the spectrum (given below the planet), we see that while at *C* the line in the spectrum is bodily displaced towards the red, as it ought to be, *the displacement at the outer edge of the ring is less than that at the inner*, and correspondingly at *A*. This shows that the particles at the outer edge are moving more slowly than at the inner.

The fact is made conspicuous by its effect upon the *inclination* of the lines: while in the spectrum of the ball the lines slope upwards towards the right, in the ring spectrum on both sides they slope the other way.

At the inner edge of the ring the observations indicated a velocity of 12½ miles a second, at the outer edge only 10, — precisely the velocities that satellites of Saturn ought to have at the corresponding distances from the planet.

It may be noted also that the inclination of the lines on the ball indicates at the edge of the planet a velocity of 6.4 miles a second, corresponding to a rotation period of $10^h 14^m.6$, — almost exactly agreeing with that deduced by Professor Hall from the observation of the spot.

The observations are extremely delicate, as the whole width of the spectrum was not quite a millimeter, the figure being magnified nearly fifty times. But Keeler's results have since been fully confirmed by Deslandres, Belopolsky, and Campbell.

Mass of rings insensible.

The investigations of Hermann Struve show that the *mass* of the rings is inappreciable; they produce no distinctly observable effect upon the motion of the satellites. To use his graphic expression, they seem to be composed of "immaterial light," — mere dust films or wreaths of fog.

Question of stability of the rings.

451. Stability of the Ring. — If the ring were solid, it certainly would not be stable; it could not endure the strains due to its rotation, nor is it certain that even the swarmlike structure makes it forever secure. There have been strong suspicions of a change in the width of the rings and their divisions, but the latest measurements hardly confirm the idea. It is not, however, improbable that the ring may ultimately be broken up.

It can hardly be doubted that the details of the ring are continually changing to some extent; thus, the outer ring, *A*, is occasionally divided into two by a very narrow black line known as "Encke's division," though more usually there is merely a darkish streak upon it not amounting to a real break in the surface. When the rings are edgewise notable irregularities are observed upon them, as if they were not accurately plane nor quite of even thickness throughout. Irregularities are reported also in the form of the shadow cast by the planet on the rings, indicating that the ring surface is not entirely flat.

But caution must be used in accepting and interpreting such observations, because illusions are very apt to occur from the least indistinctness of vision or faintness of light. Generally speaking, the writer has found that the better the seeing, the fewer abnormal appearances are noted, and the experience of other observers with large telescopes is the same.

Discovery of satellites.

452. Satellites. — Saturn has eight (possibly nine) of these attendants, the largest of which, named Titan, was discovered by Huyghens in 1655. It is easily seen with a 3-inch telescope.

D. Cassini, with his long-focus telescope (Sec. 44, note), found four others before 1700; Sir W. Herschel in 1789, with his great 4-foot reflector, discovered the two which are nearest the planet; and in 1848 the elder Bond added an eighth.

As the order of discovery does not agree with that of distance, it has been found convenient, in order to avoid confusion, to adopt names for the satellites (suggested by Sir John Herschel). They are, beginning with the most remote,

Their names.

Iapĕtus, (Hyperion), Titan ; Rhea, Dione, Tethys ; Enceladus, Mimas.

Leaving out Hyperion (which had not been discovered when the names were first assigned), they form a line and a half of a regular Latin pentameter.

Peculiarities of Iapetus.

The range of the system is enormous. Iapĕtus is at a distance of 2 225000 miles, with a period of seventy-nine days, — nearly as long as that of Mercury. On the western side of Saturn this satellite is always much brighter than at the eastern, showing that, like our own moon, it always keeps the same face towards the planet, — one half of its surface being darker than the other.

Titan.

Titan, as its name suggests, is by far the largest. Its distance is about 770000 miles and its period a little less than sixteen days. It is probably 3000 or 4000 miles in diameter. Its mass is found, from the perturbations produced by it in the motion of the other satellites, to be $\frac{1}{4600}$ of Saturn's.

The orbit of Iapetus is inclined about 10° to the plane of the rings, but all of the other satellites move sensibly in their plane, and all the five inner ones in orbits sensibly circular.

Early in 1899 Prof. W. H. Pickering announced the discovery of a ninth satellite (to which he has assigned the name of *Phœbe*), found on photographs made at Arequipa, the southern annex of the Harvard College Observatory. The data were not sufficient to determine its orbit further than that the satellite must be at a distance of several millions of miles, — much exceeding that of Iapetus, and with a period much exceeding a year.

Phœbe, a probable ninth satellite.

No confirmation has, however, yet been obtained from photographs since made. The satellite, if it exists, is too small to be *seen* with any existing telescope; but we can photograph what is not *visible*.

URANUS

453. Discovery of Uranus. — Urănus was the first *planet* ever "discovered," and the discovery created great excitement and brought the highest honors to the astronomer. It was found accidentally by the elder Herschel on March 13, 1781, while "sweeping" the heavens for interesting objects with a 7-inch reflector of his own construction. He rĕcognized it at once by its disk as something different from a star, but never dreaming of a new planet supposed it to be a peculiar kind of comet; its planetary character was not demonstrated until nearly a year had passed, when Lexell of St. Petersburg showed by his calculations that it was doubtless a planet beyond Saturn, moving in a nearly circular orbit.

Discovery of Uranus by Herschel in 1781.

It is easily visible to a good eye on a moonless night as a star of the sixth magnitude.

The name of Uranus, suggested by Bode, finally prevailed over other appellations that were proposed (Herschel had called it the "Georgium Sidus," in honor of the king).

Previous observations of the planet.

It was found on reckoning backward that the planet had been many times observed as a star and had barely missed discovery on several previous occasions. Twelve observations of it had been made by Lemonnier alone, and later they proved extremely valuable in connection with the investigations which led to the discovery of Neptune.

Data relating to its orbit.

454. Orbit of Uranus. — The *mean distance* of the planet from the sun is 19.2 astronomical units, or 1782 000000 miles. The *sidereal period* is eighty-four years and the *synodic* 369½ days, the annual advance of the planet among the stars being only a little over 4¼°. The *eccentricity* of the orbit is about the same as that of Jupiter, the sun being 83 000000 out of the center of the orbit. The *inclination* of the orbit to the ecliptic is only 46′. The light and heat received from the sun are only about $\frac{1}{370}$ of that received by us.

Diameter and bulk of the planet.

Its mass and density.

Albedo and spectrum.

455. The Planet itself. — In the telescope it shows a greenish disk about 4″ in diameter, though the measurements of See make it only 3″.3, corresponding to a diameter of only 28500 miles, which is 3400 miles less than that hitherto generally accepted and given in the tables of the Appendix. If we admit the correctness of this new measure, the *volume* comes out only forty-seven times that of the earth as against the sixty-five of the tables. The *mass* is determined much more accurately than the diameter (by the motion of its satellites) and is about 14.6 times that of the earth, the *density* of the planet (still accepting See's diameter) being 0.31 of the earth and its *surface gravity* a little greater than ours, 1.11. The *albedo* of the planet is very high, 0.62 according to Zöllner, even higher than that of Jupiter. The *spectrum* exhibits strong dark bands in the red, due, doubtless, to some unidentified substance in the planet's probably dense atmosphere. They explain the greenish tinge of the planet's light.

The planet's disk as determined by various observers about 1882, when the plane of the satellites' orbits was directed towards the earth, was obviously oval, indicating an oblateness of about $\frac{1}{14}$. At present (1902) the pole is presented to us and the disk appears round. There are no *distinct* markings on the disk, but there are faint traces of belts, which appear to lie not exactly in the plane of the satellites, but at an angle of some 15° or 20°. They are too indistinct, however, to warrant any positive assertion. Nothing has yet been observed from which the rotation of the planet can be determined.

Oblateness.

Belts.

456. Satellites. — The planet has four satellites, — Ariel, Umbriel, Titania, and Oberon, Ariel being the nearest to the planet.

The four satellites.

The two brightest, Oberon and Titania, were discovered by Sir William Herschel, who thought he had discovered four others also; he may have glimpsed Ariel and Umbriel, but it is very doubtful. They were first certainly discovered and observed by Lassell in 1851.

These satellites, especially the two inner ones, are telescopically the smallest bodies in the solar system and the hardest to see, excepting perhaps the fifth satellite of Jupiter. In real size they are, of course, much larger than the satellites of Mars, very likely measuring from 200 to 500 miles in diameter.

Their orbits are sensibly circular, and all lie in one plane, which ought to be, and probably is, coincident with the plane of the planet's equator; but the belts raise questions. They are very "close packed" also, Oberon having a distance of only 375000 miles and a period of $13^d 11^h$, while Ariel has a period of $2^d 11^h$ at a distance of 120000 miles. Titania, the largest and brightest, is at a distance of 280000 miles, a little greater than that of the moon from the earth, with a period of $8^d 17^h$.

Their orbits.

The most remarkable point about this system remains to be mentioned. The plane of their orbits is inclined 82°.2 to the plane of the ecliptic, and in that plane they revolve *backwards;* or we may say, what comes to the same thing, that their orbits are inclined to the ecliptic at an angle of 97°.8, in which case their revolution is to be considered as direct.

Great inclination of the orbits. Backward revolution of satellites.

NEPTUNE

Intractability of Uranus.

457. Discovery of Neptune. — This is reckoned as the greatest triumph of mathematical astronomy since the days of Newton. It was very soon found impossible to reconcile the old observations of Uranus by Lemonnier and others with any orbit that would fit the observations made in the early part of the nineteenth century, and, what was worse, the planet almost immediately began to deviate from the orbit computed from the new observations, even after allowing for the disturbances due to Saturn and Jupiter. It was misguided by some unknown influence to an amount almost perceptible by the naked eye; the difference between the actual and computed places of the planet amounted in 1845 to the "intolerable quantity" of nearly two minutes of arc.

Minuteness of the discrepancy between theory and observation.

This is a little more than one half the distance between the two principal components of the double-double star, ε Lyræ, the northern one of the two little stars which form the small equilateral triangle with Vega (Fig. 190, Sec. 585). A very sharp eye can perceive the duplicity of ε without the aid of a telescope.

One might think that such a minute discrepancy between observation and theory was hardly worth minding, and that to consider it "intolerable" was putting the case very strongly, but in science unexplained "residuals" are often the seeds from which new knowledge springs. Just these minute discrepancies supplied the data which sufficed to determine the position of a great world, before unknown.

De minimis curat Scientia.

Mathematical discovery of Neptune by Leverrier.

As the result of a most skilful and laborious investigation, Leverrier, a young French astronomer, wrote in substance to Galle, then an assistant in the Observatory at Berlin:

"*Direct your telescope to a point on the ecliptic in the constellation of Aquarius, in longitude 326°, and you will find within a degree of that place a new planet, looking like a star of about the ninth magnitude, and having a perceptible disk.*"

The planet was found at Berlin on the night of Sept. 23, 1846, in exact accordance with this prediction, within half an hour after the astronomers began looking for it and within 52′ of the precise point that Leverrier had indicated.

Optical discovery by Galle, 1846.

The English Adams fairly divides with Leverrier the honors for the mathematical discovery of the planet, having solved the problem and deduced the planet's approximate place even earlier than his competitor. The planet was being searched for in England at the time when it was found in Germany. It had, in fact, been already twice observed, and the discovery would necessarily have followed in a few weeks, upon the reduction of the English observations. Leverrier died in 1877, Adams in 1892. Galle alone of the discoverers survives, the aged Emeritus Professor and Director of the University Observatory at Breslau.

Share of Adams in the discovery.

458. Error of the Computed Orbit. — Both Adams and Leverrier, besides calculating the planet's position in the heavens, had deduced elements of its orbit and a value for its mass, which turned out to be seriously incorrect. The reason was that they assumed that the new planet's mean distance from the sun would follow Bode's Law, a supposition quite warranted by all the facts then known, but which, nevertheless, is not even roughly true. As a consequence, their computed *elements* were erroneous, and that to an extent which has led high authorities to declare that the mathematically computed planet was not Neptune at all, and that the discovery of Neptune itself was simply a "happy accident."

Error of computed orbit due to the assumption of Bode's Law, which here breaks down.

This is not so, however. While the data and methods employed were not by themselves sufficient to determine the planet's *orbit* with accuracy, they were adequate to ascertain the planet's *direction* from the earth; the computers informed the observers where to point their telescopes, and this was all that was necessary for finding the planet. In a similar case the same thing could be done again.

Method used gives planet's direction from earth.

459. The Planet and its Orbit.—The planet's *mean distance* from the sun is a little more than 2800 000000 miles (instead of

Data relating to planet's orbit.

being over 3600 000000, as it should be according to Bode's Law). The orbit is very nearly circular, its *eccentricity* being only 0.009. Even this, however, makes a variation of over 50 000000 miles in the planet's distance from the sun. The *period* of the planet is about 165 years (instead of 217, as it should be according to Leverrier's computed orbit) and the orbital velocity is about $3\frac{1}{3}$ miles per second. The *inclination* of the orbit is about $1\frac{3}{4}°$.

Telescopic appearance of Neptune.

Diameter of the planet smaller than generally accepted hitherto.

Neptune appears in the telescope as a small star of between the eighth and ninth magnitudes, absolutely invisible to the naked eye, but easily seen with a good opera-glass, though not distinguishable from a star with a small instrument. Like Uranus, it shows a greenish disk, having an apparent diameter, according to the measures of H. Struve, of $2''.2$. The measures of earlier observers were all much larger, and until very recently the value $2''.6$ was generally accepted, and is for the present allowed to stand in the Appendix tables. Recent measures of Struve, Barnard, and See all concur, however, in showing that this value is much too large.

Mass and density.

Accepting Struve's measures, the diameter comes out only 29750 miles, and its volume fifty-three times that of the earth; but the margin of possible error must be still quite large. The *mass*, as determined from its satellite, is about seventeen times that of the earth. Its *density* (according to Struve's diameter) comes out 0.34, and the surface gravity one and one-fourth times our own.

The planet's *albedo*, according to Zöllner, is 0.46, — a trifle less than that of Saturn and Venus.

Albedo and spectrum.

There are no visible markings upon its surface, and nothing certain is known as to its rotation.

The spectrum of the planet appears to be like that of Uranus, but of course is rather faint.

It will be noticed that Uranus and Neptune form a "pair of twins," very much as the earth and Venus do, being almost alike in magnitude, density, and many other characteristics.

460. Satellite. — Neptune has one satellite, discovered by Lassell within a month after the discovery of the planet itself. Its distance is about 223000 miles and its period 5^d21^h. Its orbit is inclined to the ecliptic at an angle of 34° 48′ and it moves *backward* in it from east to west, like the satellites of Uranus. It is a very small object, not quite as bright as Oberon, the outer satellite of Uranus. From its brightness, as compared with that of Neptune itself, its diameter is estimated as about the same as that of our own moon.

Neptune's satellite.

461. The Sun as seen from Neptune. — At Neptune's distance the sun has an apparent diameter of only a little more than one minute of arc, — about the diameter of Venus when nearest us, and too small to be seen as a disk by the naked eye, if there be eyes on Neptune. The light and heat there are only $\frac{1}{900}$ part of what we get at the earth. Still, we must not imagine that the Neptunian sunlight is feeble as compared with starlight, or even with moonlight. At the distance of Neptune the sun gives a light nearly equal to 700 full moons, — about eighty times the light of a standard candle at one meter's distance, — and is abundant for all visual purposes. In fact, as seen from Neptune, the sun would look very like a 1200 candle-power electric arc at a distance of only 12 or 13 feet.

Sunlight on Neptune.

462. Ultra-Neptunian Planets. — Perhaps the breaking down of Bode's Law at Neptune may be regarded as an indication that the solar system ends there, and that there is no remoter planet; but of course it does not make it certain. If such a planet of any magnitude exists, it is sure to be found sooner or later, either by means of the disturbances it produces in the motion of Uranus and Neptune, or else by the methods of the asteroid hunters, — although its slow motion will render its discovery in that way difficult. Quite possibly it may come in a few years as the result of the photographic star-charting operations now in progress.

Possible planets beyond Neptune.

EXERCISES

1. When Jupiter is visible in the evening do the shadows of his satellites precede or follow the satellites as they cross the planet's disk?

2. On which limb, the eastern or the western, do the satellites appear to enter upon the disk?

3. What probable effect would the great mass of Jupiter have upon the size of animals inhabiting it, if there were any?

4. How would sunlight upon Saturn compare with sunlight on the earth? How with moonlight?

5. What would be the greatest elongation of the earth from the sun as seen from Jupiter? from Saturn? from Uranus?

6. What would be the apparent angular diameter of the earth when "transiting" the sun as seen from Jupiter?

7. What is the rate in miles per hour at which a white spot on the equator of Jupiter, showing a rotation period of 9^h50^m, would pass a dark spot indicating a period of 9^h55^m?

8. Find the diameter, volume, density, and surface gravity of Neptune, accepting See's measured diameter of the planet, viz., $2''.01$, taking the planet's mass as 17 times that of the earth, the solar parallax as $8''.80$, and the mean distance of Neptune from the sun as 30.055.

Ans. { Diameter, 27200 miles.
Volume, 41 times the earth.
Density, 0.42.

CHAPTER XV

METHODS OF DETERMINING THE PARALLAX AND DISTANCE OF THE SUN

Importance and Difficulty of the Problem — Historical — Classification of Methods — Geometrical Methods — Oppositions of Mars and Certain Asteroids, and Transits of Venus — Gravitational Methods

463. In some respects the problem of the sun's distance is the most fundamental of all that are encountered by the astronomer. It is true that many important astronomical facts can be ascertained before it is solved: for instance, by a method which has been given in Sec. 371, we can determine the *relative* distances of the planets and form a map of the solar system, *correct in all its proportions*, although the unit of measurement is still undetermined, — *a map without any scale of miles.* But to give the map its use and meaning, we must ascertain the scale, and until we do this we can have no true conception of the real dimensions, masses, and distances of the heavenly bodies as compared with our terrestrial units of mass and distance. Any error in the assumed value of the astronomical unit propagates itself proportionally through the whole system, not only solar but stellar.

Relative distances in solar system easily determined.

The difficulty of the problem equals its importance. It is no easy matter, confined as we are to our little earth, to reach out into space and stretch a tape-line to the sun. In Secs. 173 and 442 we have already given the two methods of determining the sun's distance, which depend on our experimental knowledge of the velocity of light. They are satisfactory and sufficient for the purposes of the text. But methods of this kind have become available only since 1849.

Importance and difficulty of determining absolute distances.

Previously astronomers were confined entirely to purely astronomical methods, depending either upon geometrical measurement of the distance of one of the nearer planets when favorably situated, or else upon certain gravitational relations which connect the distance of the sun with some of the irregular motions of the moon, or with the earth's power of disturbing her neighboring planets, Venus and Mars.

Solar parallax 3′, according to Ptolemy.

464. Historical. — Until nearly 1700 no even approximately accurate knowledge of the sun's distance had been obtained. Up to the time of Tycho it was assumed on the authority of Ptolemy, who rested on the authority of Hipparchus, who in his turn depended upon an observation of Aristarchus (erroneous, though ingenious in its conception), that the sun's horizontal parallax is 3′, — a value more than twenty times too great.

Proof that this must be too great by Kepler and Cassini.

Kepler, from Tycho's observations of Mars, satisfied himself that the parallax certainly could not exceed 1′, and was probably much smaller; and at last, about 1670, Cassini also, by means of observations of Mars made simultaneously in France and South America, showed that the solar parallax could not exceed 10″.

Value approximately correct obtained from transits of Venus in 1761 and 1769.

The transits of Venus in 1761 and 1769 furnished data that proved it to lie between 8″ and 9″, and the discussion of all the available observations, published by Encke about 1824, gave as a result 8″.5776, corresponding to a distance of about 95 000000 miles. The accuracy of this determination was, however, by no means commensurate with the length of the decimal, and its error began to be obvious about 1860. It is now practically settled that the true value lies somewhere between 8″.75 and 8″.85, the sun's mean distance being between 92 400000 and 93 500000 miles. Indeed, it is now certain that the figure 8″.8, adopted in the text, must be extremely near the truth.

Classification of methods.

The methods available for determining the distance of the sun may be classified under three heads, — *geometrical*, *gravitational*, and *physical*. The physical methods (by means of the velocity of light) have been already discussed (Secs. 173 and 442). We proceed to present briefly the principal methods that belong to the two other classes.

GEOMETRICAL METHODS

465. The *direct* geometrical method of determining the sun's distance and parallax (by observing the sun itself at stations widely separated on the earth, in the same way that the distance of the moon is found — Sec. 196) is practically worthless. The parallax of the sun being only 8″.8, the inevitable errors of the best direct observation would be far too large a fraction of the quantity sought. Moreover, the sun, on account of the effect of its heat upon an instrument, is a very unsatisfactory object to observe.

Direct geometrical method useless.

Since, however, *we can compute at any time the distance of any planet from the earth in astronomical units*, it will answer every purpose to measure the distance *in miles* to any one of them.

466. Observations of Mars. — When Mars comes nearest to the earth its distance from us can be measured with reasonable accuracy in either of two ways:

Indirect determination by observations of Mars; two methods.

(1) By observations from *two or more stations widely separated in latitude.*

(2) By observations of the planet from a *single station near the equator* when the planet is *near its rising and setting.*

In the first case the observations may be (*a*) *meridian-circle* observations of the planet's zenith-distance, exactly such as are used for getting the distance of the moon (Sec. 196); or (*b*) they may be *micrometer or heliometer measures* of the difference of declination between the planet and stars near it; or, finally, (*c*) instead of measuring this distance directly *photographs* may be taken and measured later.

First method objectionable in requiring two observers and instruments.

Since, however, at least two different observers and two different instruments are concerned in the observations, the results are less trustworthy than those obtained by the second method.

Second method: a single observer taking advantage of the earth's rotation.

467. In this case a single observer, by measuring (best *with a heliometer* — Sec. 72) the apparent distance between the planet and small stars nearly east and west of it at the times when the planet is near the horizon, can determine its parallax with great accuracy.

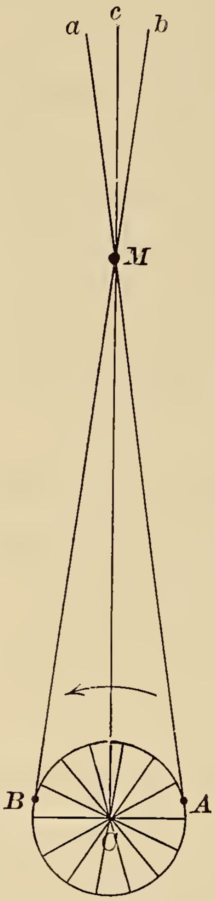


Fig. 153

Fig. 153 illustrates the principle involved.

When the observer at *A* (a point on or near the earth's equator) sees the planet *M* just rising, he sees it at *a* (a point on the celestial sphere *east* of *c*, the point where it would be seen from the center of the earth), the angle *CMA* being the planet's equatorial horizontal parallax.

Twelve hours later, when the rotation of the earth has taken the observer to *B* and the planet is setting, he sees it at *b*, displaced by parallax just as much as before, but to the *west* of *c*, its geocentric place. In other words, when the planet is rising the parallax *increases* its right ascension, and when setting *diminishes* it.

Measures of planet's distance from neighboring stars.

Suppose, now, that for the moment the orbital motion of the planet and the earth are suspended, the planet being at opposition and as near the earth as possible. If, then, when the planet is rising we measure the apparent distance, M_eS (Fig. 154), from a star, *S*, and twelve hours later measure it again, the distance M_e to M_w will be *twice the horizontal parallax of the planet.* The earth's rotation will have carried the observer a long journey, transporting him and his instrument, without disturbance, expense, or trouble, to a (virtually) different station 8000 miles away.

In practice the observations, of course, cannot be made at the moment when the planet is exactly on the horizon, but they are kept up during the whole time while the planet is crossing the heavens. Moreover, measures are made not from one star only, but from all that are in the planet's neighborhood. The orbital motion, both of the planet and of the earth, during the observations must also be allowed for; but this presents no serious difficulty.

Observations of Gill at Ascension Island.

The most important application of this method was in 1877, at Ascension Island, by Gill (now Sir David Gill of the Cape of Good Hope Observatory). He got for the solar parallax 8″.783 ± .015. The size of the planet's disk, its brightness, and its "phase" (except at the moment of opposition), however, interfere somewhat with the precision of the necessary measurements, and the great difference of brightness between it and the stars makes it difficult to use *photography*.

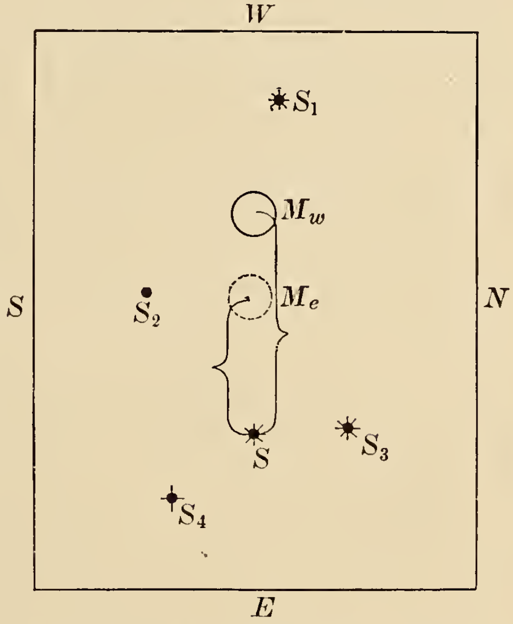


FIG. 154. — Micrometric Comparison of Mars with Neighboring Stars

Parallax by observations of asteroids.

468. Observations of Asteroids. — The asteroids do not present the same difficulties in determining their apparent places among the stars by heliometer measures or photography, since they themselves are mere starlike points. Although none of them (except Eros) come quite as near to the earth as Mars, several of them come near enough to make it possible to obtain from their observations results notably more satisfactory than those from Mars.

The heliometer observations of Iris, Sappho, and Victoria, made by Gill at the Cape of Good Hope in 1889–91, in concert with several other heliometer observers in Europe and America, gave 8″.802 ± .005. In this case, of course, the method used was neither (1) nor (2) of Sec. 466 exclusively. The apparent displacements of the planets, due both to the distance between the stations and to the motion of the stations on the whirling earth, all contribute to the result, complicating the calculation, but increasing its precision.

Eros observed in 1900–01.

We have already spoken of the case of Eros (Sec. 427). The observations of 1900–01, largely photographic, *ought*, when they have been thoroughly discussed, to give a result even more precise than that last quoted. In 1931, if the weather is good for a few days at the critical time when the planet is nearest, the opportunity will be still more favorable.

469. Transits of Venus. — When Venus is at or near inferior conjunction her distance is less than that of Mars at opposition; but she cannot be observed for parallax in the same way, because she is then in the twilight, and little stars near her cannot be seen for use as reference points. Now and then, however, she passes between us and the sun and "transits" the disk, as explained in Sec. 405. Her distance from the earth is then only 26 000000 miles, and her parallax is much greater than that of the sun. Seen by two observers at different stations on the earth, she will therefore appear to be projected on two different points of the sun's disk, and her apparent angular displacement on the sun's surface will be the *difference* between her own parallactic displacement (corresponding to the distance between the two stations) and that of the sun itself. This *relative* displacement is $\frac{723}{277}$, or 2.61, times as great as the displacement of the sun.

Transits of Venus: her displacement on sun's disk is 2.61 times her parallax.

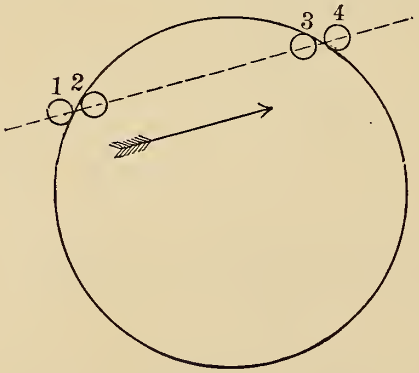


Fig. 155. — Contacts in a Transit of Venus

General principle by which solar

To determine the solar parallax, then, by means of the transit of Venus, we must somehow measure the apparent distance *in seconds of arc* between two positions of Venus on the sun's disk, *as seen simultaneously* from two widely distant stations of known latitude and longitude on the earth's surface.

parallax is determined from transit.

The methods earliest proposed and executed depend upon observations of the *instant of contact* between the planet and the sun's limb. There are four of these contacts, as shown in Fig. 155, the first and fourth *external*, the second and third *internal.*

Methods depending on observations of the contacts.

470. Halley's Method, or the Method of Durations. — Halley was the first to notice, in 1679, the peculiar advantages of the transit of Venus as a means for determining the distance of the sun, and he proposed a method which consists in simply observing the *duration* of the transit at stations chosen *as far apart in latitude* as possible.

Halley's method: durations observed from stations far apart in latitude.

This had the great advantage of not requiring an accurate knowledge of the *longitudes* of the stations, which in his time would have been very difficult to determine, nor did it require any knowledge of the *absolute*, or Greenwich, time of contact. It is only necessary to know the latitudes of the observers and that their timepieces should run accurately during the short time (five or six hours) while the planet is crossing the sun's disk. On the other hand, the stations must be in high latitudes, and the observer must see both beginning and end of the transit; if he loses either on account of clouds, the method fails.

Advantages and disadvantages.

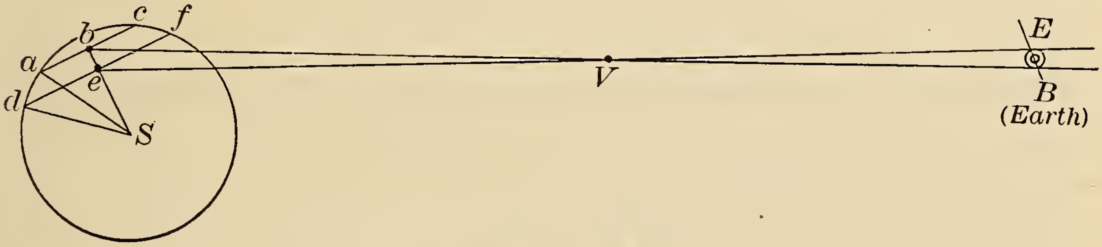


Fig. 156. — Halley's Method

Fig. 156 illustrates in a general way the principle involved: the two observers at E and B see the planet crossing the sun's disk on the chords df and ac, respectively, and from the duration

Observations should give length of transit chords with great accuracy.

and known rate of motion of the planet the length of the two chords *in seconds of arc* can be computed with more accuracy than it can be determined by any micrometer measure, provided the instant of contact can be accurately observed. But, since the angular semidiameter of the sun is known, the distances bS and eS of the two chords from the center of the sun can be computed, and their difference, eb (all in seconds of arc), and that with very great accuracy if the chords fall, as they have done in all the transits yet observed, near the edge of the sun's disk.

But, since VE and Ve are in the proportion of 277 to 723, eb (in miles) is $\frac{723}{277}$ of EB, provided the two stations are so chosen that the line EB is perpendicular to the plane of the planet's orbit (if not, due allowance must, and can, be made to get the "effective length" of EB).

We have, then, eb, both in seconds and in miles; we know, therefore, how many miles go to one second of arc at the sun's distance. It comes out about 450, and therefore (Sec. 10, eb taking the place of r) the sun's distance is about 450 miles × 206265, or 92 800000 miles.

For details of the methods of accurate calculation from the actual observations, the reader must be referred to works dealing with the special subject.

Expected accuracy not realized.

Halley expected to depend mainly on the two internal contacts, which he supposed could be observed with an error not exceeding a single second of time. If so, the observations would determine the sun's parallax within $\frac{1}{500}$ of its true value.

Effect of the planet's atmosphere.

Unfortunately, this accuracy is not found practicable. There are usually large errors caused by the imperfection of the telescope and eye of the observer, as well as atmospheric conditions. And even if these are avoided, the atmosphere of the planet introduces a difficulty that cannot be evaded; it produces a luminous ring around the edge of the planet (Sec. 401, Fig. 137), which prevents any certainty as to the precise moment when the planet's disk is tangent to the limb of the sun. The contact observations during the last two transits in 1874 and 1882 were uncertain, under the very best conditions, by at least five or six seconds.

471. Delisle's Method. — Halley's method requires stations in high latitudes, uncomfortable and hard to reach, and so chosen that both the beginning and end can be seen. And both *must* be seen or the method fails.

Delisle's method: observations of absolute instants of contact from equatorial stations widely separated.

Delisle's method, on the other hand, employs pairs of stations *near the equator*, but as nearly as possible on opposite sides of the earth, and it does not require that the observer should see both the beginning and end of the transit, — observations of either phase can be utilized for their full value, which is a great advantage, — but it requires that the *longitudes* of the stations should be known with extreme precision, since the method consists essentially in observing the *absolute time* of contact (*i.e.*, Greenwich or Paris time at both stations). It is beautifully simple and easy to understand.

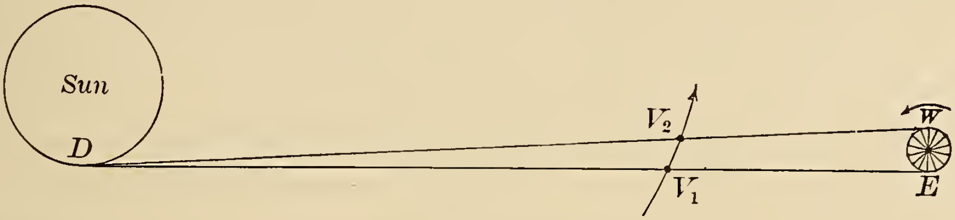


FIG. 157. — Delisle's Method

An observer at E on the equator (Fig. 157) on one side of the earth notes the moment of internal contact in Greenwich time, the planet being then at V_1; when W, on the other side of the earth, notes the contact (also in Greenwich time) the planet will be at V_2, and the angle V_1DV_2 is the earth's apparent diameter as seen from the sun, *i.e.*, *twice the sun's horizontal parallax* (Sec. 78). Now the angle at D is at once determined by the time occupied by Venus in moving from V_1 to V_2. It is simply *just the same fraction of 360° that this elapsed time is of 584 days*, the planet's synodic period. If, for example, the

Theoretical simplicity of the method.

difference of time were eleven and a half minutes between the contact as observed at E and W, we should find the angle at D to be about $17''.7$.

From all the *contact observations*, several hundred in number, made during the transits of 1874 and 1882, Newcomb gets a solar parallax of $8''.794 \pm .018$.

Parallax from heliometric and photometric measures.

472. Heliometric and Photographic Observations. — Instead of observing merely the four contacts and leaving the rest of the transit unutilized, we may either keep up a continued series of measurements of the planet's position upon the sun's disk with a heliometer, or we may take a series of photographs to be measured up at leisure. Such heliometer measures or photographs, taken in connection with the recorded Greenwich times at which they were made, furnish the means of determining just where the planet appeared to be on the sun's disk at any given moment, as seen from the observer's station. A comparison of these positions with those simultaneously occupied by the planet, as seen from another station, gives at once the means of deducing the parallax.

Unsatisfactory result.

In 1874 and 1882 several hundred heliometer measures were made (mostly by German parties), and about six thousand photographs were obtained at stations in all quarters of the earth where the transits could be seen, — more than two thousand by the different American parties. The final result of all these observations is given by Newcomb as $8''.857 \pm .023$, — differing to an unexpected degree from the figures given by other methods, and seriously discordant among themselves, as shown by the large probable error.

It looks as if measurements of this sort must be vitiated by some constant source of error as yet undetected.

On the whole, the outcome of the two transits has been to satisfy astronomers that other methods of determining the sun's parallax are to be preferred, as Leverrier maintained in 1870. It is hardly likely that transits will ever again be observed so elaborately and expensively.

473. Gravitational Methods. — The scope of our work makes it impossible to give any more than a very elementary explanation of the principles involved, since the details of investigation belong to a higher range. Of the different methods of this class we mention two only:

Gravitational methods.

(1) By the moon's *parallactic inequality*, so called because by it the sun's parallax can be determined.

By parallactic inequality of the moon.

It depends upon the fact that the sun's disturbing effect upon the moon is sensibly greater in the half of the moon's orbit nearest the sun (*i.e.*, the quarter on each side of new moon) than it is in the remoter half; and the difference depends upon the *ratio between the radius of the moon's orbit around the earth to that of the earth's orbit around the sun.* If that ratio can be determined, the radius of the earth's orbit comes out in terms of the distance of the moon from the earth, which is accurately known.

As a consequence of this difference of the sun's disturbing force on the two halves of the orbit, the moon at the end of the first quarter is about two minutes of arc ($120''$) behind the place she would occupy if there were no such inequality in the disturbing force. At the third quarter (a week after full moon) she is as much ahead.

If the moon's place could be observed as accurately as that of a star, this method would stand extremely high for precision; but the observational difficulties are serious, and the difficulty is much increased by the fact that at the first quarter we are obliged to observe the *western* limb of the moon, and at the third the *eastern*. Still, the result obtained from it agrees very well with that from the other methods.

474. (2) The second method is by the perturbations produced by the earth on the orbits of Venus and Mars (and we may now add Eros). The method depends upon the principle that the amount of these perturbations depends upon the *ratio* of the *mass of the earth (including the moon) to that of the sun;* and, further, that when this ratio of masses is known the distance of the sun follows at once from a simple equation, easily deduced.

By perturbations produced by the earth.

From Sec. 381, equation (1), we have, changing a few letters (putting $(S + E)$ for $(M + m)$ and D for r),

$$(S + E) = \frac{4\pi^2}{G} \times \frac{D^3}{T^2},$$

in which S and E are the masses of the sun and earth, G is the constant of gravitation, D is the mean distance of the earth from the sun, and T the number of seconds in a year.

Deduction of parallax from the ratio between the masses of earth and sun.

Also, for the force of gravity at the earth's surface, we have $g = G \times \frac{E}{r^2}$, whence $E = \frac{g}{G} \times r^2$, r being the radius of the earth.

Dividing the preceding equation by this, we get

$$\frac{S + E}{E} = \frac{4\pi^2}{gT^2}\left(\frac{D^3}{r^2}\right),$$

whence, $$D^3 = \left(\frac{S + E}{E}\right)\left(\frac{gr^2T^2}{4\pi^2}\right).$$

If we put $\frac{S}{E} = M$, this becomes $D^3 = \left(\frac{M + 1}{4\pi^2}\right) gT^2r^2$, in which everything is known if M is determined, g being given by pendulum observations and r by measurements of the earth's dimensions, while T is the length of the sidereal year in seconds.

The matter can also be treated differently, bringing out the sun's distance in terms of the *distance and period of the moon*, instead of g and r.

Advantages of the gravitational method. Its precision cumulative.

The great beauty of the gravitational method lies in this, — that as time goes on and the effects of the earth upon the nodes and apsides of the neighboring orbits accumulate, the determination of the earth's mass in terms of the sun's becomes continually and cumulatively more precise. Even at present the method ranks high for accuracy, — so high that Leverrier, who first developed it, would, as already mentioned, have nothing to do with the transit-of-Venus observations in 1874, declaring that all such old-fashioned ways are absolutely valueless. By this method Newcomb deduces a parallax of $8''.768 \pm 0''.010$.

475. It is to be noticed that the *geometrical* methods give the parallax of the sun *directly*, apart from all hypothesis or assumption, except as to the accuracy of the observations themselves, and of their necessary corrections for refraction, etc. The *gravitational* methods, on the other hand, assume the exactness of the law of gravitation; and the *physical method* (by the velocity of light) assumes that light travels in space with the same velocity as in our terrestrial experiments, after allowing for the retardation due to the refracting power of the air. The near accordance of the results obtained by the different methods shows that these assumptions must be very nearly correct, though perhaps not absolutely so.

Accordance of results obtained by the different methods.

We add a little table giving the distance of the sun corresponding to different values of the solar parallax, assuming the equatorial radius of the earth to be 3963.3 miles.

Distance of sun corresponding to different values of the parallax.

8″.75 corresponds to 23573 equatorial radii of the earth = 93 428000 miles.
8″.80 " " 23439 " " " " " = 92 897000 "
8″.85 " " 23307 " " " " " = 92 372000 "

Newcomb, in his *Astronomical Constants* (1896) adopts *8″.797* ± *0″.007* as the value of the solar parallax to be used in the planetary tables of the American ephemeris.

He also gives the following as the results derived by the various methods *after making allowance for probable systematic errors*, and assigns to each result the weight indicated by the number that follows it.

Newcomb's summary of parallax results.

Motion of the Node of Venus	8″.768,	10
Gill's Observations of Mars (1877)	8 .780,	1
Pulkowa Constant of Aberration (20″.492) . . .	8 .793,	40
Contact Observations of Transit of Venus	8 .794,	3
Heliometer Observations of Victoria and Sappho . .	8 .799,	5
Parallactic Inequality of the Moon	8 .794,	10
Miscellaneous Determinations of Aberration (20″.463)	8 .806,	10
Lunar Inequality of the Earth	8 .818,	1
Measures of Venus in Transit	8 .857,	1

Harkness, in his *Solar Parallax and its Related Constants* (1891), obtained as his final value 8″.809 ± 0″.006.

CHAPTER XVI

COMETS

Their Number, Designation, and Orbits — Their Constituent Parts and Appearance — Their Spectra — Physical Constitution and Behavior — Danger from Comets

General appearance of comets.

476. The comets are bodies very different from the stars and planets. They appear from time to time in the heavens, remain visible for some weeks or months, pursue a longer or shorter path, and then fade away in the distance. They are called *comets* (from *coma*, *i.e.*, "hair"), because when one of them is bright enough to be seen by the naked eye it looks like a star surrounded by a luminous fog and usually carries with it a long stream of hazy light.

Large comets are magnificent objects, sometimes as bright as Venus and visible by day, with a dazzling nucleus and a nebulous head as large as the moon, accompanied by a train which extends half-way from the horizon to the zenith, and sometimes is really long enough to reach from the earth to the sun. Such are rare, however; the majority are faint wisps of light, visible only with the telescope.

Fig. 158 is a representation of Donati's comet of 1858, one of the finest ever seen.

In ancient times comets were always regarded with terror, either as actually exerting malignant influences, or at least ominous of evil, and the notion still survives in certain quarters, although the most careful research fails to show, or even suggest, the slightest reason for it.

Their number.

477. Number of Comets. — Thus far, up to the beginning of the new century, our lists contain nearly *eight hundred*, about four hundred of which were observed before the invention of the telescope in 1609, and therefore must have been bright. Of those observed since then, only a small proportion have been

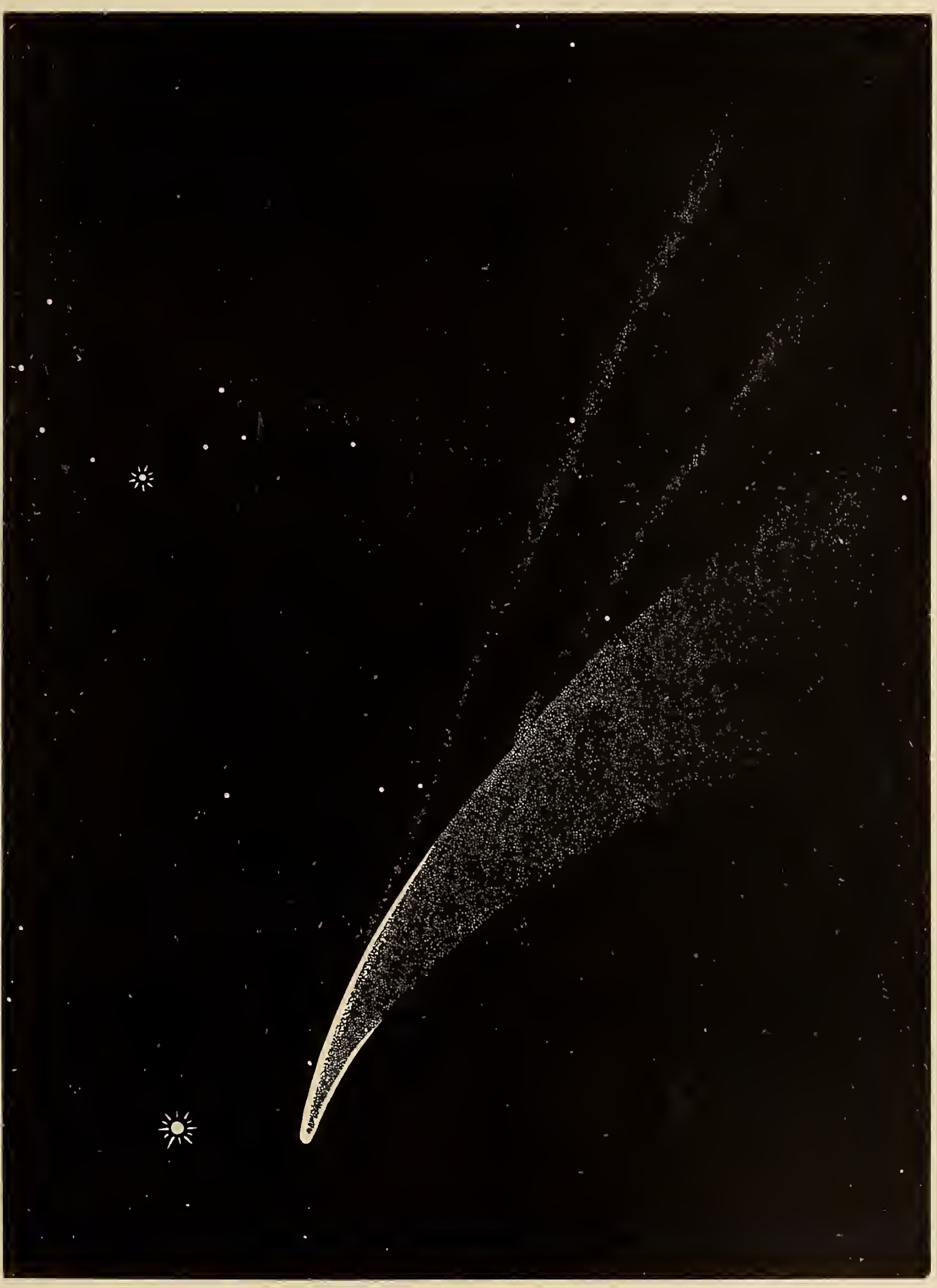


FIG. 158. — Naked-Eye View of Donati's Comet, Oct. 4, 1858

Bond

conspicuous to the naked eye, — perhaps one in five. During the first half of the present century there were nine of this rank, and in the last half four, five of which were notable. The last really brilliant comet appeared in 1882.

Since then there have been four or five that could be seen without a telescope, but only one bright enough to attract the notice of the casual observer, — Rordame's comet of 1893. In August, 1881, for a few days two conspicuous comets — one magnificent one, and the other more than respectable — shone together in the northern heavens not very far apart, a thing almost, if not quite, unprecedented.

The total number that visit the solar system must be enormous, since, although even with the telescope we can see only the comparatively few which come near the earth and are favorably situated for observation, yet not infrequently from five to eight are discovered in a single year (ten in 1898); and there is seldom a day when one is not present somewhere in the sky: often there are several.

Methods of designating comets.

478. Designation of Comets. — A remarkable comet generally bears the name of its discoverer or of some one who has acquired its "ownership," so to speak, by some important research respecting it. Thus, we have Halley's, Encke's, and Donati's comets. The common herd are distinguished only by the year of discovery, with a letter indicating the order of *discovery* in that year, as comet *a*, *b*, *c*, 1895; or, still again, by the year, with a Roman numeral denoting the order of perihelion passage. Thus, Donati's comet, which is "comet *f*, 1858," is also "comet 1858–VI," and this is the more scientific designation, and is generally used in catalogues of comets.

Comet *a* is not, however, always comet I, for comet *b* may outrun it in reaching the perihelion, and it often happens that a comet's perihelion passage does not occur in the same year as its discovery.

In some cases a comet bears a double name, as, for instance, the Pons-Brooks comet, which was first discovered by Pons in 1812, and on its return in 1883, by Brooks.

479. Discovery of Comets. — As a rule, these bodies are first found by comet hunters, who make a business of searching for them. For this purpose they usually employ a telescope known as a "comet-seeker," carrying an eyepiece of low power, with a large field of view. When first seen a comet is usually a mere roundish patch of faintly luminous cloud, which, if really a comet, will reveal its true character within an hour or two by its motion.

Their discovery.

Some observers have found a great number of these bodies. Messier discovered thirteen between 1760 and 1798, and Pons twenty-seven between 1800 and 1827. In this country Brooks, Barnard, and Swift have been especially successful. It occasionally happens, however, as with Holmes' comet of 1892, and Rordame's comet of 1893, that a comet is picked up with the naked eye by some one not an astronomer at all.

Recently three have been discovered by photography, the first by Barnard at the Lick Observatory in 1892, the second by Chase in 1898 while trying to photograph November meteors, and the third by Coddington in 1899, also at the Lick Observatory.

480. Duration of Visibility, and Brightness. — The comet of 1811 was observed for seventeen months; the great comet of 1861 for a year; and comet 1889–I was followed at the Lick Observatory for nearly two years, — the longest period of visibility yet recorded. On the other hand, the comet is sometimes visible only a week or two, and twice a comet near the sun has been photographed during a total eclipse, — never seen before or after the two minutes of totality. The average duration of visibility is probably not far from three months.

Duration of visibility.

As to *brightness*, comets differ widely. About one in four or five reaches the naked-eye limit at some point in its orbit, and a very few, say two or three in a century, are bright enough to be seen in the daytime. The comets of 1843 and 1882 were the last so observed.

Their brightness.

THEIR ORBITS

Ancient ideas as to their motion.

481. The ideas of the ancients as to the motions of these bodies were very vague. Aristotle and his school considered them to be merely exhalations from the earth, inflamed in the upper air, and therefore meteorological bodies, and not astronomical at all. Seneca, indeed, held a more correct opinion, but it was shared by few; and Ptolemy fails to recognize them as heavenly bodies in his *Almagest.*

Tycho establishes their astronomical character.

Tycho was the first to establish their rank as truly "celestial," by comparing the observations of the comet of 1577, made in different parts of Europe, and showing that its parallax was less, and its distance greater, than that of the moon.

Kepler supposed that they moved in straight lines and seems to have been more than half disposed to consider them as living beings, traveling through space with will and purpose, "like fishes in the sea."

Suggestion of parabolic orbits by Hevelius.

Hevelius in 1675 was the first to suggest that their orbits might be parabolas, and his pupil Doerfel proved this to be the case in 1681 for the comet of that year. The theory of gravitation had now appeared, and Newton soon worked out and published a method by which the elements of a comet's orbit can be determined from the observations.

Relative numbers of parabolic, elliptical, and hyperbolic orbits.

482. Relative Numbers of Parabolic, Elliptical, and Hyperbolic Orbits.—A large majority move in orbits that are sensibly *parabolas.* Out of nearly four hundred orbits computed up to 1901, more than three hundred are of this kind. About eighty-five are more or less distinctly *elliptical*, and about half a dozen seem to be *hyperbolas*, but hyperbolas differing so slightly from the parabola that the hyperbolic character is not *certain* in a single one of the cases.

Comets which have elliptical orbits of course return, if undisturbed, at regular intervals; the others visit the sun only once, and never come back.

Difficulty of recognizing a comet when it returns.

The difficulty of determining whether a particular comet is or is not periodic is much increased by the fact that comets have no characteristic "personal appearance," so to speak, by which a given individual can be recognized whenever seen, — as Jupiter or Saturn could be, for instance. It is necessary to depend almost entirely upon the elements of its orbit for the recognition of a returning comet, and this is not always satisfactory, since there are a number of cases in which several distinct comets move in orbits almost identical. (See Sec. 487.)

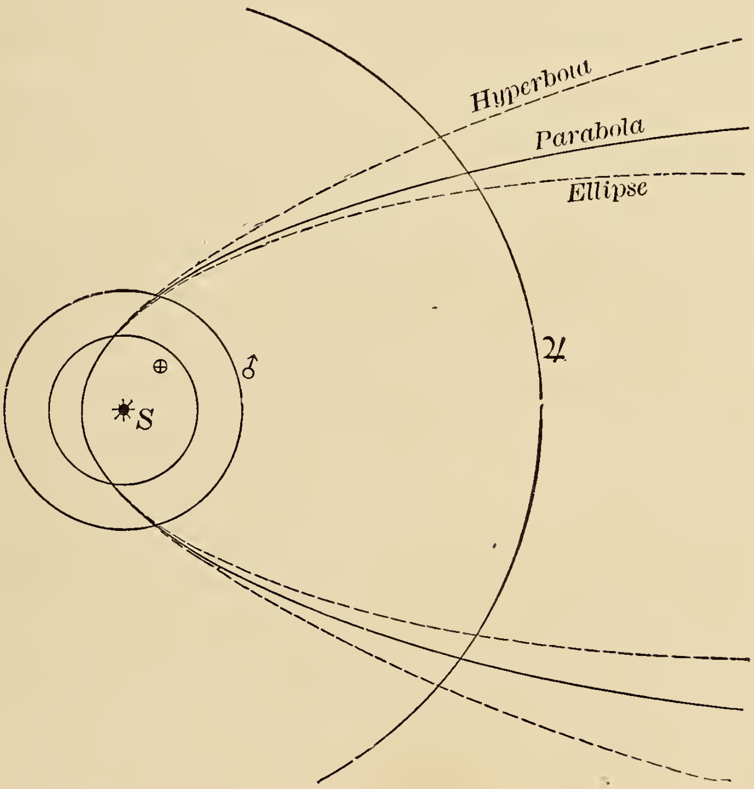


FIG. 159. — The Close Coincidence of Different Species of Cometary Orbits within the Earth's Orbit

Elements of a comet's orbit determined by three complete observations.

483. Elements of a Comet's Orbit. — As in the case of a planet, *three* perfect observations of a comet's place are theoretically sufficient to determine its entire orbit. Practically, however, it is not possible to observe a comet with anything like the accuracy of a planet (on account of its indefinite outline), nor usually with sufficient exactness to determine positively from a small number of observations whether the orbit is or is not parabolic.

Uncertainty as to major axis and period.

The *plane* of the orbit and its *perihelion distance* can, in most cases, be fairly settled without any difficulty; but the *eccentricity* and the *major axis*, with its corresponding *period*, require a long series of observations for their determination and are seldom ascertained with much precision from a single appearance of the comet. In that part of the comet's path which can be observed from the earth the three kinds of orbits usually diverge but little; indeed, they may almost coincide (as shown in Fig. 159).

Parabolic orbit has but five elements.

For a *parabolic* orbit the elements to be computed are only *five* in number, instead of seven, as in the case of an ellipse. The *semi-major axis* and *period* (which are infinite) drop out, as does the *eccentricity*, which is necessarily unity. To define the size of the orbit the perihelion distance, p, takes the place of the semi-major axis.

For the parabolic elements we have, therefore, (1) p, perihelion distance, (2) i, inclination of the orbit to the ecliptic, (3) ☊, the longitude of the ascending node, (4) ω, angle between line of nodes and perihelion, (5) T, date of perihelion passage.

It must be distinctly understood, moreover, that orbits which are "sensibly" parabolic are seldom, if ever, strictly so, — the chances are infinity to one against an exact parabola. If a comet were moving at any time in such a curve, the slightest retardation due to the disturbing force of any planet would change this parabola into an ellipse, and the slightest acceleration would make an hyperbola of it.

Effect of slight change of comet's velocity upon major axis and period in cases of orbits nearly parabolic.

It should be noted also that if a comet's orbit is *nearly* parabolic, a very slight change in the velocity of the comet's motion will cause an enormous change in the computed major axis and period. This is obvious from the equation (Sec. 320)

$$a = \frac{r}{2}\left(\frac{U^2}{U^2 - V^2}\right).$$

When V nearly equals U (as must be the case if the orbit is nearly a parabola) the denominator will be extremely small, and a very trifling change in V will make a great *percentage* of change in the difference between U^2 and V^2, and will affect a, the semi-major axis, accordingly.

Elliptic orbits.

Halley's comet.

484. The Elliptic Comets. — The first comet ascertained to move in an elliptical orbit was that known as Halley's, which has a period of about seventy-six years, its periodicity having been discovered by Halley in 1681. It has since been observed in 1759 and 1835 and is due again about 1911.

Encke's comet.

The second of the periodic comets in order of discovery is Encke's, with the shortest period known, less than three and one-half years. Its periodicity was discovered in 1819.

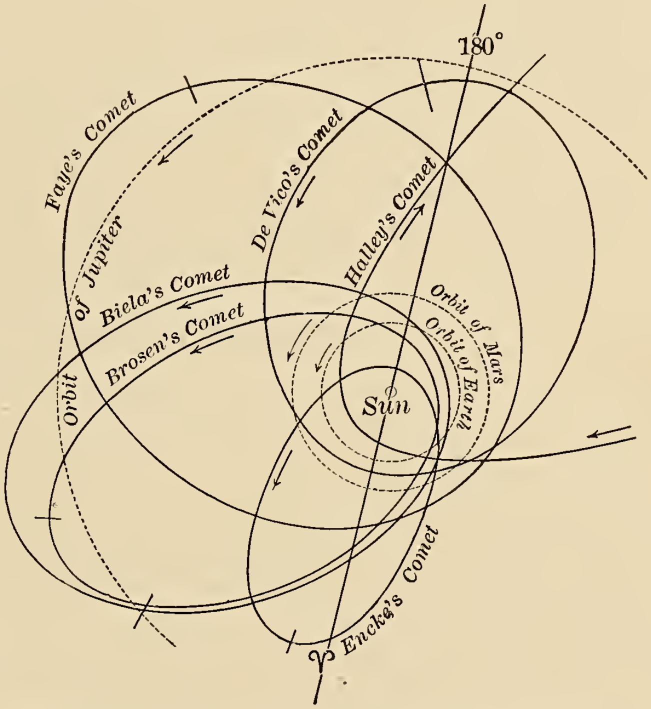


FIG. 160. — Orbits of Short-Period Comets

About twenty comets of which return has been observed.

About a dozen of the comets to which computation assigns elliptic orbits have periods so long — near or exceeding one thousand years — that their real character is still rather doubtful. About seventy-five, however, have orbits which are distinctly and certainly elliptical, and about sixty of them have periods of less than one hundred years. About twenty have been actually observed at two or more returns to perihelion; as to the rest of the sixty, several are now expected within a few years, and many have probably been lost to observation, either from disintegration, like Biela's comet (soon to be discussed), or by having their orbits transformed by perturbations, so that they no longer come within the range of observation.

Relation of short-period comets to Jupiter.

Fig. 160 shows the orbits of five of the short-period comets (as many as can be shown without confusion) and also a part of the orbit of Halley's comet. These five particular comets, and about twenty-five more, all have periods ranging from three and one-half to eight years, and they all pass very near the orbit of Jupiter. Moreover, each comet's orbit crosses that of Jupiter *near one of its nodes*, marked by a short cross line on the comet's orbit. The fact is extremely significant, showing that these comets at times come very near to Jupiter, and it points to an almost certain connection between that planet and these bodies.

Comet-families; origin of elliptic comets.

485. Comet-Families; Origin of Periodic Comets.—It is clear, as has been said, that the comets which move in parabolic orbits cannot well have originated within the limits of the solar system, but must have come from a great distance. As to those which move in elliptical orbits, it is a question whether we are to regard them as native to the system or only as "naturalized," or perhaps mere sojourners for a time; but it is evident that in some way many of them stand in peculiar relations to Jupiter and to other planets.

The comet-families of Jupiter, Saturn, Uranus, and Neptune.

The short-period comets, those which have periods ranging from three to eight years, are now recognized and spoken of as Jupiter's *family of comets*. About thirty are known already, of which fifteen have been observed twice or oftener,—some of them a dozen times. Similarly, Saturn is credited with two comets; Uranus with two, one of which is Tempel's comet, closely connected with the November meteors and due to appear in 1900, but not seen. Finally, Neptune has a family of six; among them Halley's comet, and two others which have returned a second time to perihelion since 1880.

The capture theory.

486. The Capture Theory.—The now generally accepted explanation as to the origin of these *comet-families* was first suggested by Laplace; viz., that the comets which compose them have been "captured" by the planet to which they stand related.

A comet entering the system in a parabolic orbit and passing near the planet will be disturbed and either accelerated or retarded. If it is accelerated, then according to equation (4) (Sec. 320), the major axis will become negative, the orbit will be changed to an hyperbola, and the comet will never be seen again. But if the comet is *retarded*, the semi-major axis will become finite and the orbit will be made *elliptical*, so that the comet will return at each revolution to the place where it was first disturbed; it will become a *periodic comet*, with its orbit passing near to the orbit of the disturbing planet.

Effect of retardation or acceleration to transform a parabolic orbit.

It will not, however, as students sometimes imagine, revolve around its capturer like a satellite. The focus of its new and diminished orbit still remains at the sun.

But this is not all. After a certain time the planet and the comet will be sure to come together again at or near this place. The result then *may be* an acceleration which will enlarge the comet's orbit, or even transform it to a parabola or hyperbola; but it is an even chance at least that the result may be a *retardation* and that the orbit and period may thus be further diminished. This may happen over and over again, until the planet's orbit falls so far inside that of the planet that it suffers no further disturbance to speak of.

Subsequent encounters of comet and planet.

Given time enough and comets enough, the ultimate result would necessarily be such a comet family as really exists. It is not *permanent*, however; sooner or later, if a captured comet is not first disintegrated, it will almost certainly encounter its planet under such conditions as to be thrown out of the system.

A recent investigation, however, by Callandreau, upon the disintegration of comets by the action of the sun and the planet Jupiter, shows that the limit of distance at which such an effect is possible is quite considerable, and that the breaking up of a comet ought not to be very unusual. He suggests that

Callandreau's investigation on the breaking up of comets.

the number of the comets of Jupiter's family has probably thus been largely increased by the division of single comets into several, — a suggestion which greatly relieves very serious objections that have been urged against the capture theory.

Comet-groups

487. Comet-Groups. — There are several instances in which a number of comets, certainly distinct, chase each other along almost exactly the same path, at an interval usually of a few months or years, though they sometimes appear simultaneously. The most remarkable of these *comet-groups* is that composed of the great comets of 1668, 1843, 1882, and 1887. These have all come in from the direction (nearly) of Sirius and have receded nearly on that line, passing close around the sun and actually *through the corona*. As Professor Comstock has pointed out, they are all of them, if their computed orbits can be trusted, now (1902) bunched together in a space hardly bigger than the sun, at a distance of about 150 radii of our orbit, and are moving away together very slowly.

Members of a comet-group have a common origin; to be carefully distinguished from comet-families.

It is, of course, nearly certain that the comets of such a group have a common origin, perhaps from the disruption of a single comet by the attraction of the sun or a planet, in accordance with the suggestion of Callandreau just mentioned.

The distinction between comet-*families* and comet-*groups* must be carefully noted: in the former the orbits agree only in passing close to that of the capturing planet; in the latter the orbits are nearly identical, at least in the part near the sun.

Perihelion distance of comets.

488. Perihelion Distance, etc. — The *perihelion distances* of comets differ greatly. Eight of the 300 computed orbits approach the sun within less than 6 000000 miles, and four have a perihelion distance exceeding 200 000000. A single comet, that of 1729, had a perihelion distance of more than four astronomical units, or 375 000000 miles; this is one of the half dozen possibly hyperbolic comets, and must have been an enormous one to be visible at such a distance. There may, of course, be any number of comets with still greater perihelion distances, because, as a rule, we are able to see only such as come reasonably near to the earth's orbit,—probably but a small percentage of the total number that visit the sun. It has been computed that something like six thousand come within the orbit of Jupiter every year.

Inclination of cometary orbits.

The *inclinations* of cometary orbits range all the way from zero to 90°, but most of the short-period comets have orbits of small inclination, as might be expected, since such comets would be much more likely to suffer capture than those that cross the planes of the planetary orbits at a high angle.

Direction of motion.

As regards the *direction of motion*, the six hyperbolic comets and all the elliptical comets having periods of less than one hundred years move *direct*, excepting only Halley's comet and Tempel's comet of 1866. The rest show no decided preponderance either way.

Comets are visitors to the solar system.

489. Comets are Visitors.—The fact that the orbits of most comets are sensibly parabolic, and that their planes have no evident relation to the ecliptic, apparently indicates (though it does not absolutely demonstrate) that these bodies do not in any proper sense belong to the solar system. They come to us with just the velocity they would have if falling towards the sun from an enormous distance, and they leave the system with a velocity which, if no force but the sun's attraction acts upon them, will carry them away to an infinite distance, or until they encounter the attraction of some other sun.

Their motions are just what might be expected of ponderable masses moving in empty space between the stars under the law of gravitation.

Difficulty with hypothesis that they come from stellar regions.

There are difficulties with the theory that the comets come to us from space *among the stars*, chiefly depending upon the now certain fact that the solar system is traveling at the rate of several miles a second (Sec. 543) and that, therefore, comets composed of matter *met* by us ought to have a relative velocity, with respect to the sun, so great as to produce numerous hyperbolic orbits, whereas we find few such, if any. Then, too, there ought to be a marked concentration of the axes of cometary orbits near the direction towards which the sun is moving.

The home of the comets.

While the investigations of the late Professor Newton of New Haven partially relieve the difficulty, astronomers still feel it; and many are disposed to think that our solar system, in its journey through space, is accompanied by far-distant, outlying clouds of nebulous matter, which are the source and original "home of the comets," to borrow Professor Peirce's expression.

Peculiar behavior of Encke's comet.

490. Acceleration of Encke's Comet. — With one remarkable exception, the motions of comets appear to be just what would be expected of masses moving in free space under the law of gravitation. The single exception is in the case of Encke's comet, which, since its first discovery in the last century (its periodicity was not discovered until 1819), has been continually quickening its speed and shortening its period. In 1819 its period was 1205 days. Between 1820 and 1860 each successive period shortened about two and one-half hours; from 1860 to 1870 the shortening was only one and three-fourths hours to each revolution, and since then it has increased to about two hours. The period at present is about fifty-four hours shorter than in 1819, and the mean distance from the sun is nearly a quarter of a million of miles less than then.

No perturbations by any known body will account for such an acceleration, and thus far no reasonable explanation has been suggested as even possible, except that something encountered in its motion through interstellar space retards the comet, just as air retards a rifle bullet.

Paradoxical increase of speed as result of resistance.

At first sight it seems almost paradoxical that a resistance should accelerate a comet's speed, but referring to Sec. 320 we see that any diminution of the velocity will also diminish the semi-major axis. This will reduce the period, which is proportional to $\sqrt{a^3}$, by a greater percentage than it will reduce the circumference of the orbit, which is simply proportional to a; as a consequence there will be an increase of velocity above what the comet had in the larger orbit. A comet gains more speed *by falling nearer to the sun* than it loses by the direct effect of the resistance. If this action continues without cessation, the ultimate result must be a spiral winding inward until the comet strikes the surface of the sun.

When this peculiar behavior was first discovered by Encke it was ascribed to the action of a resisting medium and adduced as proof of the existence of the "luminiferous ether." But since no other comets exhibit the same effect, and the effect upon Encke's comet itself varies in amount from time to time, it is now generally attributed to something encountered along the orbit of this particular body; possibly the passage through some cloud of meteors, or disturbances by some unknown body in the asteroidal regions.

THE COMETS THEMSELVES

Physical characteristics.

491. Physical Characteristics of Comets. — The orbits of these bodies are now thoroughly understood, and their motions are calculable with as much accuracy as the nature of the observations permit; but we find in their physical constitution and behavior some of the most perplexing and baffling problems in the whole range of astronomy, — apparent paradoxes which have not yet received a satisfactory explanation.

Non-planetary peculiarities.

While comets are evidently subject to the attraction of gravitation, as shown by their orbits, they also exhibit evidence of being acted upon by powerful *repulsive* forces emanating from the sun. While they shine partly by reflected light, they are also certainly *self-luminous*, their light being generated in a way not yet thoroughly explained. They are the *bulkiest* bodies known, except the nebulæ, in some cases thousands of times larger than the sun or stars; but in mass they are "airy nothings," and one of the smaller asteroids probably rivals the largest of them in weight.

492. The Constituent Parts of a Comet. — (*a*) The *essential* part of a comet — that which is always present and gives the comet its name — is the *coma*, or nebulosity, a hazy cloud of faintly luminous transparent matter.

The coma.

The nucleus.

(*b*) Next, we have the *nucleus*, which, however, makes its appearance only when the comet is near the sun, and is wanting in many comets. It is a bright, more or less starlike point near the center of the coma, and is usually the object "observed on" in noting a comet's place. In some cases the nucleus is double, or even multiple.

The train, tail, or beard.

(*c*) The *tail*, or *train*, is a stream of light which commonly accompanies a bright comet and is sometimes present even with a telescopic one. As the comet approaches the sun the tail follows it, but as the comet moves away from the sun it precedes, and by the ancients was then called the *beard*. Speaking broadly, the train is always *directed away from the sun*, though its precise form and position are determined partly by the comet's motion. It is practically certain that it consists of *extremely rarefied matter*, which is thrown off by the comet and powerfully repelled by the sun. It certainly is not — like the smoke of a locomotive or the train of a meteor — matter simply left behind.

Jets and envelopes.

(*d*) *Jets and Envelopes.* The head of a brilliant comet is often veined by jets of light, which appear to be spirted out from the nucleus; and sometimes it throws off a series of concentric envelopes like hollow shells, one within the other. These phenomena, however, are seldom observed in any but brilliant comets.

493. Dimensions of Comets. — The volume, or bulk, of a comet is often enormous, — almost beyond conception if the tail is included in the estimate. The head, or *coma*, is usually from 40000 to 150000 miles in diameter; a comet less than 10000 miles in diameter would stand little chance of discovery, and comets exceeding 150000 miles are rather unusual, though there are a considerable number on record.

Dimensions of heads of comets.

The head of the comet of 1811 at one time measured nearly 1 200000 miles, — more than forty per cent larger than the diameter of the sun itself. Holmes' comet of 1892 had at one time a diameter exceeding 700000 miles, but no visible nucleus at that time. A few weeks later it looked like a mere hazy star. The comet of 1680 had a head 600000 miles across, and that of Donati's comet of 1858 was 250000 miles in diameter.

Contraction of head when near the sun.

The diameter of the head changes all the time, and what is singular is, that while the comet is approaching the sun, the head ordinarily *contracts*, expanding again as it recedes. The diameter of Encke's comet shrinks from about 300000 miles when it is 130 000000 miles from the sun to a diameter not exceeding 12000 or 14000 miles when at perihelion, a distance of 33 000000 miles, the variation in bulk being more than 10000 to 1. No satisfactory explanation is known, but Sir John Herschel has suggested that the change may be merely optical, — that near the sun a part of the nebulous matter is evaporated by the solar heat and so becomes invisible, condensing and reappearing again when the comet reaches cooler regions.

Diameter of the nucleus.

The *nucleus* usually has a diameter ranging from a mere point less than 100 miles in diameter up to 5000 or 6000, or even more. Like the comet's head, it also changes in diameter, even from day to day. The variations, however, do not seem to depend in any regular way upon the comet's distance from the sun, but rather upon its activity in throwing off jets and envelopes.

Dimensions of the train.

Its usual form.

The *tail* of a comet, as regards simple magnitude, is by far its most imposing feature. Its length is seldom less than 5 000000 or 10 000000 miles; it frequently attains 50 000000, and there are several cases in which it has exceeded 100 000000. It is usually more or less fan-shaped, so that at the outer extremity it is millions of miles across, being shaped roughly like a cone projecting behind the comet from the sun, and more or less bent like a horn, as shown in Fig. 158. The volume of the train of

Mass of comets extremely small.

the comet of 1882, 110 000000 miles in length, some 200000 miles in diameter at the comet's head, and with a diameter of 10 000000 or 12 000000 at its extremity, exceeded the bulk of the sun itself more than eight thousand times.

494. Mass of Comets. — While the volume of comets is thus enormous, their *mass* is apparently insignificant, — in no case at all comparable even with that of our little earth.

Nature of the evidence.

The evidence on this point, however, is purely negative; it does not enable us in any case to determine how great the mass really is, but only *how great it is not; i.e.*, it only proves that the comet's mass is *less* than a certain very small fraction of the earth's, but does not warrant us in setting any lower limit.

The evidence is derived from the fact that no sensible perturbations have ever been produced in the motions of the planets or their satellites even when comets have come very near them; and yet in such a case the comet itself is "sent kiting" in a new orbit, showing that gravitation is fully operative between the comet and the planet.

Lexell's comet in 1770, and Biela's comet on several occasions, came so near the earth that the length of the comet's period was greatly changed, while the year was not altered by so much as a single second; and it would have been changed by many seconds if the comet's mass were as much as $\frac{1}{100000}$ of that of the earth.

Brooks' comet of 1886 actually passed between Jupiter and the orbit of its first satellite. None of the satellites were sensibly disturbed, but the comet's orbit was changed from an ellipse with a period of over thirty years to one of a period with less than seven.

May equal mass of an iron ball 150 miles in diameter.

At present this mass ($\frac{1}{100000}$ of the earth's mass) is very generally assumed as a probable *upper limit* for even a large comet. It is about ten times the mass of the earth's atmosphere and is about equal to the mass of a ball of iron 150 miles in diameter, but how much smaller the limit may really be no one can say.

495. Density of Comets. — The mean density is necessarily extremely low, the mass of the comet being so small and the volume so great. If the head of a comet 50000 miles in diameter has the very improbable mass of $\frac{1}{1000000}$ of that of the earth, its *mean density* is only about $\frac{1}{6000}$ part of that of the air at the earth's surface, — a degree of rarefaction reached by only the very best air-pumps.

Mean density of comets very low. Comparable with an air-pump vacuum.

The extremely low density of comets is shown also by their transparency. Small stars are often seen directly through the head of a comet 100000 miles in diameter, even very near its nucleus, and with hardly a perceptible diminution of luster. There are, however, in such cases indications of a very slight refraction of the light passing through the comet, causing a barely sensible displacement of the star.

Transparency of comets.

As for the tail, the density of this must be almost infinitely lower than that of the head, — far below the best vacuum we can make by any means of science. It is nearer to an airy nothing than anything else we know of.

Still lower density of the train.

Another point should be referred to. Students often find it hard to conceive how such impalpable "dust clouds" can move in orbits like solid masses and with such enormous velocities; they forget that in a vacuum a feather falls as swiftly as a stone. Interplanetary space is a vacuum, far more perfect than anything we can produce by artificial means, and in it the lightest bodies move as freely and swiftly as the densest, since there is nothing to resist their motion. If all the earth were suddenly annihilated, except a single feather, the feather would keep on and pursue the same orbit, with the unchanged speed of $18\frac{1}{2}$ miles a second.

Dust clouds traverse interplanetary space as swiftly as solid bodies.

496. Nature of Comets. — We must bear in mind, however, that the low *mean* density of the comet does not necessarily imply that the density of its constituent parts is small. A comet may be in the main composed of small heavy bodies and still have a very low *mean* density, provided they are widely

Low mean density not incompatible with high density of constituent parts.

enough separated. There is much reason, as we shall see, foı supposing that such is really the case, — that the comet is largely composed of small meteoric sand grains (say pinheads, many feet apart), each carrying with it a certain quantity of enveloping gas, in which light is produced either by electric discharges or by some different action due to the rays of the sun.

Comets probably swarms of small solid masses.

As to the size of the particles opinions vary widely: some maintain that they are large rocks; Professor Newton calls a comet a "gravel bank"; others think it a mere "dust cloud" or "smoke wreath."

The unquestionable and close connection between comets and meteors, which we shall soon discuss, almost compels some "meteoric hypothesis," and, at present at least, no other theory is maintained by any high authorities.

The light of comets not reflected sunlight though largely due to action of solar rays.

497. The Light of Comets. — To some extent this is reflected sunshine, but in the main it is light emitted by the comet itself under the stimulus of solar action. That the light depends in some way upon the sun is shown by the fact that its intensity follows approximately the same law as the brightness of a planet, and is usually proportional to $\frac{1}{R^2\Delta^2}$, in which R is the comet's distance from the sun and Δ its distance from the earth.

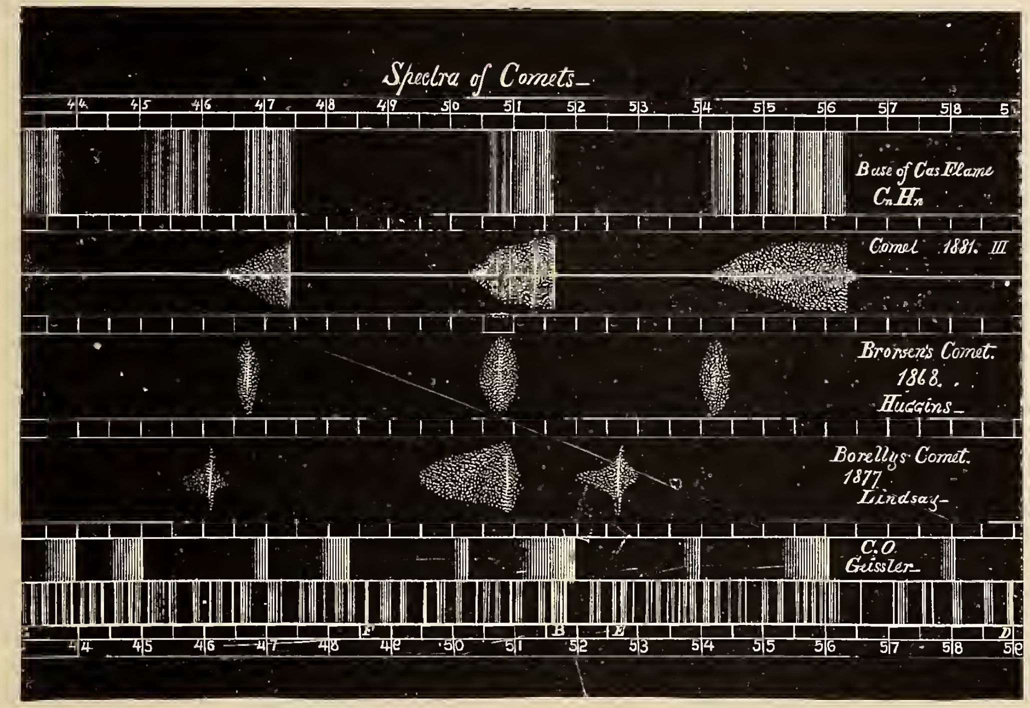
A comet as it recedes from the earth does not simply grow smaller, retaining the same apparent intrinsic brightness, as would be the case with an *independently* self-luminous body, but grows fainter and disappears on account of faintness.

Capricious variations.

Not infrequently, however, the light of a comet varies capriciously, brightening and fading without apparent cause, within a few days or even a few hours.

The ordinary comet spectrum.

498. Spectra of Comets. — The spectrum is usually a faint continuous spectrum, on which are superposed certain bright bands, five of them in the visible spectrum; there are others in the ultra-violet, observable only by photography. Of the five visible bands, two are very faint, so that ordinarily but three can be seen. The spectrum is identical with that of the blue cone at the base of a Bunsen-burner flame, which is always found where hydrocarbon gases are in a state of combustion, and is generally ascribed to *acetylene*. (See Fig. 161, comet, 1881–III.) The spectrum of a comet is *not* mainly, as sometimes stated in text-books, the spectrum of *carbon monoxid*, though some slight peculiarities in the comet's spectrum suggest the presence, at times, of this gas also.


Spectra of Comets_
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Base of Gas Flame
C_nH_n
Comet 1881. III
Brorsen's Comet.
1868.
Huggins_
Borellys Comet.
1877
Lindsay-
C.O.
Geissler_
F
b
E
D

Fig. 161. — Comet Spectra

For convenience in engraving, the *dark* lines of the solar spectrum in the lowest strip of the figure are represented as *bright*

The faint continuous spectrum is due, in part at least, to reflected sunlight, as shown by the fact that some of the principal Fraunhofer lines have been *photographed* in it, though they cannot be *seen*.

If the nucleus is bright, its spectrum also appears like a narrow streak, nearly continuous, running through the spectrum of the head, as shown in the figure. At least ninety per cent of all the comets thus far observed have given this hydrocarbon (acetylene?) spectrum.

Anomalous spectra.

If the comet is one that does not approach the sun within the distance of 100 000000 miles or so (such comets are not numerous), the hydrocarbon bands are sometimes missing, replaced in some cases by unidentified bands of a different wave-length, as in the case of Brorsen's comet and Borrelly's comet of 1877 (Fig. 161).

The spectrum of Holmes' comet of 1892, which never came inside the earth's orbit, showed no bands or lines at all, either bright or dark, but was simply continuous.

Bright lines in spectrum of 1882-II.

If, on the other hand, the comet approaches the sun within 8 000000 or 10 000000 miles, the hydrocarbon bands grow relatively faint, and the yellow line of sodium becomes dominant, as in Wells' comet, 1882-I, and the great comet, 1882-II.

The latter, indeed, which almost grazed the surface of the sun, showed numerous bright lines of other substances (probably *iron* for one).

It has been maintained by Sir Norman Lockyer that the comet's spectrum changes regularly and progressively with distance from the sun, the bands not only altering their appearance, but slightly shifting their position; but the evidence for this is not conclusive.

Question as to cause of luminosity.

As to the cause of luminosity, it is practically agreed that it cannot be due to any *general heating* of the mass of the comet, of which the mean temperature, on the contrary, is probably extremely low. The explanation now most favored attributes the light to electric discharges between the solid (?) particles through the gases which envelop them, — discharges due to *inductive* action of the sun on the "cometic" cloud rushing towards it from regions of space, where the electric potential is presumably different from that of the sun itself. At present we can assign no certain reason for such difference, but, on the other hand, there is not any known reason for assuming a uniform electric potential through all space. (See Sec. 502.)

Comet not mainly composed of hydrocarbons.

It is, perhaps, necessary to remark that while the hydrocarbon bands of the spectrum demonstrate the presence of hydrocarbons in the comet, they do not at all prove that the comet is *mainly* composed of them, nor even that they constitute a considerable portion of its *mass*. It is much more likely that the minute solid or liquid particles constitute ninety per cent of the whole.

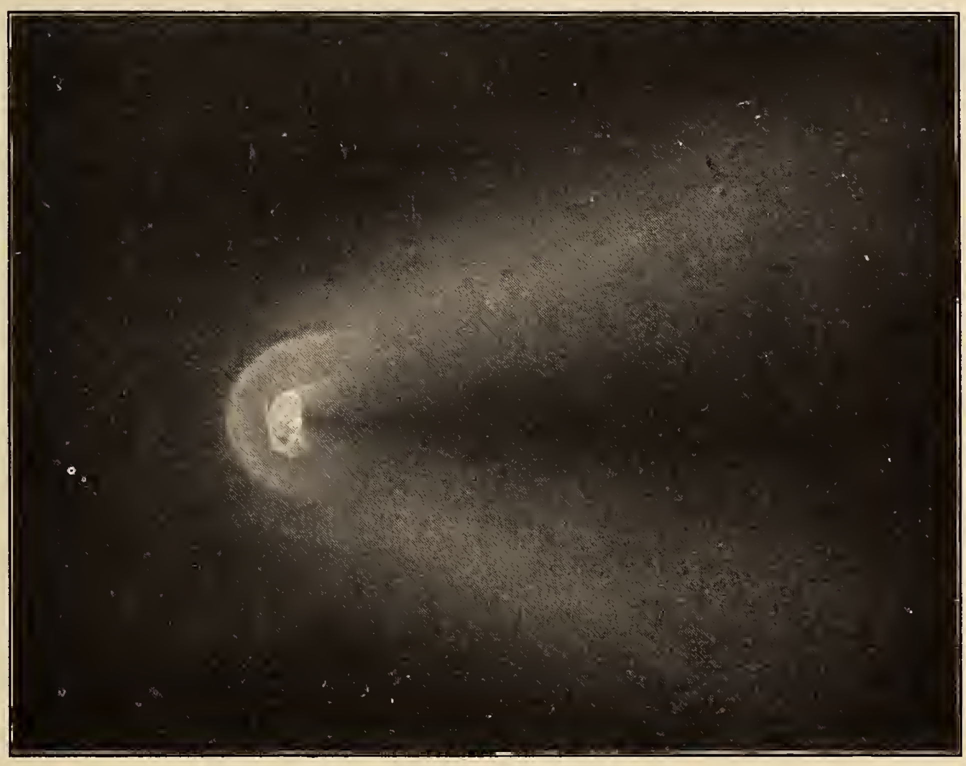

FIG. 162. — Head of Donati's Comet
Bond

Phenomena resulting from approach to the sun.

499. Phenomena that accompany the Comet's Approach to the Sun. — When a comet is first discovered it is usually, as has been already said, a mere round nebulosity, a little brighter near the middle. As it approaches the sun it brightens rapidly, and the nucleus appears. Then on the sunward side the nucleus appears to emit luminous jets, or to throw off more or less symmetrical envelopes, which follow each other at intervals of a few hours, expanding and growing fainter, until they are lost in the general nebulosity of the head.

Envelopes and jets.

Fig. 162 shows the envelopes as they appear in the head of Donati's comet of 1858. At one time seven of them were visible at once; very few comets, however, exhibit the phenomena with such symmetry. More frequently the emissions from the nucleus take the form of mere jets and streamers.

Formation of tail by repulsion from nucleus and sun.

500. Formation of the Tail. — The tail appears to be formed of material first projected from the nucleus towards the sun and afterwards repelled both by nucleus and sun, as illustrated by Fig. 163. At least, this theory has the great advantage over all others which have been proposed (there have been many of them) that it not only accounts for the phenomenon in a general way, but admits of being worked out in detail and verified mathematically, by comparing the actual size and form of the comet's tail at different points in the orbit with that indicated by theory; and the accordance is usually satisfactory.

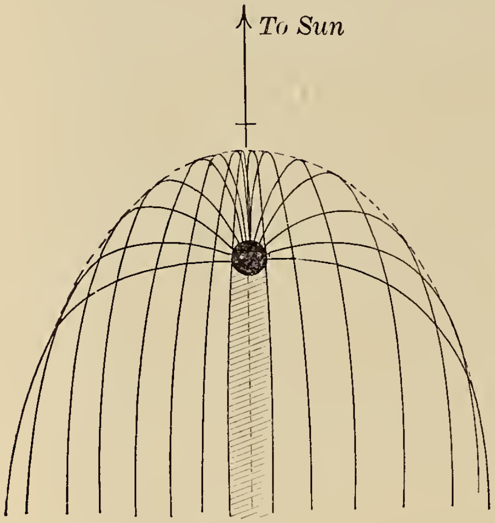

Fig. 163. — Formation of a Comet's Tail by Matter expelled from the Head

According to this theory, the tail is simply an assemblage of repelled particles, each moving in its own hyperbolic orbit[1] around the sun, the separate particles having very little connection with, or effect upon, each other and being almost entirely emancipated from the control of the comet's head.

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[1] Since the assumed repulsive force upon a particle virtually diminishes the sun's attraction upon it, it also virtually diminishes its parabolic velocity (*i.e.*, if under this diminished attraction the particle had fallen from an infinite distance, its parabolic velocity would be less than if gravitation had acted unmodified). In the formula of Sec. 320, U^2, if the comet is moving in a parabola, therefore becomes less than V^2 for the particles that compose the tail; and the semi-major axis, a, for the subsequent orbit of such particles, becomes *negative*, converting their orbits into hyperbolas.

Since the force of the projection from the comet is seldom very great, all these orbits lie nearly in the plane of the comet's orbit, and the result is that the tail is usually a sort of a flat, *hollow*, curved, horn-shaped cone, open at the large end. The edges of the tail, near the comet at least, therefore usually appear much brighter than the central part.

The tail usually a flattened, curved cone.

501. Curvature of the Tail, and Tails of Different Types. — The tail is curved, because the repelled particles, after leaving the comet's head and receding from the sun, retain their original motion, and in consequence are arranged, not along a straight line drawn from the sun to the comet, but on a curve convex to the direction of the comet's motion, as shown in Fig. 164, — the stronger the repulsion, the less the curvature.

Explanation of curvature.

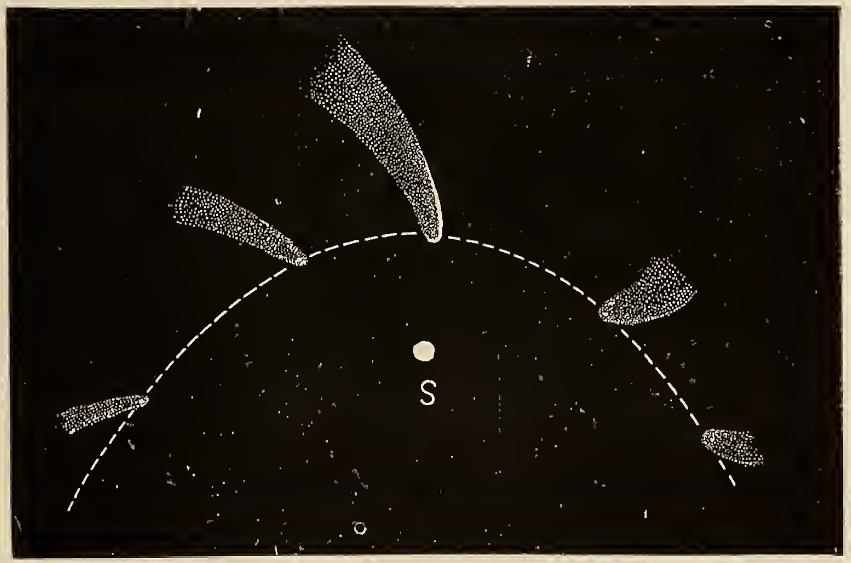

FIG. 164. — A Comet's Tail at Different Points in its Orbit near Perihelion

Bredichin of Moscow has found that in this respect the trains of comets may be classified under three different types:

The three types of comet's tails.

First, the *long*, *straight rays:* they are composed of matter upon which the solar repulsion is from twelve to fifteen times as

The hydrogenous tail.

great as gravitational attraction, so that the particles leave the comet with a relative velocity of 4 or 5 miles a second, which is afterwards continually increased until it becomes enormous. The nearly straight rays, shown in Fig. 158, tangent to the principal tail of Donati's comet, belong to this class. For plausible reasons, connected with the low density of hydrogen, Bredichin considers them to be composed of that substance, possibly set free by the decomposition of hydrocarbons. They are rather uncommon, and in no case since the promulgation of the theory have been bright enough to allow a spectroscopic test of their nature.

The hydrocarbon tail.

The second type is the curved, plumelike train, like the principal train of Donati's comet. In trains of this type, supposed to be due to hydrocarbon vapors, the repulsive force varies from 2.2 times the gravitational attraction for particles on the convex edge of the train to half that amount for those on the inner edge. Trains of this class show the hydrocarbon spectrum through all their extent.

Tails due to metallic vapors.

Third. A few comets show tails of still a third type, — short, stubby brushes, violently curved, and due to matter upon which the repulsive force is feeble as compared with gravity. These are assigned to metallic vapors of considerable density, sodium perhaps, possibly sometimes iron.

Nature of the repulsive force: the electrical theory.

502. The Repulsive Force. — The nature of the force which repels the particles of a comet is, of course, only a matter of speculation. There is probably at present a decided preponderance of opinion in favor of the idea that it is electrical. In this case the repulsion upon small particles, being a surface action, would be more effective in proportion as the particle was smaller, and this is in accordance with the apparent fact that the molecules of hydrogen, hydrocarbon gas, and metallic vapors are sorted out, so to speak, to form the three different types of tails.

But the experiments of Nichols and Hull in this country and of Lebedew in Russia, made independently in 1901, tend to confirm a long-standing surmise that it may be due to the *direct action* of the waves of solar radiation upon extremely small particles of matter.

Repulsion due to direct action of light-waves.

Maxwell, years ago, showed that as a consequence of his electromagnetic theory of light (then new, but now almost universally accepted), a particle receiving light-rays ought to be repelled by a force the amount of which he computed. For particles of sensible magnitude the calculated force is insignificant as compared with the solar attraction, but for particles — say, a hundred thousandth of an inch in diameter — it many times exceeds that attraction. Various unsuccessful attempts have been made to detect such a force experimentally, but at last the physicists seem to have overcome the difficulties, and their result practically agrees with Maxwell's prediction. This theory is supplementary to the electrical rather than contradictory, as the repulsive force of light is due to an electromagnetic reaction, and it is not unlikely that the particles repelled may carry electric charges.

Evaporation theory.

It has also been attempted to account for the repulsion by an *indirect* action resulting from the heating of the surfaces of the almost infinitesimal particles on the side next to the sun.

There is no reason to suppose that the matter driven off to form the tail is ever recovered by the comet. It probably remains in space, to be picked up by any large masses which the particles may meet.

Disintegration of comets.

Whenever a comet comes near to the sun or to one of the larger planets, it is subjected to forces which tend to pull it to pieces, and, as the mutual attraction between its particles is extremely feeble, it sometimes happens that it is separated into several portions, as was the case with Biela's comet in 1846, with the great comet of 1882, and with Brooks' comet of 1889. Indeed, it seems likely that all along its course it loses portions of its substance, so that at each successive return to perihelion it becomes smaller and finally ceases to exist as a recognizable "body," the scattered particles traveling by themselves until they fall upon some larger body as "shooting-stars."

Unexplained phenomena.

503. Unexplained and Anomalous Phenomena. — A curious phenomenon, not yet explained, is the dark stripe which in the case of a large comet nearing the sun runs down the center of the tail, looking very much as if it were a shadow of the comet's head. It is certainly not a shadow, however, because it usually makes more or less of an angle with the sun's direction. It is well shown in Fig. 162. When the comet is at a greater distance from the sun this central stripe is usually bright, as in Fig. 165. Indeed many, perhaps most, small comets, instead of the usual hollow, horn-shaped tail, show only this narrow streak, smaller in diameter than the comet's head, — as if the material repelled by the sun followed around the coma and left it only at the point remotest from the sun.

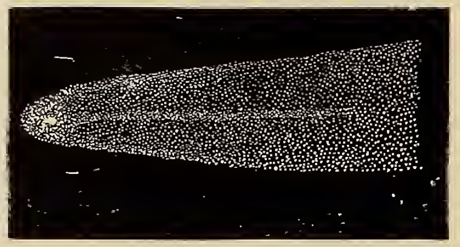

Fig. 165. — Bright-Centered Tail of Coggia's Comet, June, 1874

Not infrequently, however, comets possess *anomalous* tails, — usually in addition to the normal tail, but sometimes substituted for it, — tails directed sometimes straight towards the sun and sometimes nearly at right angles to that direction.

Peculiar features of the great comet of 1882.

The great comet of 1882 also carried with it for a time a faintly luminous "sheath," which seemed to envelop the comet itself and that portion of the tail near the head, projecting 2° or 3° forward towards the sun. For some days, moreover, it was accompanied by little clouds of cometary matter, which left the main comet, like smoke puffs from a bursting bomb, and traveled off at an angle until they faded away. None of these appearances *contradict* the theory outlined above, but they cannot be said to be explained by it, — evidently we have not yet the whole story.

Photography of comets.

504. Photography of Comets. — It is not unlikely that photography will give us light on the subject, for the sensitive plate reveals in the tail of the comet (not in the head) many interesting details which are wholly invisible to the eye; partly, it is likely, because of the cumulative action of the feeble light during a long photographic exposure, and partly, also, because the light of a comet's tail probably resembles that of the positive "brush" from a charged electrode in being very rich in ultra-violet rays, which act powerfully in photography, but do not affect the eye.

The first photograph of a comet was obtained by Bond in 1858,—only a partial success and but little known. The next was in 1881, when Henry Draper in New York and Huggins in England photographed Tebbutt's comet, and in 1882 the great comet was well photographed by Gill in South Africa. Fig. 166 is a series of photographs of Swift's comet of 1892 by Barnard. The tail was barely visible to the naked eye, and the peculiar features exhibited in the photograph were not visible at all.

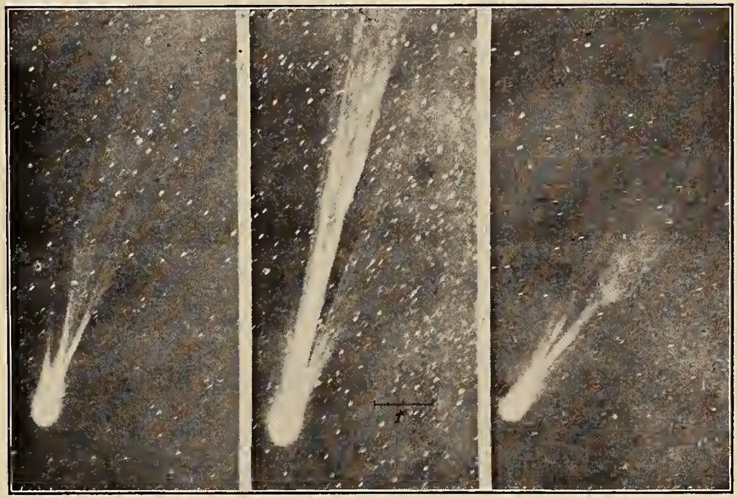

Fig. 166.—Swift's Comet of 1892

Rordame's comet of 1893.

Fig. 167 is from Hussey's beautiful photograph of Rordame's comet of 1893, for which we are indebted to the kindness of Professor Holden.

Since in photographing a comet the camera is kept pointed at the head, which is moving more or less rapidly among the stars, the star images, during the long exposure, are drawn out into parallel streaks, as seen in the photograph. The little irregularities are due to faults of the driving clock and vibrations of the telescope and atmosphere.

The knots and streamers, which in the photographs characterize the comet's tail, were none of them visible in the telescope

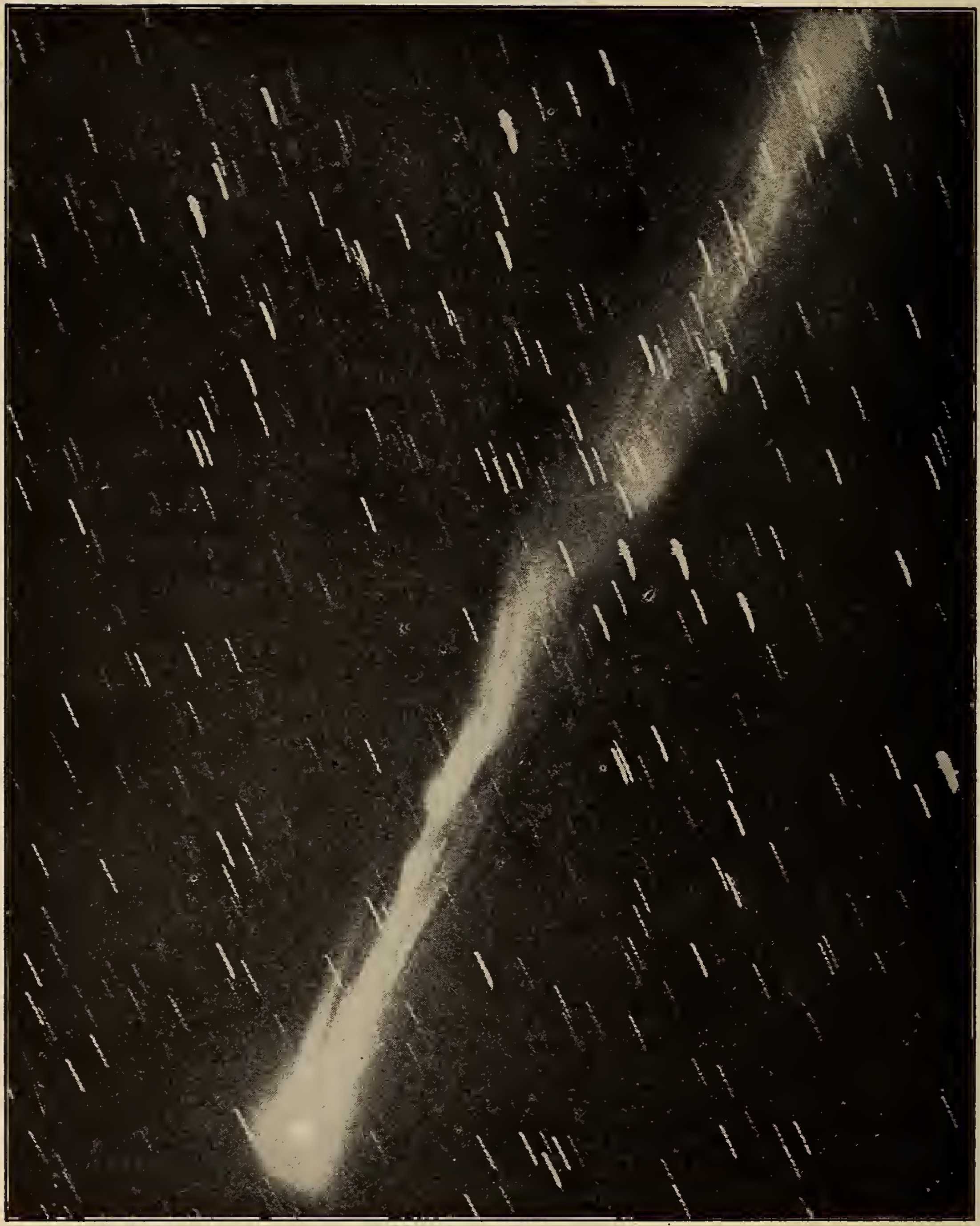

FIG. 167. — Rordame's Comet, July 13, 1893

From photograph by W. J. Hussey, at the Lick Observatory and differ from those shown upon plates preceding and following. Other plates of Rordame's comet, made on the same evening a few hours earlier and later, indicate that these knots were swiftly receding from the comet's head at a rate exceeding 150000 miles an hour.

Rapid motion of knots in tail of comet.

Fig. 168 is a photograph (also by Barnard) of Gale's comet (May, 1894). It was moving through a crowd of stars.

Gale's comet.

In three cases already mentioned (Sec. 479) comets have been *discovered* by photography.

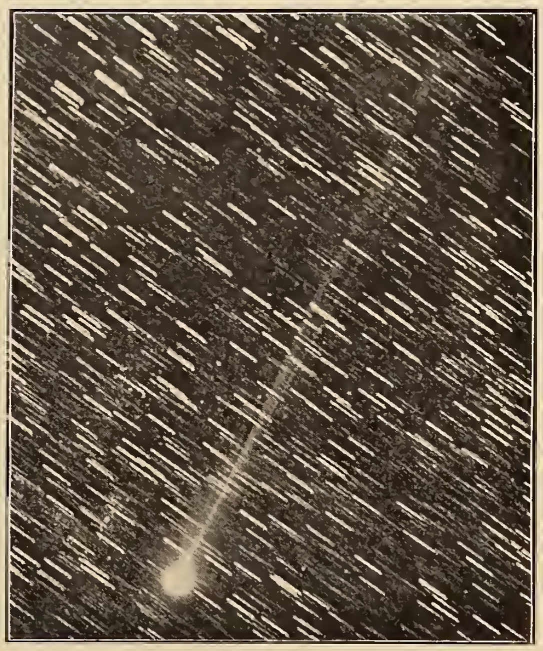

Fig. 168. — Gale's Comet, May 5, 1894

505. Danger from Comets. — We close the chapter with a few remarks upon a subject which has been much discussed.

It has been supposed that comets might do us harm in two ways, — either by actually striking the earth or by falling into the sun, and thus producing such an increase of solar heat as to burn us up.

Possibility of collision with comet.

As regards collision with a comet, there is no question that the event is possible. In fact, if the earth lasts long enough, it is practically sure to happen; for there are several comets whose orbits pass nearer to our own than the semidiameter of the comet's head, and at some time the earth and comet, if the comet lasts long enough, will certainly come together.

As to the consequence of such a collision it is impossible to speak positively, for want of sure knowledge of the constitution of the comet. If the theory which has been presented is true, everything depends on the size of the separate particles which form the main portion of the comet's mass. If they weigh tons, the bombardment experienced by the earth when struck

Probability that collision would be harmless to the earth.

by a comet would be a very serious matter; if, as seems much more likely, they are for the most part smaller than pinheads, the result would be simply a splendid shower of shooting-stars. In 1861 the earth actually passed unnoticed through the tail of the great comet of that year.

Such encounters will, however, be very rare; if we accept the estimate of Babinet, they ought to occur once in about 15 000000 years in the long run.

A danger of a different sort has been suggested, — that if a comet were to strike the earth, our atmosphere would be poisoned by the mixture with the gaseous components of the comet. Here, again, the probability is that on account of the low density of the cometary matter no sufficient amount would remain in the air to do any mischief at the earth's surface.

Possible fall of comet into the sun.

506. Effect of the Fall of a Comet into the Sun. — As to this, it may be stated that, except in the case of Encke's comet, there is no evidence of any action going on that would cause a now existing periodic comet to strike the sun's surface; it is, however, doubtless possible, perhaps not improbable, that a comet may sometime enter the system from without, so accurately aimed as to hit the sun.

Probably no harm except to the comet.

But in that case it is not likely that the least mischief would be done. If a comet with a mass equal to $\frac{1}{1000000}$ of the earth's mass were to strike the sun's surface with the parabolic velocity of nearly 400 miles a second, the energy of impact converted into heat would generate about as many calories of heat as the sun radiates in eight or nine hours. If this were all instantly effective in producing increased radiation at the sun's surface (increasing it, say, eightfold, for even a single hour), harm would doubtless follow; but it is practically certain that nothing of the sort would happen. The cometary particles would pierce the photosphere and liberate their heat mostly *below the solar surface*, simply expanding, by some slight amount, the sun's diameter, and so adding to its store of potential energy about as much as it ordinarily expends in a few hours and postponing, by so much, the date of its final solidification. There might, and very likely would, be a flash of some kind at the solar surface as the shower of cometary particles struck it, but probably nothing that the astronomer would not take delight in watching.

EXERCISES

1. What would be the mean density, compared with air, of the spherical head of a comet 100000 miles in diameter and having a mass $\frac{1}{100000}$ that of the earth, assuming the density of the earth to be 5.53 times that of water and the density of water 773 times that of air? *Ans.* About $\frac{1}{49000}$.

2. What would be the diameter of such a comet if compressed to a density the same as that of the earth? *Ans.* 171 miles.

3. Can the dimensions of a comet's tail be determined with much accuracy? If not, why not?

4. How can it happen that comets whose orbits nearly coincide within a distance of 100 000000 miles from the sun may have periods differing by hundreds of years? For example, the comets of 1880 and 1882, of which the first has a computed period of only 33 years, and the other of more than 600.

5. In the case of two cometary orbits very nearly parabolic, and having the same very small perihelion distance, how would the ratio of their major axes be affected by a small difference in their perihelion velocities? (See Sec. 320, remembering that, as the orbits are nearly parabolic, V^2 must be very nearly equal to U^2 when the comets pass perihelion.)

6. If the repulsive force of the sun upon a particle of a comet's tail were just equal to the gravitational attraction (Sec. 502), what would be the path of that particle? *Ans.* A straight line.

7. If the repulsive force exceeded the gravitational attraction, what would be the nature of the path?

Ans. An orbit *convex* toward the sun, hyperbolic if the repulsion varied inversely as the square of the distance, the sun being in the focus outside the curve, *i.e.*, at F'' in Fig. 119, Sec. 314.

8. What would be the path if the repulsive force were only very small as compared with the gravitational attraction?

Ans. An orbit of slightly greater major axis and period than that of the comet itself.

9. Will a given comet (say Encke's) have precisely the same orbit on successive returns?

10. Why can we not infer with certainty that two comets which have orbits practically identical are themselves identical?

11. Can we, from spectroscopic observations of a comet, infer the relative proportions of the luminous and non-luminous substances present in the comet?

12. Is it probable that a comet can continue permanently in the solar system as a comet? If not, why not, and what will become of it?

Potsdam Astrophysical Observatory

CHAPTER XVII

METEORS AND SHOOTING-STARS

Aërolites: their Fall and Physical Characteristics; Cause of Light and Heat; Probable Origin — Shooting-Stars: their Number, Velocity, and Length of Path — Meteoric Showers: the Radiant; Connection between Comets and Meteors

METEORS

507. Meteorites, or Aërolites. — Occasionally bodies fall upon the earth out of the sky, coming to us from outer space. Until they reach our air they are invisible, but as soon as they enter it they blaze out, become conspicuous, and the pieces which fall from them are called *meteorites*, *aërolites*, or simply *meteoric stones.*

Circumstances of fall of aërolites.

If the fall occurs at night, a ball of fire is seen, which moves with an apparent velocity depending upon the distance of the meteor and the direction of its motion, and is generally followed by a luminous train, which sometimes remains visible for many minutes after the meteor itself has disappeared. The motion is usually somewhat irregular, and here and there along its path the fire-ball throws off sparks and fragments and changes its course more or less abruptly. Sometimes it vanishes by simply fading out in the distance, sometimes by bursting like a rocket.

Delay of sound of explosion.

If the observer is near enough, the flight is accompanied by a heavy continuous roar, like that of a passing railway train, accentuated now and then by violent detonations; the noise is frequently heard 50 miles away, especially the final explosion. The observer, however, must not expect to hear the explosion when he sees it. Sound travels only about 12 miles a minute, so that there is often an interval of several minutes between the visible bursting and its report.

If the fall occurs by day, the luminous appearances are mainly wanting, though sometimes a white cloud is seen, and even the train may be visible. In a few cases, aërolites have fallen almost silently, and without warning.

Size of aërolites.

508. The Aërolites themselves. — The mass that falls is sometimes a single piece, but more usually there are many fragments, sometimes to be counted by thousands. At the Pultusk "fall," in 1869, the number was estimated to exceed 100000, mostly very small. The pieces weigh from 500 pounds to a few grains, the aggregate mass occasionally amounting to more than a ton. The largest single mass, so far as known, is one that fell at Knyahinya in 1866, weighing 647 pounds.

Aërolites mostly stones.

By far the greater number of aërolites are stones, but a few — one or two per cent of the whole number — are pieces of nearly pure iron more or less alloyed with nickel.

Number of meteorites collected since 1800.

The total number of meteorites which have fallen and been gathered into our cabinets since 1800 is about 275. The only instances in which purely iron meteorites have been actually seen to fall and are represented by specimens in our cabinets are the eight following, viz.:

Iron meteorites of which the fall was observed.

Agram, Croatia, Austria	1751
Dickson County, Tennessee, U.S.	1835
Braunau, Bohemia	1847
Victoria West, South Africa	1862
Nedagollah, Arabia	1870
Rowton, England	1876
Mazapil, Mexico	1885
Cabin Creek, Arkansas, U.S.	1886

There are about as many more which contain large quantities of iron and by some authorities have been reckoned as "irons"; nearly all meteorites contain a large percentage of the metal, either in the metallic form or as sulphid.

About 30 of the 275 fell within the United States, the most remarkable being those of Weston, Conn., in 1807; New

Concord, Ohio, in 1860; Amana, Iowa, 1875; Emmet County, Iowa, 1879 (largely iron); and Cabin Creek, Ark., 1886.

Total number in cabinets.

Our cabinets at present contain specimens of somewhat more than three hundred meteors which have been seen to fall, besides a nearly equal number of other bodies, — mostly masses of iron which, from the circumstances of their finding and the peculiarities of their constitution, are supposed to be of meteoric origin.

The finest collection in the world is that at Vienna. The collection of the British Museum and that at Paris are also noteworthy; and in this country the cabinet of Yale University is especially rich.

Appearance of meteorites: the crust.

509. Appearance and Constitution of the Meteorites. — The most characteristic external feature of an aërolite is the thin black crust which covers it, usually, but not always, glossy like varnish. It is formed by the fusion of the surface in the meteor's swift motion through the air, and in some cases penetrates deeply into the mass through veins and fissures. It is largely composed of oxid of iron and is almost always strongly magnetic. The crusted surface usually exhibits pits and hollows, called "thumb-marks" because they look like prints produced by thrusting the thumb into a piece of putty. These cavities are explained by the burning out of certain more fusible substances during the meteor's flight.

Internal structure.

On breaking, the stone is sometimes found to be comparatively fine grained, but usually is made up of crystalline lumps and globules, and sometimes has a considerable portion of solid iron scattered throughout the mass in grains as large as a pinhead or bird shot.

Chemical elements: peculiar minerals.

Twenty-seven of the chemical elements, including argon and helium, have been found in meteorites, but not a single *new element.* Many of the *minerals* of which the meteorites are composed present a great resemblance to terrestrial minerals of volcanic origin, but there are also many which are peculiar and not found on the earth.

The occasional presence of carbon is to be especially noted; and in a meteor which fell in Russia in 1887, the carbon appeared to be in a crystalline form, identical with the black diamond, though in particles exceedingly minute.

Fig. 169 is from a photograph of a fragment of one of the meteoric stones which fell at Gross Divina, Hungary, in 1837; weight about twenty-four pounds.

Path and motion.

510. Path and Motion. — When a meteor has been well observed from a number of different stations a considerable distance apart, its path with reference to the surface of the earth can be computed.

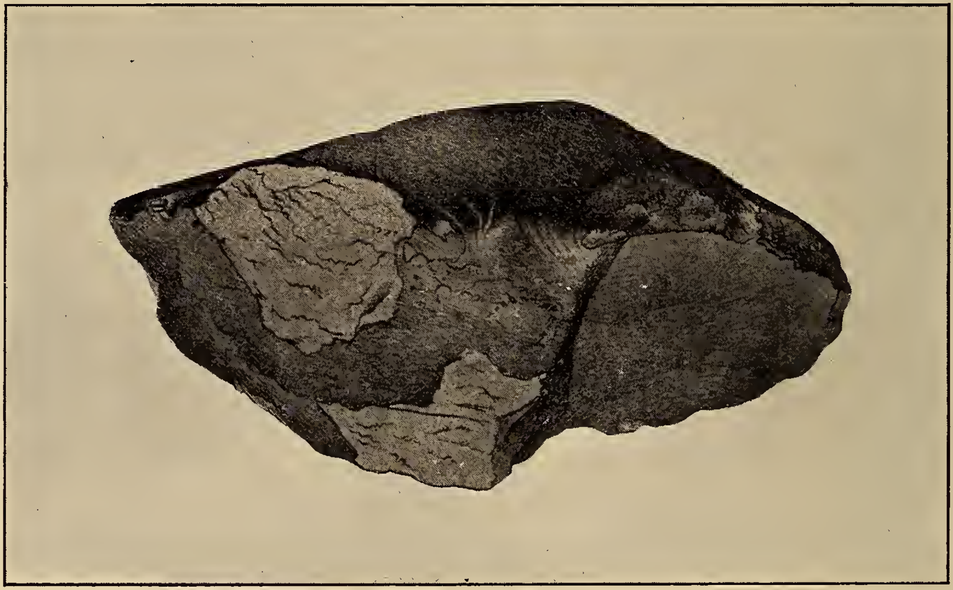

Fig. 169. — The Gross Divina Meteorite

Elevation when first seen.

It is found that it usually first appears at an altitude of about 80 or 100 miles and disappears at a height of from 5 to 10. The length of the path is generally between 50 and 500 miles, though in some cases it has been much greater. In 1860 one passed from over Lake Michigan across the country and fell into the sea beyond Cape May; and in 1876 a great meteor traversed the country from Kansas to northern Pennsylvania.

Length of path.

Velocity.

The velocity ranges from 10 to 40 miles a second in the earlier part of its course, but is very rapidly and greatly reduced by the resistance of the atmosphere, so that when the surface of the earth is reached it is often not more than 400 or 500 feet a second. In one case (a meteor that fell near Upsala, Sweden, in January, 1869) several of the stones struck upon the ice of a lake and rebounded without breaking the ice or damaging themselves.

Meteorites are visitors from distant regions.

The *average* velocity with which these bodies enter the air seems to be very near the parabolic velocity of 26 miles a second, due to the sun's attraction at the earth's distance, — just as should be the case, if, like the comets, they come to us from distant regions of space.

Observation of meteors.

511. Observation of Meteors. — The object of the observer should be to obtain as accurate an estimate as possible of the *altitude* and *azimuth* of the meteor at moments which can be identified, and also of *the time* occupied in traversing definite portions of the path.

Determination of meteor's altitude and azimuth.

By night the stars furnish the best reference points from which to determine its position. By day one must take advantage of natural objects and buildings to define the meteor's place, the observer marking the precise spot where he stood when the meteor disappeared behind a chimney, for instance, or was seen to burst just over a certain branch in a tree. By taking a surveyor's instrument to the place afterwards it is then easy to translate such data into *altitude* and *bearing.*

Time of flight.

As to the time of flight, which is required in order to determine the meteor's velocity, it is usual for the observer to begin to repeat rapidly some familiar verse of doggerel when the meteor is first seen, reiterating it until the meteor disappears. Then, by rehearsing the same before a clock, the number of seconds can be pretty accurately determined.

512. Explanation of Heat and Light. — These are simply due to the intense condensation of the air before the swiftly moving meteor, and consequent destruction of the meteor's energy. The resistance, due to condensation, amounts in many cases to the back-pressure of hundreds of pounds upon a square inch; and most of the energy of the meteor destroyed in this way is transformed into heat, largely imparted to the air, but to a considerable extent expended upon the surface of the meteor, fusing it and producing the crust.

Heating due to condensation of air in front of meteor.

If a moving body whose mass is M kilograms, and its velocity V kilometers per second, is stopped by a resistance, its energy is almost entirely converted into heat, and the number of *calories* (Sec. 267) developed is given (approximately) by the equation

Formula for amount of heat developed.

$$Q = 120\, MV^2.$$

In bringing to rest a body having a mass of one kilogram and a velocity of forty-two kilometers, or 26 miles a second, the quantity of heat developed is enormous, — nearly 212000 calories, — vastly more than sufficient to fuse it, even if it were made of the most refractory material. As Lord Kelvin has shown, the *thermal effect* of the rush through the air is the same as if the meteor were immersed in a blowpipe flame having a temperature of many thousand degrees; and it is to be noted that this *temperature is independent of the density of the air* through which the meteor is passing. The *quantity of heat* developed in a given time is greater, of course, where the air is dense, but the *temperature* produced in the air itself at the surface where it encounters the moving body is the same whether it be dense or rare.

Virtual temperature of air encountered is that of a blowpipe flame.

This rise of temperature is due to the fact that the gaseous molecules strike the surface of the meteor as if the meteor were at rest and the molecules themselves were moving with speed correspondingly increased. (According to the kinetic theory of gases, the "temperature" of a gas depends entirely upon the mean velocity square of its molecules.)

When the moving body has a velocity of one and one-half kilometers per second the *virtual temperature* of the surrounding air is about that of red heat. When the velocity reaches thirty kilometers per second the amount of heat developed is 15^2, or 225, times as great, and the surface is acted upon as if the surrounding gas were a blowpipe flame, as has been said; the surface of the meteor is fused and the liquefied portion is continually swept off by the rush of air, condensing as it cools into the luminous dust that forms the train. The fused surface is continually renewed until the velocity falls below two kilometers a second, or thereabouts, when it solidifies and forms the crust.

Formation of crust and train.

As a general rule, therefore, the fragments are hot if found soon after their fall; but if the stone is a large one and falls nearly vertically, so as to have a short path through the air, the heating effect will be confined to its surface, and, owing to the low conducting power of stone, the center may still remain intensely cold for some time, retaining nearly the temperature which it had in interplanetary space. It is recorded that one of the fragments of the Dhurmsala, India, meteorite, which fell in 1860, was found in moist earth half an hour or so after the fall *coated with ice*.

Stones sometimes cold when found.

One unexplained feature of the meteoric trains deserves notice. They often remain luminous for a long time, sometimes as much as half an hour, and are carried by the wind like clouds. It is impossible to suppose that such a cloud of impalpable dust remains white-hot for so long a time in the cold upper regions of the atmosphere, and the question of its enduring luminosity or phosphorescence is an interesting and a puzzling one.

Unexplained phosphorescence of trains.

513. The Origin of Meteors. — The high velocity with which many enter our atmosphere makes it quite certain that they at least had not a terrestrial origin. A body projected from the earth could never return with higher velocity than that of projection, and any velocity exceeding about *7 miles a second* (the parabolic velocity at the earth's surface — Sec. 319) would carry

Meteors come to us as astronomical bodies.

the body permanently away from the earth, never to return unless after many revolutions around the sun. Most meteors, if not all, come to us as astronomical bodies, moving like planets or comets; as to their origin, we can only speculate.

Meteorites can be classified.

At the same time we find in our cabinets many distinct *classes* of them, and in each class all the meteors which compose it resemble each other so closely as to suggest the idea that they must have had a common source, or at one time formed portions of a single mass; but where and when?

Theories of their origin.

Some have maintained that they were projected from lunar volcanoes, ages ago perhaps (for lunar volcanoes are now inactive), and that since that time they have been moving around the sun like planets, until now encountered by the earth. Others refer them to similar imagined volcanic eruptions from the earth in some past age, and others consider them as proceeding from the disintegration of comets.

Iron meteorites perhaps of stellar origin.

As to the iron meteorites, some believe that they have been ejected from the *sun* or from a *star*, basing the opinion upon the remarkable fact that these meteoric irons are usually "soaked full" of occluded gases, — hydrogen, helium, and carbon oxids, which can be extracted from them by well-known methods. It is argued that the iron could have absorbed these gases only when immersed in a hot dense atmosphere saturated with them, — a condition existing, so far as known, only on the sun and stars.

Professor Newton's results: some meteorites perhaps asteroids.

An investigation by the late Professor Newton, however, shows that about ninety per cent of the aërolites, for the determination of whose orbits we have sufficient data, were moving around the sun before their encounter with the earth in paths not parabolic, but resembling those of the short-period comets, or more eccentric asteroids, and nearly all direct, suggesting a planetary rather than a stellar origin; they might possibly be minute outriders of the asteroid family.

Lord Kelvin suggested many years ago that meteors may have acted in conveying germs of life from one part of the universe to another, — a suggestion, however, not generally accepted, since they seem to have passed through conditions of temperature which must have destroyed all life.

Theory of meteors as carriers of life improbable.

514. Number of the Aërolites. — As to the number of these bodies which strike the earth, it is difficult to make a trustworthy estimate. We generally add to our cabinets each year specimens of from two to six meteors which have been seen to fall. But for one that is found, even of the meteors whose flight has been observed, a dozen are missed; and if we include all that were not seen, or that fell unobserved on the ocean or in regions from which no report could come, the sum total must be very great. Schreibers estimated the number at seven hundred a year. Reichenbach puts it at three or four thousand, but this is probably excessive.

Number of aërolites.

SHOOTING-STARS

515. Their Nature and Appearance. — These are the swift-moving, evanescent, starlike points of light which may be seen every few moments on any clear moonless night. They make no sound, nor (perhaps with one exception, to be noted later) has anything been known to reach the earth's surface from them, not even in the greatest "meteoric showers."

Shooting-stars: probably minute aërolites, but may be dust clouds.

For this reason it may be well to retain provisionally the old distinction between them and the large meteors from which aërolites fall. It is quite probable that the distinction has no real ground, that shooting-stars are just like other meteors, except in size, being so small that they are entirely consumed in the air; but then, on the other hand, there are some things which favor the idea that the two classes of bodies differ in constitution about as asteroids do from comets.

516. Number of Shooting-Stars. — Their number is enormous. A single ordinary observer averages from four to eight an hour; one used to observation, well situated and on a moonless night, will see at least twice as many; Schmidt of Athens sets the average number at fourteen. If the observers are sufficiently numerous and so organized as to be sure of noting all that are visible from their station, about eight times as many will be counted.

Their number ten to twenty million daily.

On this basis Professor Newton has estimated that the total number which enter our atmosphere daily must be *between ten and twenty million*, the average distance between them being over 200 miles; and besides those which are visible to the naked eye there is an immensely larger number so small as to be observable only with the telescope. Dr. See estimates the number of these as at least one hundred million daily.

Average hourly number in morning twice as great as in evening.

The average hourly number about six o'clock in the morning is double the hourly number in the evening, and the meteors move much swifter, the reason being that in the morning we are on the *front* of the earth as regards its orbital motion, while in the evening we are in the rear. (The earth's orbital motion is always directed towards a point on the ecliptic about *90° west of the sun.*) In the evening, therefore, we see only such as overtake us. In the morning we see all that we either meet or overtake. This proportion of morning and evening meteors is precisely what it should be if they come to us indiscriminately from all directions and with the parabolic velocity of 26 miles a second.

Determination of path, etc.

517. Elevation, Path, and Velocity. — By observations made at stations 30 or 40 miles apart (best by photography) it is easy to determine these data with some accuracy whenever meteors identifiable at the two or more stations make their appearance. It is found that on the average the shooting-stars appear at a height of about 74 miles and disappear at an elevation of about 50 miles, after traversing a course of 40 or 50 miles, with a velocity of from 10 to 30 miles a second, — about 25 on the average. They do not begin to be visible at so great a height as the aërolitic meteors, and they are more quickly consumed and therefore do not penetrate our atmosphere to so great a depth, — fortunately for us.

518. Brightness, Material, etc. — Now and then a shooting-star rivals Jupiter or even Venus in brightness. A considerable number are like first-magnitude stars, but the great majority are faint. The bright ones generally leave trains, which sometimes endure from five to ten minutes and then fold up and are wafted away by the air currents, which at 40 miles above the earth's surface ordinarily have velocities of from 50 to 75 miles an hour.

Brightness.

Trains.

The swift meteors are usually of green or bluish tinge, while those that move slowly are generally red or yellow.

Color.

Occasionally it has been possible to get a "snap shot," so to speak, at the *spectrum* of a meteor, and in it the bright lines of sodium and (probably) magnesium are fairly conspicuous among many others which cannot be identified by a hasty glance.

Spectrum.

Since these bodies are consumed in the air, all we can hope to get of their material is their ashes. In most places its collection and identification is hopeless; but Nordenskiold thought that it might be found in the polar snows. In Spitzbergen he therefore melted several tons of snow, and on filtering the water he actually detected in it a sediment containing minute globules of oxid and sulphid of iron. Similar globules have also been found in the products of deep-sea dredging. They *may* be meteoric; but what we now know of the distance to which smoke and fine volcanic dust is carried by the wind makes it not impossible that they may be of purely terrestrial origin.

Meteoric ashes.

519. Probable Mass of Shooting-Stars. — We have no way of determining the exact mass of such a body; but from the light it emits, as seen from a known distance, an estimate can be formed not likely to be widely erroneous.

Probable mass extremely small.

A good ordinary incandescent lamp consumes about 150 foot-pounds of energy per minute for each candle-power. Assuming for the moment that the ratio of the total *light* emitted (*luminous* energy) to the total energy consumed is the same for a meteor as for an electric lamp, we can compute the total energy of a meteor which shines with known brightness for a given time at a known distance.

Data for calculation of mass.

Suppose, for instance, that the shooting-star is at an average distance during its flight of 30 miles from the observer and appears as bright as a 16 candle-power lamp $\frac{1}{4}$ of a mile away, and shines for five seconds ($\frac{1}{12}$ minute). The total luminous energy then equals

$$150 \times 16 \times \tfrac{1}{12} \times \left(\frac{30}{\frac{1}{4}}\right)^2 = 2\,880000 \text{ foot-pounds.}$$

Calculation of the mass of a shooting-star.

Suppose its velocity, V, to be 20 miles, or 105600 feet, per second. For the energy, E, in foot-pounds, of a moving body whose velocity is V *feet* per second and its mass *in pounds* M, we have

$$E = \tfrac{1}{2}\,\frac{MV^2}{g} = \frac{MV^2}{64} \text{ (nearly), whence, } M = 64 \times \frac{E}{V^2}.$$

Finally, then, in the case before us,

$$M = 64 \times \frac{2\,880000}{105600^2} = \frac{1}{61} \text{ pounds, or 115 grains (nearly).}$$

This represents fairly the observed conditions for a very bright shooting-star.

If a meteor converted *all* its energy into light,—*i.e.*, if its luminous efficiency were higher than that of a lamp,—this would give the mass much too great. On the other hand, if the meteor were only feebly luminous, the mass thus determined would be much too small.

Average mass probably only a fraction of an ounce.

It seems likely that an average meteor and a good electric lamp do not differ widely in their luminous efficiency, and on this basis observations indicate that ordinary shooting-stars weigh only a fraction of an ounce,—from a grain or two up to 100 or 150 grains. Some authorities, however, estimate the mass considerably higher. It all turns on the assumed "luminous efficiency" of the shooting-stars.

520. Effects produced by Meteors and Shooting-Stars. — (1) Meteors *add continually to the mass of the earth.* If we assume 20 000000 a day, each weighing $\frac{1}{60}$ of a pound, the total amount would be about 50000 tons a year; and if the specific gravity of the meteoric dust averages the same as that of granite, it would take about eight hundred million years for the deposition of a layer 1 inch thick on the earth's surface.

Effects due to fall of meteoric matter practically insensible.

(2) *They diminish the length of the year in three ways:* (*a*) by acting as a resisting medium, and so really *shortening the major axis of the earth's orbit* (just as the orbit of Encke's comet is shortened); (*b*) by *increasing the mass of the earth and sun*, and so increasing the attraction between them; (*c*) by increasing the size of the earth, thus slackening its rotation, *lengthening the day*, and so making fewer days in the year.

Calculation shows, however, that on the preceding assumption as to the mass of the meteors, the combined effect would hardly amount to more than $\frac{1}{1000}$ of a second in a million years.

(3) *Each meteor brings to the earth a certain amount of heat*, developed in the destruction of its motion. According to the best estimates, however, all the meteors that fall upon the earth in a year supply no more heat than the sun does in about one tenth of a second.

(4) *They must necessarily render space imperfectly transparent* if they pervade it throughout in any such numbers as in the domain of the solar system; but this effect, though doubtless real, is also so small as at present to defy calculation.

METEORIC SHOWERS

521. There are occasions when the shooting-stars, instead of appearing here and there in the sky at intervals of several minutes, appear in showers of thousands; at such times they do not move at random, but all their paths diverge or *radiate* from a single point in the sky, known as the *Radiant; i.e.*,

Meteoric showers.

The radiant.

their paths produced backward all pass through or near that point, though they do not usually start there. Meteors which appear near the radiant are apparently stationary, or describe paths which are very short, while those in the more distant regions of the sky pursue longer courses.

Nomenclature of meteoric showers.

The radiant keeps its place among the stars sensibly unchanged during the whole continuance of the shower, — for hours or days, it may be, — and the shower is named according to the place of the radiant among the constellations. Thus, we have the Leonids, or meteors whose radiant is in the constellation of Leo, the Andromedes (or Bielids), the Perseids, the Lyrids, etc.

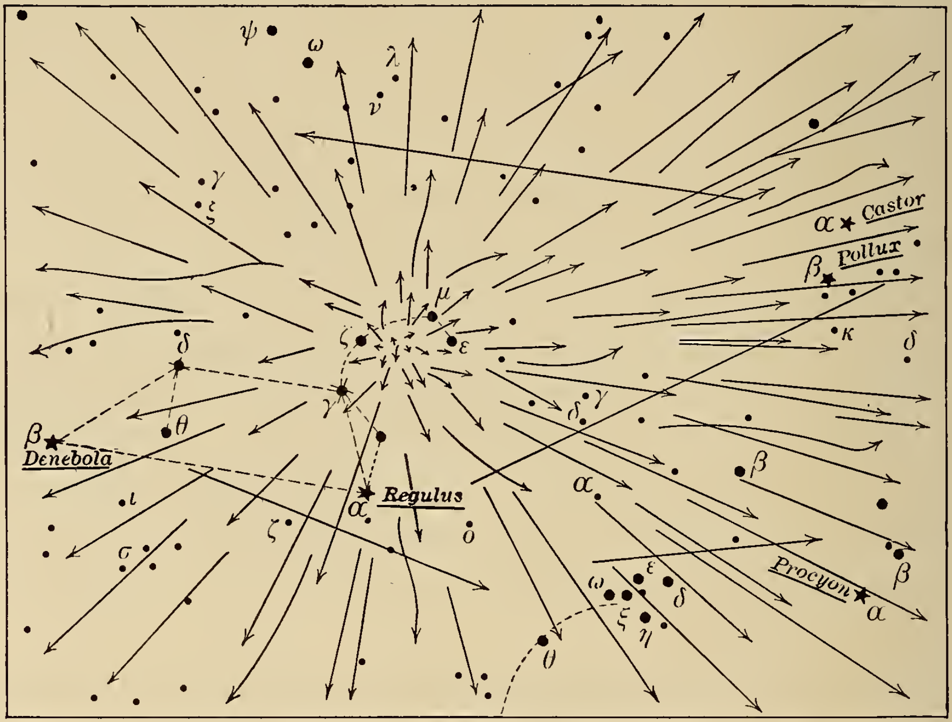

Fig. 170. — The Meteoric Radiant in Leo, Nov. 13, 1866

Fig. 170 represents the tracks of a large number of the Leonids of 1866, showing the position of the radiant near ζ Leonis. It shows also the tracks of four meteors observed during the same time, which did not belong to the shower.

The radiant an effect of perspective.

The radiant is a mere effect of *perspective.* The meteors are all moving in lines nearly parallel when encountered by the earth, and the radiant is simply the perspective *vanishing point* of this system of parallels; their paths all appear to converge, like the rails of a railway track for an observer looking upon it from a bridge. The position of the radiant on the celestial sphere depends entirely upon the *direction of the motion of the meteors relative to the observer.* For various reasons, however, the paths of the meteors, on account of irregularities in their form and surfaces, are not exactly parallel or straight, and in consequence the radiant is not a mathematical point, but a spot or patch in the sky, often covering an area of 3° or 4°.

The Leonid shower of 1833.

Probably the most remarkable of all the meteoric showers that have ever occurred was that of the Leonids, on November 12, 1833. The number at some stations was estimated as high as 200000 an hour for five or six hours. "The sky was as full of them as it ever is of snowflakes in a storm" and, as an old lady described it, looked "like a gigantic umbrella."

Fixed dates of meteoric showers.

522. Dates of Meteoric Showers. — Meteoric showers are evidently caused by the earth's encounter with a swarm of the little bodies, and since this swarm or flock pursues a regular orbit around the sun, the earth can meet it only when she is at the point where her orbit cuts the path of the meteors; this, of course, must always happen at or near the same time of the year, except as in the process of time the meteoric orbits shift their positions on account of perturbations. The Leonid showers, therefore, appear about November 15, and the Andromedes about the 24th; but both dates are slowly changing, the Leonids coming gradually earlier and the Andromedes later. Since 1800 the former have shifted from November 12 to the 15th, and the latter from the 28th to the 24th since 1872.

Annual recurrence of the Perseids.

In some cases the meteors are distributed along their whole orbit, forming a sort of ring and rather widely scattered. In that case the shower recurs every year and may continue for several weeks, as is the case with the Perseids, or August meteors. On the other hand, the flock may be concentrated, and then a notable shower will occur only on the day when the earth and the meteors arrive together at the orbit crossing. This is the case with both the Leonids and the Andromedes, though the latter are already getting pretty widely scattered. The showers then occur, not every year, but only at intervals of several years, and always on or near the same time of the month. For the Leonids the interval is about thirty-three years, and for the Andromedes usually thirteen, but sometimes only six or seven.

Conditions different with the Leonids and Andromedes.

Characteristics of different meteor swarms.

The meteors which belong to the same group have certain family resemblances. The Perseids are yellow and move with medium velocity. The Leonids are very swift (we *meet* them), and they are of a bluish green tint, with vivid trains. The Andromedes are sluggish (they overtake the earth), are reddish, being less intensely heated than the others, and usually have only feeble trains.

Principal radiants now recognized.

About one hundred meteoric radiants are now recognized and catalogued. The most conspicuous of them, except those already named, are the following: the *Draconids*, January 2; *Lyrids*, April 20; *Aquariids I*, May 6; *Aquariids II*, July 28; *Orionids*, October 20; *Geminids*, December 10.

Stationary radiants.

523. Stationary Radiants. — When a meteoric shower persists for days and even weeks, as do the Perseids for instance, the radiant, as a rule, gradually shifts its position among the stars, on account of the change in the direction of the earth's motion, — as it ought to, since the place of the radiant depends upon the *combination* of the earth's motion with that of the meteors.

Mr. Denning of Bristol (England), for many years an assiduous observer of meteors, claims, however, to have discovered numerous cases in which the radiant of a long-continued shower remains absolutely *stationary;* and he presents as typical the Orionids, which scatter along from about October 10 to 24, all the time, according to his observations, keeping their radiant close to the star ν Orionis.

Difficult to explain.

No satisfactory explanation of such fixity of the radiant yet appears, though certain mathematical investigations by Turner of Oxford (on the disturbing effect of the earth upon meteors passing near her) look promising and may resolve the problem; but some high authorities still remain skeptical as to the fact.

Meteorite which fell during shower of Bielids.

524. The Mazapil Meteorite. — As has been said, during these showers no sound is heard, no sensible heat perceived, nor have any masses ever reached the ground; with the one exception, however, that on Nov. 27, 1885, a piece of meteoric iron fell at Mazapil, in northern Mexico, during the shower of Andromedes, or "Bielids," which occurred that evening.

Whether the coincidence was accidental or not, it is interesting. Many high authorities speak confidently of this piece of iron as being a piece of Biela's comet itself.

This brings us to one of the most remarkable discoveries of nineteenth-century astronomy.

CONNECTION BETWEEN COMETS AND METEORS

Olmsted's recognition of meteoric swarms as cometlike.

525. At the time of the great meteoric shower of 1833, Professors Olmsted and Twining of New Haven were the first to recognize the radiant and to point out its significance as indicating the existence of a swarm of meteors revolving around the sun in a permanent orbit; Olmsted even went so far as to call the body a "comet." Others soon showed that, in some cases at least (Perseids), the meteors must be distributed in a complete ring around the sun, and Erman of Berlin developed a method of computing the meteoric orbit when its radiant is known.

In 1864 Professor Newton of New Haven showed by an examination of the old records that there had been a number of great meteoric showers in November, at intervals of

Newton's prediction of shower of 1866.

thirty-three or thirty-four years, and he predicted confidently a repetition of the shower on Nov. 13 or 14, 1866. The shower occurred as predicted and was observed in Europe; and it was followed by another in 1867, which was visible in America, the meteoric swarm being extended in so long a procession as to require more than two years to cross the earth's orbit. Neither of these showers, however, was equal to the shower of 1833. The researches of Newton, supplemented by those of Adams, the discoverer of Neptune, showed that the swarm moves in a long ellipse with a thirty-three-year period.

Failure in 1900.

A return of the shower was expected in 1899 or 1900, but failed to appear, though on Nov. 14–15, 1898, a considerable number of meteors were seen, and in the early morning of Nov. 14–15, 1901, a well-marked shower occurred, visible over the whole extent of the United States, but best seen west of the Mississippi, and especially on the Pacific coast. At a number of stations several hundred Leonids were observed by eye or by photography, and the total number that fell must be estimated by tens of thousands. The display, however, seems to have nowhere rivaled the showers of 1866–67, and these were not to be compared with that of 1833.

Second shower of 1901.

It is not impossible that another minor shower may be observed in 1902.

Cause of failure in 1900.

The calculations of Downing and Stoney show that the failure in 1900 was probably due to perturbations of the meteors by the action of Jupiter, Saturn, and Uranus during their absence from the neighborhood of the sun, causing the main body to pass at a distance of nearly 2 000000 miles below the orbit of the earth.

Schiaparelli's identification of orbit of Perseids.

526. Identification of Meteoric Orbits with Cometary. — The researches of Newton and Adams had awakened lively interest in the subject, and Schiaparelli, a few weeks after the Leonid shower, published a paper upon the Perseids, or August meteors, in which he brought out the remarkable fact that they are moving in the same orbit as that of the bright comet of 1862, known as Tuttle's comet. Shortly after this Leverrier published his orbit of the Leonid meteors, derived from the observed position of the radiant in connection with the periodic time assigned by Adams; and almost simultaneously, but without any idea of a connection between them, Oppolzer published his orbit of Tempel's comet of 1866, and the two orbits were at once seen to be practically identical. Now a single coincidence might be accidental, but hardly two.

Leverrier and Oppolzer on orbit of Leonids.

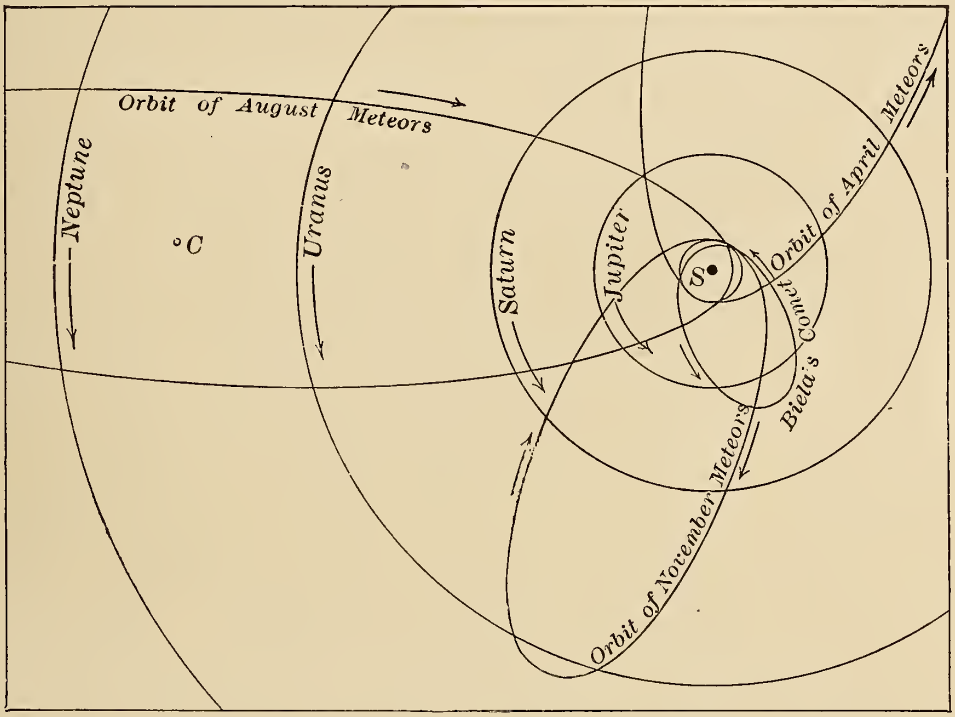

Fig. 171. — Orbits of Meteoric Swarms

Andromedes and Biela's comet.

Five years later came the shower of the Andromedes, following in the track of Biela's comet, and among more than one hundred of the distinct meteor swarms now recognized Prof. Alexander Herschel finds five others which are similarly related each to its special comet. It is no longer possible to doubt that there is a real and close connection between these meteors and their attendants. Fig. 171 represents four of these cometo-meteoric orbits.

Nature of connection between comets and meteors not yet determined.

527. Nature of the Connection. — This cannot be said to be ascertained. In the case of the Leonids and the Andromedes the meteoric swarm *follows* the comet, but this does not seem to be so in the case of the Perseids, which scatter along more or less abundantly every year.

The prevailing belief at present seems, on the whole, to be that the comet itself is only the thickest part of a meteoric swarm, and that the clouds of meteors scattered along its path result from its *disintegration.*

Transformation of meteoric swarm into a ring.

It is easy to show that if a comet really is such a swarm it is likely to break up gradually more and more at each return to perihelion, and at every near approach to one of the larger planets, dispersing its constituent particles along its path until the compact swarm has become a diffuse ring. The different parts of the comet are at different distances from the sun, and there is almost no sensible mutual attraction between them, the mass is so minute. The attraction of the sun or planet is therefore likely to cause the separation that has been referred to.

Rings older than compact swarms.

The longer the comet has been moving around the sun, the more uniformly the particles will be distributed. The Perseids are supposed, therefore, to have been in the system for a long time, while the Leonids and Andromedes are believed to be comparatively new-comers. Leverrier, indeed, has gone so far as to indicate the year A.D. 126 as the time at which Uranus captured Tempel's comet and brought it into the system (as illustrated by Fig. 172). But the theory that meteoric swarms are the product of cometary disintegration assumes that comets are compact aggregations when they enter the system, which is by no means certain.

The meteoritic hypothesis.

528. Sir Norman Lockyer's Meteoritic Hypothesis. — Within the last twenty years Sir Norman Lockyer has been enlarging greatly the astronomical importance of meteors. The probable meteoric constitution of the zodiacal light, as well as of Saturn's rings, and of the comets, has long been recognized; but he goes much further and maintains that all the heavenly bodies are either meteoric swarms, more or less condensed, or the final products of such condensation. Upon this hypothesis he attempts to explain the evolution of the planetary system, the phenomena of temporary and variable stars, the various classes of stellar spectra, the forms and structure of the nebulæ, — in fact, pretty much everything in the heavens from the aurora borealis to the sun. As a working hypothesis his theory is unquestionably suggestive and has attracted much attention, but it encounters serious difficulties in details and cannot be said to be as yet "accepted."

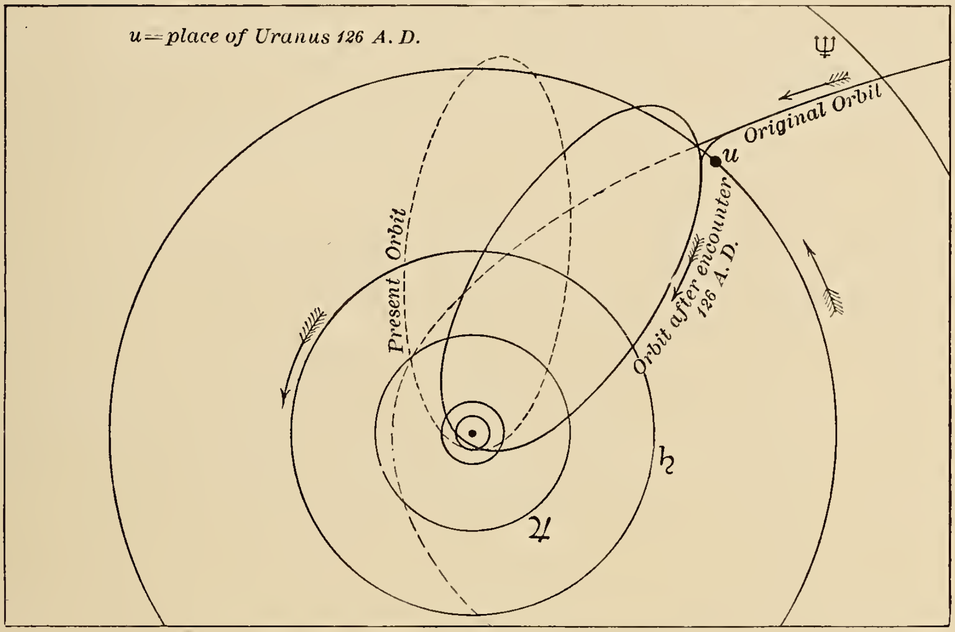
u = place of Uranus 126 A. D.
Ψ
Original Orbit
u
Present Orbit
Orbit after encounter 126 A. D.
♄
♃

FIG. 172. — Origin of the Leonids

EXERCISES

1. If a compact swarm of meteors were now to enter the system and be deflected by the attraction of some planet into an elliptical orbit around the sun, would the swarm continue to be compact? If not, what would be the ultimate distribution of the meteors?

2. What is the probable relative age of meteoric *swarms* and meteoric *rings* as members of the solar system?

3. Assuming that the earth encounters 20 000000 meteors every 24 hours, what is the average number in a cubic space of 1000 000000 cubic miles (*i.e.*, a cube 1000 miles on each edge)? *Ans.* About 250.

4. If space were occupied by meteors uniformly distributed 100 miles apart on three sets of lines perpendicular to each other, how many would be encountered by the earth in a day? *Ans.* 78 700000.

NOTE. — In this cubical arrangement the *average* distance between the meteors much exceeds 100 miles. If they were packed as closely as possible, consistently with the condition that the distance between two neighbors *should nowhere be less than 100 miles*, the number would be increased by nearly forty per cent.

Lick Observatory

CHAPTER XVIII

THE STARS

Their Nature, Number, and Designation — Star-Catalogues and Charts — The Photographic Campaigns — Proper Motions, Radial Motions, and the Motion of the Sun in Space — Stellar Parallax

The solar system an island in space.

529. Our solar system is an island in space, surrounded by an immense void inhabited only by meteors and comets. If there were any body a hundredth part as large as the sun within a distance of a thousand astronomical units, its presence would be indicated by disturbances of Uranus and Neptune, even if it were itself invisible.

Distance of nearest star.

The nearest star, so far as known at present, is at a distance of more than 275000 astronomical units, — so remote that, seen from it, our sun would look about like the pole-star, and no telescope ever yet constructed would be able to show a single one of all the planets of the solar system.

The stars are suns, but differ greatly in size and intrinsic brilliance.

That the stars are *suns*, *i.e.*, bodies of the same nature as our own sun, composed largely of the same substances and under similar physical conditions, is shown by their spectra. Each star has its incandescent photosphere surrounded by a gaseous envelope, and while in a general way their spectra resemble each other as human faces do, each has its own peculiarities of detail. Small as they appear to us, they are many of them immensely larger and hotter than the sun; others, however, are smaller and cooler, and some hardly shine at all. They differ enormously among themselves in mass, bulk, and brightness, not being as much alike as individuals of a single race usually are, but differing as widely as whales from minnows.

Number visible to the naked eye.

530. Number of the Stars. — Those that are visible to the eye, though numerous, are by no means countless. If we examine a limited region, as, for instance, the bowl of "The Dipper," we shall find that the number we can see within it is not very large, — hardly a dozen, even on a very dark night.

In the whole celestial sphere the number of stars bright enough to be distinctly seen by an average eye is between six and seven thousand, and that only in a perfectly clear and moonless sky; a little haze or moonlight will cut down the number by fully one half. At any one time not more than two thousand or twenty-five hundred are fairly visible, since, of course, one half are below the horizon and near it the small stars (which are vastly the most numerous) disappear. The total number which could be seen by the ancient astronomers *well enough to be observable with their instruments* is not quite eleven hundred.

Total number visible by telescopes.

With even the smallest telescope the number is enormously increased. A common opera-glass brings out at least one hundred thousand, and with a $2\frac{1}{2}$-inch telescope Argelander made his *Durchmusterung* of stars north of the equator, three hundred and twenty-four thousand in number. The Yerkes telescope, 40 inches in diameter, probably reaches over one hundred million.

The constellations.

531. Constellations. — The stars are grouped in so-called "constellations," many of which are extremely ancient, all those of the zodiac and all those near the northern pole being of prehistoric origin. Their names are, for the most part, drawn from the Greek and Roman mythology, many of them being connected in some way or other with the Argonautic expedition.

In some cases the eye, with the help of a lively imagination, can trace in the arrangement of the stars a vague resemblance to the object which gives name to the constellation, as in the case of Draco for instance, but generally no reason is obvious for either name or boundaries.

Of the sixty-seven constellations now generally recognized, forty-eight have come down from Ptolemy, the others having been formed since 1600 by later astronomers, in order to embrace stars not included in the old constellations, and especially to provide for the stars near the southern pole. Many other constellations have been proposed at one time or another, but have since been rejected as useless or impertinent, though about a dozen have obtained partial acceptance and still hold a place upon some star-maps.

Sixty-seven now recognized: forty-eight Ptolemaic.

Originally certain stars were reckoned as belonging to more than one constellation, but at present this is no longer the case: the entire surface of the celestial sphere is divided up between recognized constellations. There is, however, no decisive definition of their respective boundaries, and different authorities disagree at many points. Argelander is now generally accepted as the authority for the northern constellations and Gould for the southern.

Constellation boundaries.

A thorough knowledge of these artificial star groups and of the names and places of the stars that compose them is not at all essential, even to an accomplished astronomer; but it is a matter of great convenience and of real interest to an intelligent person to be acquainted with the principal constellations[1] and to be able to recognize at a glance the brighter stars, — from fifty to one hundred in number. This amount of knowledge is easily obtained in a few evenings by studying the heavens in connection with a good celestial globe or star-map, taking care, of course, to select evenings in different seasons of the year, so that the whole sky may be covered.

Knowledge of constellations desirable, but not essential to an astronomer.

532. Methods of designating Individual Stars. — (*a*) *By Names.* About sixty of the brighter stars have names in more or less common use.

Designation of stars: by names.

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[1] In his *Uranography*, a booklet of about fifty pages, published by Ginn & Company, the author has given a brief description of the various constellations and directions for tracing them. The star-maps which accompany it are quite sufficient for this purpose, though not on a scale large enough to answer for detailed study. For reference purposes, Professor Upton's *Star Atlas* (issued by the same publishers) is recommended, or the still more elaborate (and expensive) ones of Argelander, Heis, or Klein.

A majority of these names are of Greek or Latin origin (*e.g.*, Capella, Sirius, Arcturus, Procyon, Regulus, etc.); others have Arabic names (Aldebaran, Vega, Rigel, Altair, etc.). For the smaller stars the names[1] are almost entirely Arabic.

By place in constellation.

(*b*) *By the Star's Place in the Constellation.* This was the usual method employed by Ptolemy and Tycho Brahe.

Spica, for instance, is the star in the spike of wheat which Virgo carries; *Cynosure* is Greek for "the tail of the dog" (in ancient times the constellation which we now call Ursa Minor was a dog); *Capella* is the goat which Auriga, the charioteer, carries in his arms. Hipparchus, Ptolemy, in fact all the older astronomers, including Tycho Brahe, used this method to indicate particular stars, speaking, for instance, of "the star in the head of Hercules," or in the "right knee of Boötes" (Arcturus).

By constellation and letter.

(*c*) *By Constellation and Letter.* In 1603 Bayer, in publishing his star-map, adopted an excellent plan, ever since followed, of designating the stars in a constellation by the letters of the Greek alphabet. The letters generally (not always) were applied in the order of brightness, α being the brightest star of the constellation and β the next brightest; but they are sometimes (as in the case of "The Dipper") assigned to the stars in their order of position rather than in that of brightness.

When the naked-eye stars of a constellation are so numerous as to exhaust the letters of the Greek alphabet the Roman letters are next used, and then, if necessary, we employ numbers which Flamsteed assigned a century later.

At present every naked-eye star can be referred to and identified by its letter or Flamsteed number in the constellation to which it belongs.

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[1] Allen's *Star-Names and their Meanings* (G. E. Stechert Company, New York) is the best work on the subject; full of curious and interesting information relating to the names themselves, and to the various legends connected with them and with the constellations.

By number in a star-catalogue.

(*d*) *By Catalogue Number.* The preceding methods all fail in the case of telescopic stars. To such we refer as number so-and-so of some one's catalogue; thus, "Ll., 21185" is read "Lalande, 21185," and means the star so numbered in Lalande's catalogue. At present about eight hundred thousand different stars are contained in our numerous catalogues, so that (except in the Milky Way) every star visible in a 3-inch telescope can be found and identified in one or more of them.

Synonyms.

Synonyms. Of course all the bright stars which have names have letters also and are sure to be found in every catalogue which covers their part of the heavens. A star notable for any reason has, therefore, usually many "aliases," and sometimes care is necessary to avoid mistakes on this account.

Ancient and medieval catalogues.

533. Star-Catalogues. — These are lists of stars, arranged in some regular order, giving their positions (*i.e.*, their right ascensions and declinations, or longitudes and latitudes), and usually also indicating their so-called magnitudes or brightness.

The first of these star-catalogues was made about 125 B.C. by Hipparchus of Bithynia (the first of the world's great astronomers), giving the longitude and latitude of 1080 stars. This catalogue was republished by Ptolemy 250 years later, the longitudes being corrected for precession, though not quite correctly.

The next of the old catalogues of any value was that of Ulugh Beigh, made at Samarcand about A.D. 1450. It was followed in 1580 by the catalogue of Tycho Brahe, containing 1005 stars, the last constructed before the invention of the telescope.

The "Durchmusterungs."

The modern catalogues are numerous, — already counted by the hundred. Some give the places of a great number of stars rather roughly, merely as a means of *identifying* them when used for cometary observations or other similar purposes. To this class belongs Argelander's *Durchmusterung* of the modern heavens, which contains over 324000 stars, — the largest number in any one catalogue thus far published. This has since been supplemented by Schoenfeld's *Southern Durchmusterung*, on a similar plan.

Fundamental Catalogues.

Then there are the "Fundamental Catalogues," like the Pulkowa and Greenwich catalogues, which give the places of a few hundred stars only, but as accurately as possible, in order to furnish reference points in the sky.

Zones.

The so-called "Zones" of Bessel, Argelander, Gould, and many others are catalogues covering limited portions of the heavens, containing stars arranged in zones about a degree wide in declination and running through some hours in right ascension.

The Gesellschaft catalogue.

An immense catalogue is now in process of publication under the auspices of the German Astronomische Gesellschaft, and will contain accurate places of all stars above the ninth magnitude north of 15° south declination. The observations, by numerous coöperating observatories, have occupied twenty years, but are at last finished, and three quarters of the different parts of the catalogue are already published. The Cordova catalogue and Cordova "Zones," together with the catalogues and *Photographic Durchmusterung* of the Cape of Good Hope Observatory, cover the rest of the southern heavens.

Fundamental star places determined by meridian-circle.

534. The Determination of Star Places for Catalogues. — The observations from which a star-catalogue is constructed have until lately been usually made with the meridian-circle. For the fundamental catalogues comparatively few stars are observed, but all with the utmost care and on every possible opportunity, during several years, with every precaution to eliminate all instrumental and observational errors.

Secondary places by differential observations.

In the more extensive catalogues most of the stars have been observed only two or three times, and everything is made to depend upon the accuracy of the places of the "fundamental stars," which are assumed as correct. The instrument in this case is used only "differentially" to measure the comparatively small difference between the right ascension and declination of the fundamental stars and those of the stars to be catalogued.

Photography now used.

At present, by means of photography, the catalogues are being extended to stars much fainter than those observable by meridian-circles. On the photographic plates the positions of the smaller stars are determined by reference to larger stars which appear upon the same plate, and the catalogues now in process of construction from the photographic campaign will contain between one and two million stars down to the eleventh magnitude.

Reduction of mean place to apparent place and *vice versa.*

535. Mean and Apparent Places of the Stars.—The modern star-catalogue contains the *mean* right ascension and declination of its stars at the beginning of some designated year, *i.e.*, the place the star would occupy on that date if there were no *equation of the equinoxes*, *nutation*, *aberration*, or *proper motion.* To get the actual (apparent) right ascension and declination of a star *for some given date* (which is what we always want in practice), the catalogue place must be "reduced" to that date, *i.e.*, it must be corrected for precession, aberration, etc. The operation with modern tables and formulæ is not a very tedious one, involving perhaps five minutes work, but without it the catalogue places are useless for accurate purposes. *Vice versa*, the observations of a fixed star with the meridian-circle do not give its *mean* right ascension and declination ready to go into the catalogue, but the observations, before they can be tabulated, must be reduced backwards from the apparent place observed to the mean place for some chosen "epoch."

Star charts.

536. Star Charts and Stellar Photography.—For certain purposes accurate *star charts* are even more useful than catalogues. The old-fashioned way of making such charts was by plotting the results of zone observations, but at present it is being done, by means of photography, vastly better and more rapidly. A coöperative campaign began in 1889, the object of which is to secure a photographic chart of all the stars down to the

The photographic campaign.

fourteenth magnitude. The work is now (1902) fully three fourths done. Eighteen different observatories have participated in the work. From these chart plates extensive catalogues are also being made, as already mentioned.

No limit yet found to faintness of stars that can be photographed.

One of the most remarkable things about the photographic method is that with a good instrument there appears to be no limit to the faintness of the stars that can be photographed; by increasing the time of exposure, smaller and smaller stars are continually reached. With the ordinary plates, and exposure times not exceeding twenty minutes, it is now possible to get distinct impressions of stars that the eye cannot possibly see with the telescope employed.

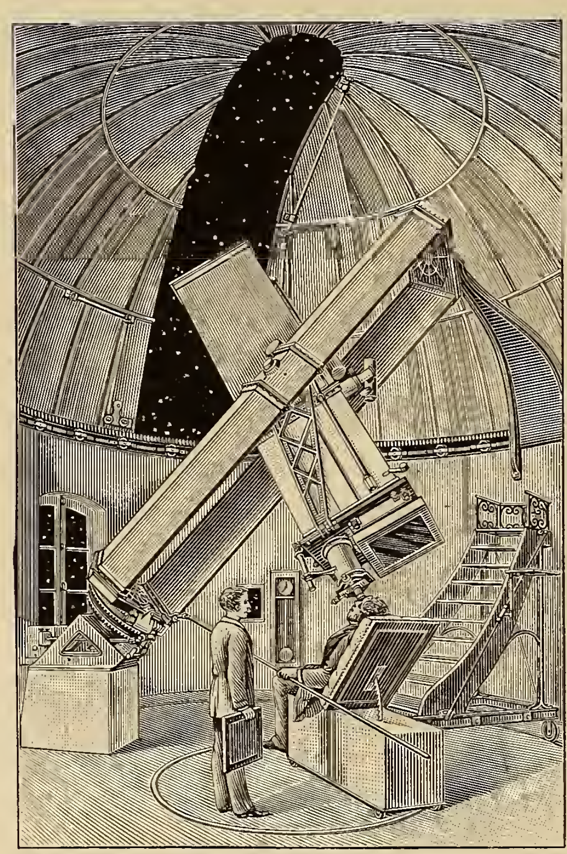

Fig. 173. — Photographic Telescope of the Paris Observatory

The Paris photographic telescope.

Fig. 173 is a representation of the Paris instrument of the Henry Brothers, which was the first employed in such work and was adopted as the typical instrument for the charting operation. It has an aperture of about 14 inches and a length of about 11 feet, the object-glass being specially corrected for the photographic rays. A 9-inch visual telescope is inclosed in the same tube, so that the observer can watch the direction of the instrument during the whole operation.

The instruments used at the other observatories differ in mechanical arrangements, but all have lenses of the same aperture and focal length, the scale of all the photographs being 1′ to a millimeter, — the same as that of Argelander's charts.

As already mentioned, these charts furnish the material for a very extensive catalogue.

Other large photographic telescopes.

Several other very large photographic telescopes have already been constructed. The Bruce telescope, presented to the Harvard College Observatory by the late Miss Bruce of New York, has for its objective a four-lens photographic doublet 2 feet in diameter, but with a focal length of only 11 feet, — the same as those mentioned above, — so that its negatives are on the same scale. While the ordinary photographic lens will cover an area of only about two degrees square, this covers from five to six degrees square, and with a very much diminished time of exposure. It has been sent to the Harvard subsidiary observatory at Arequipa, Peru, where it is employed in the photography and spectroscopy of the southern heavens.

The new telescopes at Greenwich and the Cape of Good Hope have the same aperture, but are much longer. Both have visual finders 18 inches in diameter.

The enormous instrument at Meudon (near Paris) has also two telescopes combined, — a visual telescope of 32 inches aperture and a photographic of 25 inches, each 55 feet focal length.

Still more recently the Potsdam Astrophysical Observatory has mounted an immense instrument, shown in the frontispiece, the photographic object-glass of which has a diameter of 31½ inches, with a focal length of 43 feet, and the visual object-glass a diameter of 20 inches. This long-focus instrument will, however, be used mainly for other purposes than charting.

STAR MOTIONS

The so-called fixed stars really moving.

537. In contradistinction from the planets, or "wanderers," the stars are called "fixed," because they keep their relative positions and configurations sensibly unchanged for centuries. Delicate observations, however, separated by sufficient intervals of time, show that the fixity is not absolute. Nearly two hundred years ago (in 1718) it was discovered by Halley that Arcturus and Sirius had changed their places since the days of Ptolemy, having moved southward, the first by a full degree and the other about half as much. Indeed, even to the naked eye, these two stars no longer fit certain alignments described by Ptolemy.

Modern observations show clearly that the stars are really all in motion, "drifting" upon the celestial sphere. Not only so, but the spectroscope now makes it possible to measure their rate of motion towards or from the earth, and it appears on the whole that their velocities are of the same order as those of the planets: they are flying through space incomparably more swiftly than cannon-shot, and it is only because of their inconceivable distance from us that they seem to go so slowly.

Common motions due to earth's motions only apparent.

538. Common Motions. — If we compare a star's position (*i.e.*, its right ascension and declination) as determined to-day by a meridian-circle with that observed one hundred years ago, it will always be found to have altered considerably. The change, however, is *mainly* due, not to any real change in the position of the star, but to precession, nutation, and aberration, already discussed (Secs. 165–171).

These depend upon variation in the direction of the earth's axis and upon the swiftness of her orbital motion and are not real changes of the star's direction from the earth. They are only *apparent* displacements and are called "common" motions because they are shared alike by all stars in the same region of the sky. They do not in the least affect their apparent configurations and angular distances from each other.

Proper motions determined by comparison of old with recent star-catalogues.

539. Proper Motions. — But after allowing for all these common motions of the stars, it generally appears that in the course of a century the stars have really changed their places *with reference to each other*, each having a *motus peculiaris*, or "proper motion" of its own, the word "proper" being here the antithesis of "common." Of two stars side by side in the same telescopic field of view the proper motions may be very different in amount, or even directly opposite, while the common motions, due to precession, etc., are, of course, sensibly identical.

About 175 stars are at present known to have a proper motion exceeding 1″ annually, but the number is being constantly increased by additions from among the fainter stars. Even the largest of these proper motions (always expressed in seconds of arc) is very small.

The maximum at present known (discovered in 1898) is that of a little star of the eighth magnitude, known as "G.C.Z., V, No. 243" (*i.e.*, Gould's Cordova Zones, Fifth Hour, No. 243), which drifts 8″.7 yearly. The next in magnitude, and for a long time at the head of the list, is that of the seventh-magnitude star, 1830 Groombridge, the so-called "runaway star," which has an annual drift of 7″. Neither of these stars is visible to the naked eye. It will take two hundred years for the first of them to drift a distance equal to the moon's apparent diameter.

Maximum known 8″.7 yearly.

As might be expected, the proper motions of the bright stars *average* higher than those of the faint ones, since, on the whole, the bright stars are nearer; but the faint stars are so much more numerous that among them many drift faster than any of the fewer bright ones.

Average motion greater for the nearer stars.

The *average* proper motion of the first-magnitude stars is about $\frac{1}{4}''$ annually, and that of the sixth-magnitude stars (the smallest visible to the naked eye) is about $\frac{1}{25}$ of a second.

These motions are always sensibly rectilinear.

Table IV of the Appendix, in connection with other matters, gives the proper motions of about forty of the nearer stars which also, as a rule, are the stars having the larger proper motions.

Hitherto the determination of proper motions has rested almost entirely upon the comparison of remotely dated star-catalogues, but it is likely that hereafter much more rapid progress will be made by the comparison of photographic charts, in which consideration of the *common* motions is unnecessary, as these affect alike all the stars on each negative.

Advantage of photography in determining proper motion.

540. Real Motions of Stars. — The proper motion of a star gives us very little knowledge as to the star's real motion in miles unless we know the star's distance, nor even then unless we also know its rate of motion towards or from us. The

Real motion of a star.

Reduction of proper motion to miles requires knowledge of distance.

proper motion derived from the comparison of the catalogues of different dates is only the angular value of that part of the whole motion which is *perpendicular to the line of vision*, the "cross" or "thwartwise" motion, as it may be called. A star moving directly towards or from the earth has no proper motion, *i.e.*, no change of apparent place to be detected by comparing observations of its position.

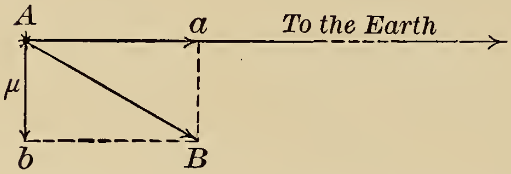

FIG. 174. — Components of a Star's Proper Motion

Fig. 174 illustrates the matter. If a star really moves in a year from *A* to *B*, it will seem to an observer at the earth to have traversed the line *Ab*, and the proper motion (in seconds of arc) will be $206265 \times \frac{Ab}{\text{distance}}$. Since *Ab* cannot possibly be greater than *AB*, we are able in some cases to fix a minor limit to the star's velocity.

According to the determination of Brünnow, accepted until lately, the distance of 1830 Groombridge is a little over two million astronomical units; and therefore, since *Ab* subtends an angle of 7″ at the earth, its length must be at least $\frac{7 \times 2\,000000}{206265}$ astronomical units, which, reduced to miles and divided by the number of seconds in a year, corresponds to a velocity exceeding 200 miles a second.

More recent observations by Kapteyn make the distance of this star considerably less — about 1 400000 astronomical units — and proportionally reduce the cross motion, *Ab*, to about 140 miles a second.

For the star of greatest proper motion, G.C.Z., V, 243, the cross motion comes out about 80 miles per second, so that the "runaway star" still holds the record for real swiftness.

Formula for cross motion.

The formula for this "cross" or "thwartwise" motion (*Ab* in Fig. 174) is

$$\Theta \text{ (miles per second)} = 2.944 \frac{\mu}{p} = 0.903\, y \times \mu,$$

where μ is the annual proper motion of the star, p its parallax (both in seconds of arc), and y its distance in "light-years." (See Secs. 546 and 547.)

In many cases a number of stars in the same region of the sky have proper motions practically identical, making it almost certain that they are in some sense neighbors and really connected, — very likely by community of origin. In fact, it seems the rule rather than the exception that stars which are apparently near each other and about alike in brightness are really comrades. They show, as Miss Clerke expresses it, a distinctly "gregarious" tendency. In certain cases, however, there are groups of stars in which some conspicuous members have different proper motions from the others, and these discordant motions will in time destroy the configuration. The "Dipper" of Ursa Major is a case in point. The two extreme stars, α and η, are, according to Flammarion, moving in a nearly opposite direction from the others, so that about one hundred thousand years ago the "Dipper" was no dipper at all, and will not be one a hundred thousand years hence. The other stars of the group maintain their configuration.

Gregarious tendency of stars.

Proper motions in Ursa Major.

541. Motion in the Line of Sight, or "Radial Velocity."[1] — Observations of the proper motions of stars furnish no information as to the rate at which the stars are receding or approaching; but if a star is bright enough to give an observable spectrum, its radial velocity can be determined by means of the spectroscope and the application of the Doppler-Fizeau principle (Sec. 254). If the star is receding, the lines of its spectrum will be shifted towards the red, and towards the blue if it is coming nearer. The shift is ascertained by arranging the telespectroscope (Sec. 244) so that by a comparison prism the observer shall have, close together or superposed, the spectrum of the star he is dealing with and of some substance (hydrogen, sodium, iron, or titanium) whose lines are present as dark lines in

Spectroscopic determination of radial velocity.

Application of the Doppler-Fizeau principle.

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[1] We shall follow the French usage in employing the term "radial velocity" (*vitesse radiale*) to denote the rate at which a body is changing its distance from the observer. The equivalent expression, "motion in line of sight," is rather clumsy.

the star spectrum; he can then appreciate and measure any displacement of the stellar lines, as illustrated by Fig. 98, Sec. 254.

First success by Huggins in 1867.

Sir William Huggins, in 1867, was the first to apply this method, and obtained some very interesting results (especially the determination of the radial motion of Sirius), quite sufficient to establish the feasibility of his method. From the insufficient power of his instruments, however, they can now be regarded only as approximations.

Visual observations unsatisfactory.

The work was followed up for several years at Greenwich and some other places, but so long as *visual* observations were depended upon the results were not very satisfactory. Visual observations of this kind are extremely difficult; the star spectra are very faint, the displacements of the lines very minute, and the lines themselves often broad and hazy and ill adapted for accurate measurement.

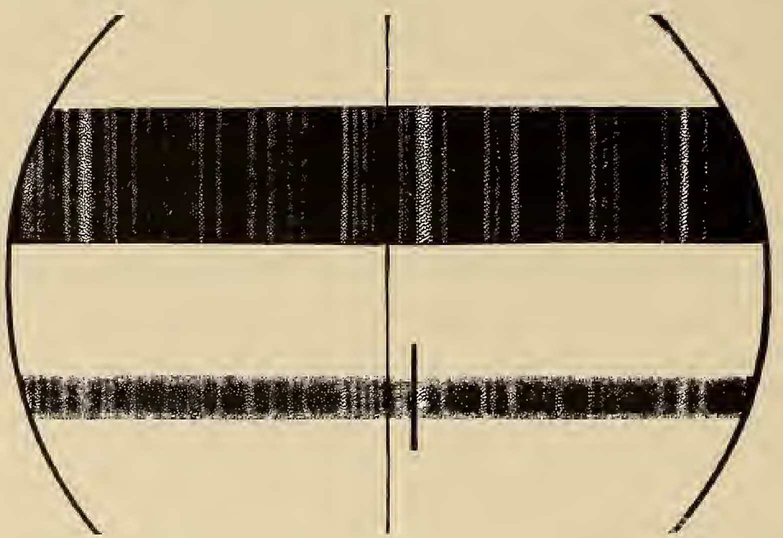

FIG. 175. — Spectrum of *α* Aurigæ compared with Hydrogen
Vogel

Keeler's work on nebulæ.

In the case of the nebulæ, however, which give spectra containing sharp, bright lines, Professor Keeler of the Lick Observatory has made visual observations which fairly compete with photographic work.

Application of photography. Vogel's work.

542. Spectrographic Determination of Radial Velocity. — The unsatisfactory results of visual observations led Vogel in 1888–89 to apply photography, and with immediate success. In this case the difficulties arising from the faintness of the star spectra can be largely overcome by prolonged exposure, and all necessary measurements can be made at leisure under the microscope.

Fig. 175 (borrowed by permission from Frost's translation of Scheiner's *Astronomical Spectroscopy*) shows very perfectly the actual appearance of part of the negative of the spectrum of α Aurigæ (Capella) and the corresponding part of the solar spectrum as seen under the microscope with which the measurements are made. The solar spectrum is, of course, on a separate plate, but this plate and the star negative are clamped together so as to make the lines correspond and facilitate the identification of lines in the star spectrum. (Note in passing the perfect correspondence between the spectrum of this star and that of the sun.) The sharp black line which crosses the narrow star spectrum is the "Hydrogen γ" bright line in the spectrum of a Geissler tube placed in the cone of rays about 2 feet above the slit-plate and illuminated by electricity for a few seconds at different times during the long exposure (an hour or so) which is required for the star spectrum.

One sees easily that in this case the star line is shifted slightly to the right, but it appears to be so poorly defined that accurate measurement would be difficult. For the methods by which this difficulty is overcome, and for the corrections required on account of the motion of the earth and other causes, the reader is referred to the book from which the figure is taken.

Fig. 176 is from a photograph of the new Potsdam spectrograph, attached to the great telescope shown in the frontispiece. It is a much more powerful and perfect instrument than that used by Vogel in the work above mentioned.

In 1889 the probable error of a determination of radial velocity was about a mile a second; at present (1902) it does not exceed one fourth of a mile for the best observers and instruments under favorable conditions.

Table V of the Appendix presents the results of Vogel for the fifty-one stars that he had been able to deal with up to 1892, with the addition of one or two from other observers. His telescope had an aperture of only 11 inches, which limited him to the brighter stars. It has now been replaced by the much larger instrument, figured in the frontispiece.

Maximum radial velocity so far measured about 60 miles a second.

The maximum velocity indicated by his observations of 1888–89 is that of α Tauri, 30.1 miles a second, receding. The next in order is that of γ Leonis, 24.1 miles, approaching. Belopolsky, at Pulkowa, has since found for ζ Hercules the higher velocity of 44 miles, approaching; and Campbell, at the

Lick Observatory, finds for μ Cassiopeiæ the still higher velocity of 61 miles, also approaching, and for δ Leporis and θ Canis Majoris a nearly equal receding velocity.

Since 1890 the same line of work has been taken up successfully by many observers, especially by Belopolsky at Pulkowa, and in this country by Keeler and Campbell at the Lick Observatory, and by Frost at the Yerkes.[1] Fig. 177 is enlarged from one of Frost's photographs of the spectrum of Polaris (a positive in this case) compared with that of the metal *titanium*,

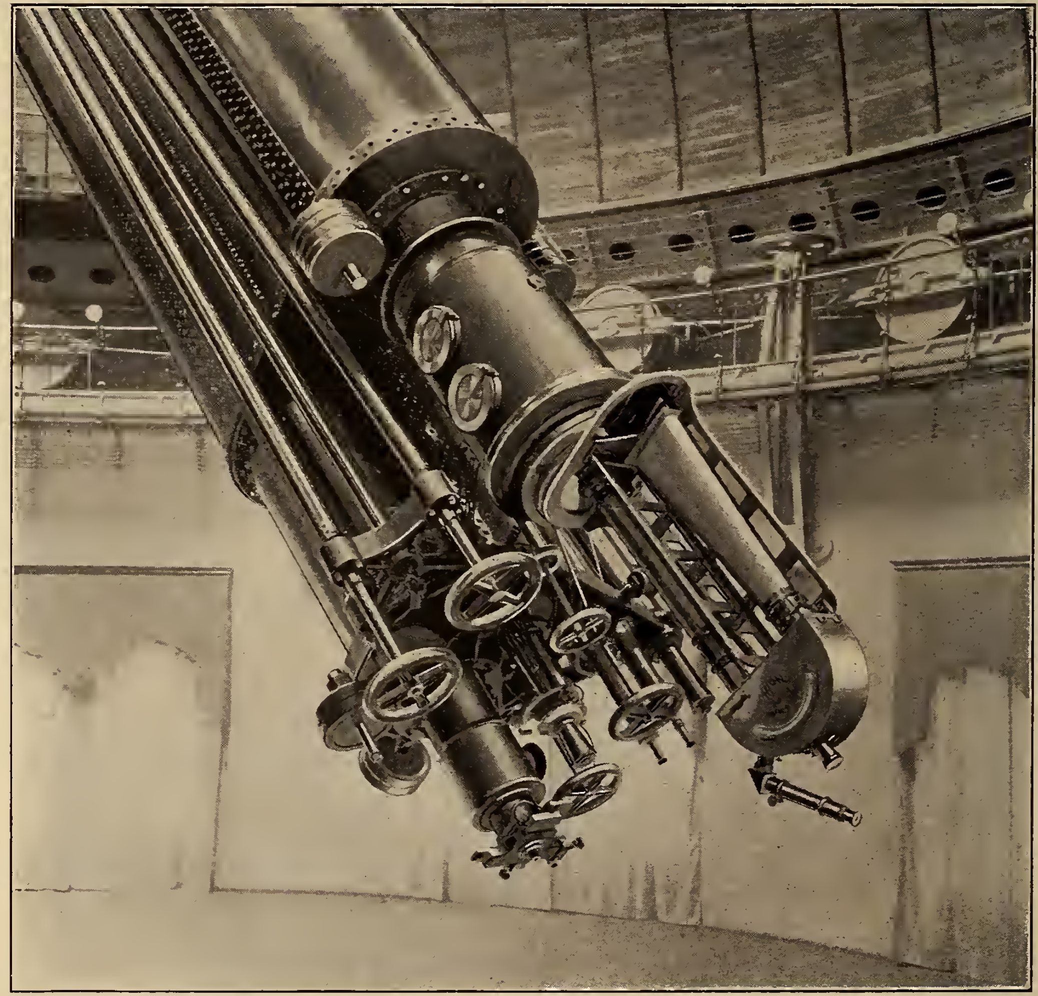

Fig. 176. — The New Potsdam Spectrograph

which has been found specially advantageous for such comparisons. In this case, the radial velocity of the star is so small (about 8 miles a second) that the shift of the lines in the figure is hardly perceptible to the eye.

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[1] See Fig. 180, on page 505, for the Yerkes spectrograph.

Causes which modify results for radial motion.

Observations by Humphreys and Mohler of Baltimore in 1895 (already mentioned in Sec. 256) show that under heavy pressure the spectrum lines of many elements shift slightly towards the red, very much as if the luminous object were receding. The shift under a given pressure is, however, different for different substances and for different lines of the same substance. It is always minute, never, even under a pressure of ten or twelve atmospheres, exceeding the displacement that would be due to a receding velocity of 1 or 2 miles a second, but it is quite sufficient to require to be examined and taken into account in all applications of Doppler's principle.

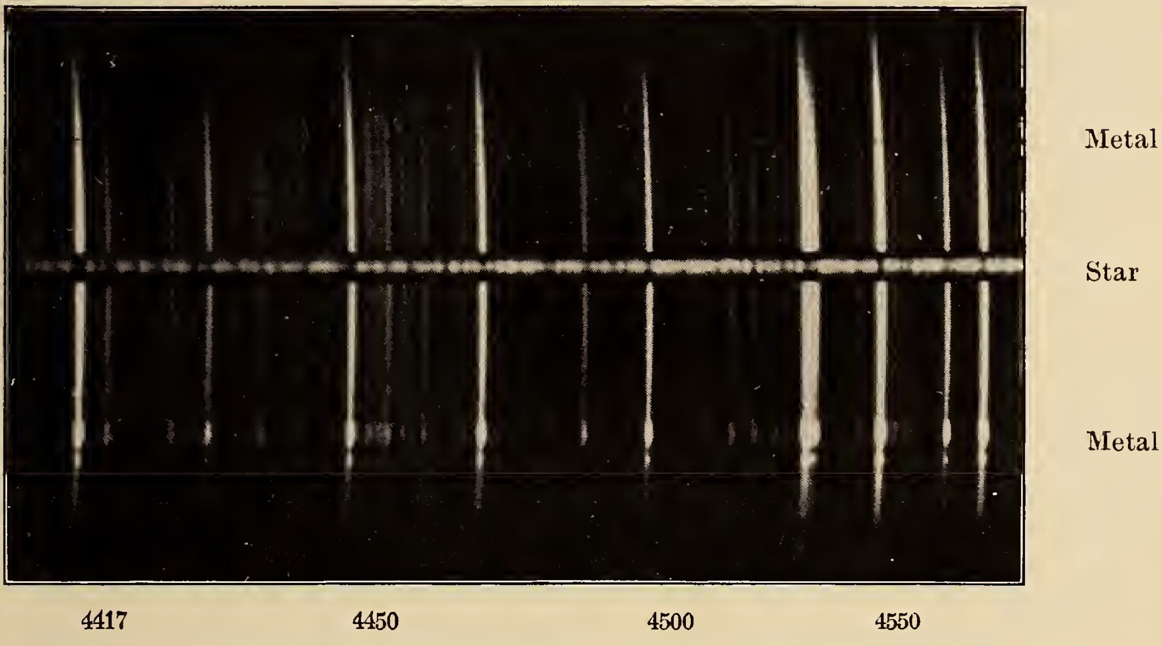

Fig. 177. — Spectrum of Polaris compared with Titanium
Frost

The sun's motion in space.

543. The Sun's Way, or Motion of the Solar System. — The sun, like other "stars," is traveling through space, taking with it the earth and the planets.

Sir William Herschel was the first to investigate and determine the direction of this motion, more than a century ago. The principle involved is this: the apparent motion of a star relative to the sun is made up of its own real motion combined with the sun's motion *reversed*. If the stars, therefore, were absolutely at rest, they would apparently all drift bodily in a direction opposite to the sun's real motion. If, as is the fact, they themselves are in motion, and if their motions are indiscriminately in all possible directions (an assumption probable as an approximation to the truth, but which can hardly be proved as yet), there will be, *on the whole*, a similar drift. Those in that quarter of the sky towards which we are approaching will, *on the whole*, open out from each other, and those in the rear will close up behind us, while in the region of the sky between, they will, on the whole, drift backwards, — just as one walking in a park filled with people moving indiscriminately in different directions would, on the whole, find that those in front of him appear to grow larger,[1] and the spaces between them to open out, while at the sides they would drift backwards, and in the rear close up.

Determination of its direction by means of proper motions of stars.

Determination by radial motions.

Again, from the *radial motions* of the stars spectroscopically measured a result can be obtained. In the portion of the heavens towards which the sun is moving the stars will, on the whole, seem to approach, and in the opposite quarter to recede.

The individual motions, proper and radial, lie in all directions; but when we deal with them by the thousand the individual is lost in the general, and the prevailing drift appears.

Apex of the sun's way: approximate position.

About twenty different determinations of the point in the sky towards which the sun's motion is directed have been thus far made by various astronomers. There is a reasonable accordance of results, and they all show that the sun, with its attendant planets, *is moving towards a point on the borders of the constellation of Hercules, having, according to Newcomb, a right ascension of about 277°.5, and a declination of about 35°.* This point is called the *Apex of the sun's way.*

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[1] Theoretically, of course, the stars towards which we are moving must appear to grow *brighter* as well as to drift apart, but this change of brightness, though real, is entirely imperceptible within a human lifetime.

Uncertainty as to exact position.

There is, however, a curious systematic difference between the results obtained by comparing the proper motions of stars that drift very slightly with those that drift more rapidly. Dividing them into four groups, some 550 of the slower stars give for the "apex" a declination of about 42°, those of the next grade of about 40°, those still nearer about 35°, while those that are nearest us, or at least have the largest proper motion, push the point still nearer to the equator, to a declination of about 30°. The right ascension deduced for the apex shows no such systematic discordance.

This probably indicates that the motions of the stars are not absolutely indiscriminate, but that those that are near to us have some common drift of their own.

Velocity about 11 miles a second.

As to the velocity of the sun's motion in space, the spectroscopic results, which are on the whole more trustworthy since they involve no assumption as to the distance of the stars, indicate that it is *about 11 miles a second*, which probably is very near the truth.

No central sun.

544. The Imagined "Central Sun." — We mention this subject simply to say that there is no satisfactory foundation for the belief in the existence of such a body. The idea that the motion of our sun and of the other stars is a revolution around some great central sun is a very fascinating one to certain minds, and one that has been frequently suggested. It was seriously advocated half a century ago by Maedler, who placed this center of the universe at Alcyone, the principal star in the Pleiades.

It is certainly within bounds to deny that there is any conclusive evidence of such a motion, and it is still less probable that the star Alcyone is its center, if the motion exists. (But see Sec. 609, last paragraph.)

So far as we can judge at present, it is most likely that the stars are moving, not in regular closed orbits around any center whatever, but rather as bees do in a swarm, — each for itself, under the action of the predominant attraction of its nearest neighbors. The *solar* system is an absolute monarchy, with the sun supreme. The great *stellar* system appears to be a republic, without any such central and dominant authority.

The stellar system a republic.

PARALLAX AND DISTANCE OF THE STARS

Heliocentric or annual parallax.

545. Heliocentric or Annual Parallax. — This has already been defined (Sec. 78) as the difference between the direction of a star seen from the sun and from the earth, which difference, if the star is not at the pole of the ecliptic, varies throughout the year with an *annual* periodicity. In the case of a star the *geocentric* or "diurnal" parallax is absolutely insensible, — hopelessly beyond all present power of measurement.

When, therefore, we speak of a *star's* parallax the *heliocentric* parallax is to be understood. Moreover, unless otherwise clearly indicated, the *maximum* value of the star's heliocentric parallax is always meant. Twice a year the earth is so situated that the sun and star are 90° apart in the sky, when the longitude (celestial) of the sun is 90° greater or less than that of the star. At that moment the radius vector of the earth's orbit is perpendicular to a line drawn from the earth to the star, and the star's annual parallax has its greatest possible value.

Maximum value reached twice a year.

The parallax of a star may therefore be defined, as the term is ordinarily used, to be the *angle subtended by the semi-major axis of the earth's orbit when viewed perpendicularly from the star.* In Fig. 178, R is the distance from the earth to the sun, D from the sun to the star, and the angle p is the star's parallax. If we can measure p'' (*i.e.*, p in seconds of arc), the distance of the star at once follows from the equation

Definition of star's parallax.

Relation between star's distance and parallax.

$$D = \frac{R}{\sin p''} = R \times \frac{206265}{p''},$$

R being the radius of the earth's orbit, 93 000000 miles.

Parallactic orbit of a star.

546. The Star's Parallactic Orbit. — We may look at the matter differently. In accordance with the principle of relative motion (Sec. 354), every star really at rest (leaving aberration out of the account at present) must appear to us to move in a little "parallactic orbit" 186 000000 miles in diameter, the precise counterpart of the earth's orbit, and having its plane parallel to the ecliptic; in this little orbit the star keeps always *opposite to the earth.* If the star is at the pole of the ecliptic, we see this parallactic orbit as a circle; if in the ecliptic, edgewise, as a short straight line; in intermediate (celestial) latitudes, as an oval. The semi-major axis of this apparent parallactic orbit is, of course, the star's parallax. (In Fig. 178 the dotted oval is the parallactic orbit of the star *s*, as seen from the solar system.) If the star is drifting (proper motion), this motion will, of course, be combined with the parallactic movement, but the two can easily be separated by calculation.

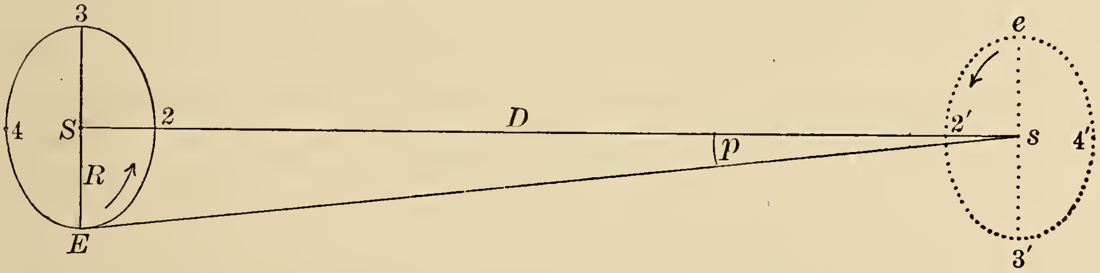

Fig. 178. — Heliocentric Parallax

Maximum parallax at present known, 0″.75 for *a* Centauri.

For *a* Centauri, which is our nearest neighbor among the stars so far as we know at present, the parallax is only 0″.75, and there are but seven other stars which are known to have a parallax as large as 0″.3. Indeed, the whole number of those which are fairly determined to have a parallax of 0″.1 and over is less than forty.

The light-year.

547. Unit of Stellar Distance; the Light-Year. — The distances of the stars are so enormous that the radius of the earth's orbit, the astronomical unit hitherto employed, is too small for a convenient measure. It is better, and now usual, to take as the unit of stellar distance the so-called "light-year," *i.e.*, the distance which light travels in a year, — about sixty-three thousand[1] times the distance of the earth from the sun.

A star with a parallax of one second is at a distance of 206265 astronomical units. Dividing this by 63000, we find its distance in light-years, $D_y = \frac{3.26}{p''}$.

Formula for distance in light-years.

A star with a parallax of half a second is at a distance of 6.52 light-years, and a star with a parallax of one tenth of a second is at a distance of 32.6 light-years.

No star as near as three light-years.

So far as can be judged from the scanty data available, it appears that few, if any, stars are within a distance of three light-years from the solar system, — not one has thus far been discovered; that the naked-eye stars are probably, for the most part, within two or three hundred light-years; and that many of the telescopic stars must be some thousands of years away.

Table IV of the Appendix contains a list of the parallaxes thus far best determined.

Difficulty of measuring stellar parallax.

548. The Determination of Stellar Parallax. — It is obvious, therefore, that while simple enough in principle, the measurement of a star's parallax is practically one of the most delicate and difficult of all astronomical operations; and there is no way at present of evading the difficulty or "flanking the position," so to speak (as in the case of the solar parallax), by measuring the parallax of some near object or utilizing our knowledge of the speed of light.

Bessel's success in 1838.

Many attempts were made by early astronomers to measure the parallax of stars, but with no real success until Bessel, in 1838, succeeded in determining the parallax of 61 Cygni, a little star of the sixth magnitude, which had for some time been interesting astronomers on account of its great proper motion of 5″ a year. He made his observations with a heliometer and ascertained its parallax to be about half a second, but more recent determinations bring it down to 0″.40.

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[1] This number is found by dividing the number of seconds in a sidereal year, 31 558149, by 499, the number of seconds required by light to travel from the sun to the earth. The exact quotient is 63243. The light-year bears to the astronomical unit almost exactly the same ratio as the mile to the inch.

Henderson and *α* Centauri.

Almost simultaneously Henderson announced the parallax of *α* Centauri as 0″.9, according to meridian-circle observations at the Cape of Good Hope. The star has a large proper motion and is one of the brightest in the heavens, but is not visible in our latitudes. It still holds its place as our nearest neighbor, though later observations show that its parallax is only 0″.75.

Two methods of procedure have thus far been used, known as the *absolute method* and the *differential method.*

The absolute method.

549. The Absolute Method. — This consists in making with a meridian-circle, or some equivalent instrument, numerous observations of the star's apparent right ascension and declination throughout the year, especially at the two seasons when the parallax has its largest value. These observations are then carefully corrected for aberration, precession, and nutation, also for the star's proper motion, and for any known errors due to the action of the seasons on the instrument. If the observations and their corrections are perfectly accurate, they will give a set of slightly different positions for the star during the year, which, when plotted, will all fall on the circumference of a little oval, — the star's "parallactic orbit." One half the angular length of the orbit will be the star's parallax.

Embarrassed by large corrections.

But the corrections to be applied to the observations are enormous compared with the parallax itself. While the parallax is only a fraction of a second, the aberration corrections run up to 41″. The instrumental correction is especially troublesome, because it runs its course yearly, just as the parallax does, and any outstanding error confounds itself with the parallax in a manner almost inextricable. Hence, comparatively little success has attended operations of this sort, though it was by such observations that the parallax of *a* Centauri was first detected, as already stated.

The differential method.

550. The Differential Method. — This consists in determining the position of the suspected star at different times during the year, not absolutely, but *with reference to the smaller stars apparently near it, though presumably at a great distance beyond.* This almost entirely obviates the difficulty due to aberration, precession, and nutation, since these affect all the stars concerned in the operation nearly alike; the observations therefore need correction only for the *difference* between the aberration, etc., of the investigated star and that of each of its neighbors, and this small differential correction can be easily computed with very great accuracy. To a considerable extent also the method evades the effect of refraction and that of temperature disturbances upon the instrument, since any displacement of the instrument does not perceptibly affect the *relative position* of the stars seen through it. *Per contra*, the method measures, not the whole parallax of the star investigated, but only the *difference between its parallax and that of the stars with which it is compared.*

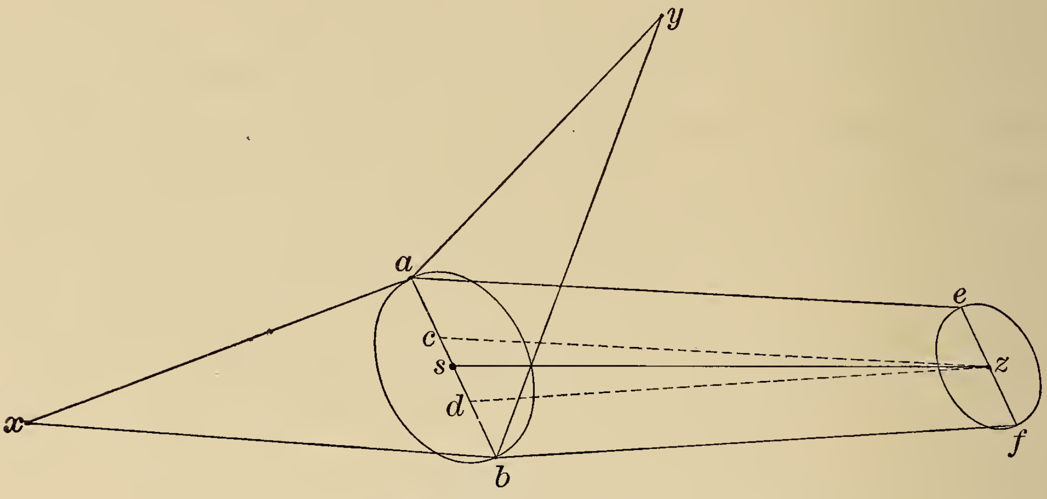

FIG. 179. — Differential Measurement of Parallax

It avoids large corrections.

Suppose, for instance (Fig. 179), that in the same telescopic field of view we have the star *s*, which is near us, the stars *x* and *y*, which are so remote that they have no sensible parallax at all, and the star *z*, which is more remote than *s* but has a sensible parallax of its own. *s* and *z* will describe their parallactic orbits every year, just alike in form, and parallel, but the orbit of *s* will be much larger than *z*'s. If now, during the year, we continually measure the distance and the direction from *x* or *y* to the apparent places of *s* and *z* in their parallactic orbits, the results will give us the true dimensions of their two orbits. If, however, we had taken *z* as the reference point from which to measure the parallactic motion of *s*, we should have found less than the true value, as is obvious from the figure. Considering only the measures made at the moments when *s* and *z* are at the extremities of their parallactic orbits, the lines really measured from the star *z* to *s* will be *ea* and *fb*. If we assume that the parallax of *z* is insensible, *i.e.*, that its parallactic orbit is a mere point, these measured lines must be used in computation as if they were reckoned from the point *z* and were *zc* and *zd*; the major axis of the parallactic orbit of *s* would then come out as *cd* instead of *ab*, and the *computed* parallax *cs* will be less than the true parallax *as* by the amount *ac*, which equals *ez*, the parallax of *z*.

Parallax measured is only the difference of parallax between stars.

It follows that if the measurements are absolutely accurate, the parallax deduced by this method can never be too large, but may be too small, — the distance of the star will be more or less exaggerated. If, however, the small reference stars are so remote that their parallax and proper motions are really insensible (*i.e.*, less than $0''.01$), the changes in the relative position of the star under investigation, after the correction for its proper motion has been applied, will be due simply to its parallax.

Parallax too small: distance too great.

551. Instrumental Work. — If the comparison stars are very near the one under investigation, the necessary measurements may be made with the filar micrometer; but if the distance exceeds a few minutes, we must resort to the heliometer

Instrument: filar micrometer or heliometer.

(Sec. 72), with which Bessel first succeeded, or we may employ photography, which the late Professor Pritchard at Oxford has done with some success. This has the advantage over the heliometer that the actual observations (in taking the photographs) can be made much more rapidly than the heliometer measures, and the subsequent measurement of the photograph can be made under the microscope at leisure; moreover, the suspected star can be compared with a considerable number of others, while the heliometer is usually limited to two or three, on account of the long time required to make each complete measurement. On the other hand, when the suspected star is much brighter than the reference stars its photographic image is so large and hazy as to render the measures of its position difficult and uncertain.

Photography also used.

On the whole, the *differential method*, notwithstanding the fundamental objection which has been mentioned, is at present much more trustworthy than the absolute.

Negative parallax: comparison stars nearer than the suspected star.

Negative Parallax. Now and then it happens that the observations appear to show a small *negative* parallax for the star. This may indicate simply insufficient accuracy of observation or computation, or, if the observations have been made by the differential method, it may mean that the investigated star is really *beyond* the comparison stars and therefore has a larger parallax than they, so that the difference between its parallax and that of the comparison stars comes out negative. Of course a real "negative parallax" is impossible. It would mean, as some one has said, that "the star is somewhere on the other side of nowhere."

552. Selection of Stars to be examined for Parallax. — It is obviously necessary to choose for observations of this sort stars that are presumably near. The most important indication of proximity is *a large proper motion.* It is easy to see that if the observer were brought nearer to a star, the rapidity of its apparent drift across the sky would be increased; and accordingly it

Large proper motion the best indication of probable nearness.

is practically certain that the stars of large proper motion *average* nearer than those for which the motion is smaller, though the indication is not to be depended on in any individual case: if a star happened to be moving directly towards or from us, its proper motion would be zero, however near it might be. Brightness is, of course, also confirmatory, but nearly all the bright stars have been already attacked. Their number is not very great, and the majority of them turn out to be much farther away than 61 Cygni. Among the millions of faint stars it is quite likely that some few individuals at least will be found nearer than even *a* Centauri.

Parallax determined by spectroscopic observations of binary stars.

553. Possible Spectroscopic Method of the Future. — It will appear later that in the case of certain binary stars (Sec. 587), which have the plane of the orbit nearly directed to the sun, the spectroscope will enable us to determine the actual speed with which they move in this orbit and the true dimensions of the orbit in miles. Combining this with the *apparent dimensions of their orbits in seconds of arc*, it will be possible to compute their distance far more accurately than by any direct measure of their parallax. But it will be many years yet before the necessary measures can become available in more than one or two instances, since the orbital periods of the binaries which have an apparent orbit of measurable size are mostly long, ranging from twenty years to several hundred, and the orbits that lie in a favorable position are not numerous.

Data too scanty for broad generalizations.

554. General Conclusions. — It is obvious that the data so far obtained are too scanty to warrant many general conclusions. We have not a sufficient number of well-defined parallaxes to furnish a safe basis for averages, — say forty or fifty only, — and the smaller ones among them are subject to a probable error of at least twenty-five per cent. Consequently all calculations as to mass, brightness, etc., of *individual stars* are likely to differ widely from the truth. It is, however, already clear that, taken in classes, the stars of large proper motion average the nearest, Caution in applying average results to individual stars.

the distance being roughly inversely proportional to the apparent rapidity of their drift; and, further, that on this assumption the majority of the stars must be at a distance greater than fifty light-years, and the remoter stars in all likelihood many times more distant still.

Something similar appears to be true in the relation between the brightness of the stars and their distance.

Kapteyn, from stellar parallaxes already determined, also finds evidence of a roughly uniform distribution of stars in space, leaving the Milky Way out of the question.

Another interesting fact appears, viz., that the stars which show a spectrum like the sun's are on the average much nearer than the so-called "Sirian" stars.

But the student must be warned again that he cannot safely apply to individuals the results of such general averages. Any small star is not at all unlikely to be really brighter and larger than a bright star near it, just as in the case of two persons, one a youth and the other a man of mature age, it would be unsafe to predict from the mere difference of age which would have the longer life.

EXERCISES

1. Assuming the parallax of 61 Cygni as 0″.40, and that it is approaching the sun at the rate of 34.5 miles a second, how many years will it be before its brightness is increased *10 per cent* by the diminution of its distance? *Ans.* 2050 years.

2. Assuming the distance of 61 Cygni as 8.15 light-years, and that its radial and "thwartwise" velocities are 34.5 and 38 miles a second, respectively, find how near the star will come to the sun if it keeps up this motion uniformly, and how long it will take to reach this point of nearest approach.

Ans. { Nearest approach, 6.03 light-years.
Time, 19900 years.

3. Make the same calculation for α Aquilæ, assuming its parallax as 0″.20, its proper motion as 0″.65 annually, and the rate at which it is approaching as 24 miles a second.

4. What would be the time required to make the journey to Sirius (parallax $0''.38$) at the rate of 60 miles a second, and the fare at 1 cent a mile?

5. Deduce the formulæ of Sec. 540, viz.:

$$\Theta \text{ (miles per second)} = 2.944 \frac{\mu}{p} = 0.903\, y \times \mu,$$

using the data of Secs. 546–547.

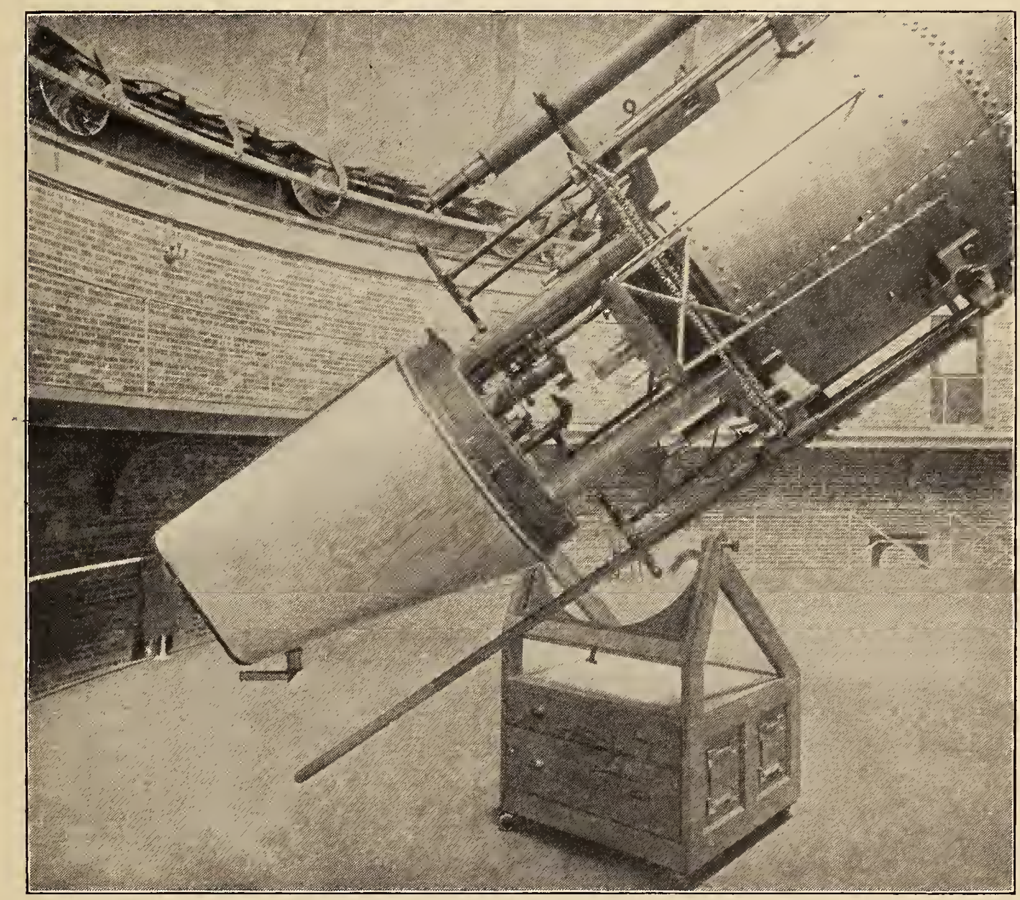

Fig. 180. — Bruce Spectrograph of the Yerkes Observatory (1902)

CHAPTER XIX

THE LIGHT OF THE STARS

The Light of the Stars — Magnitudes and Brightness — Color and Heat — Spectra — Variable Stars

555. Star Magnitudes. — The term "magnitude," as applied to a star, refers simply to its brightness and has nothing to do with its apparent angular diameter. Hipparchus and Ptolemy arbitrarily graded the visible stars into six magnitudes, the stars of the sixth being the faintest visible to the eye, while the first magnitude comprises about twenty of the brightest. There is no known reason why *six* classes should have been constituted rather than eight or ten, unless perhaps the physiological one that ordinary eyes do not easily recognize differences of brightness sufficiently small to warrant a more refined subdivision.

The six magnitudes of naked-eye stars.

After the invention of the telescope the same system was extended to the smaller stars, but without any general agreement, so that the magnitudes assigned by different observers to telescopic stars in the early part of the century differ enormously. Sir William Herschel, especially, used very high numbers, his twentieth magnitude being about the same as the fourteenth on the scale now generally used, which nearly corresponds with that which was adopted by Argelander in his great *Durchmusterung* (Sec. 533).

Fractional magnitudes.

Of course the stars classed together under one magnitude are not exactly alike in brightness, but shade from the bright to the fainter, so that exactness requires the use of fractional magnitudes. It is now usual to employ decimals giving the brightness of a star to the nearest tenth of a magnitude. Thus, a star of 4.3 magnitude is a shade brighter than 4.4, and so on.

A peculiar notation was employed by Ptolemy and used by Argelander in his *Uranometria*[1] *Nova*. It recognizes *thirds* of a magnitude as the smallest subdivision. Thus, 2, 2,3, 3,2, and 3 express the gradations between second and third magnitude, 2,3 being applied to a star whose brightness is a little inferior to the second, and 3,2 to one a little brighter than the third. *Note the comma;* 3,2 (3 *comma* 2) must not be confounded with 3.2 (3 *point* 2): the first means $2\frac{2}{3}$ magnitude, the other $3\frac{2}{10}$ magnitude.

Argelander's peculiar notation.

556. Stars Visible to the Naked Eye. — Heis enumerates the stars clearly visible to the naked eye in the part of the sky north of 35° south declination, as follows:

1st magnitude	14	4th magnitude	313
2d "	48	5th "	854
3d "	152	6th "	2010
Total			3391

Number of visible stars of the different magnitudes.

According to Newcomb, the number of stars of each magnitude is such that united they would give, roughly speaking, about the same amount of light as that received from the aggregate of those of the next brighter magnitude. But the relation is very far from exact and fails entirely for the magnitudes below the eleventh, the smaller stars being much less numerous than this law would make them.

557. Light-Ratio and Scale of Magnitude. — It was found by Sir John Herschel about 1830 that an average star of the first magnitude is just about *one hundred times as bright as one of the sixth*, and that for the naked-eye stars a corresponding ratio had been roughly maintained by former observers through the whole scale of magnitudes, the stars of each magnitude being approximately two and one-half times as bright as those of the next fainter.

First-magnitude star equals one hundred of sixth magnitude.

Still, on the star-maps of Argelander, Heis, and others long accepted as standards there are notable deviations from a consistent uniformity, and in 1850 Pogson proposed the

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[1] The term "Uranometria" has come to mean a catalogue of naked-eye stars, like the catalogues of Hipparchus, Ptolemy, and Ulugh Beigh.

The "Absolute Scale." Light-ratio equals $\sqrt[5]{100}$.

"Absolute Scale" *of star magnitudes*, adopting *the fifth root of one hundred*, 2.512 +, as the uniform "light-ratio," adjusting the first six magnitudes to fit as nearly as possible the magnitudes of Argelander's *Durchmusterung*, and then carrying forward the scale among the telescopic stars. Until about 1885 this scale had not been much used; but it has been adopted in the new "Uranometrias" made at Cambridge, U.S., and Oxford, and is now rapidly supplanting all the arbitrary scales of former observers. On this scale Aldebaran and Altair are very nearly typical "first-magnitude" stars, and the two "pointers" and Polaris are practically "second-magnitude" stars.

Aldebaran a standard first-magnitude star.

558. Relative Brightness of Different Star Magnitudes. — In this scale the "light-ratio" (*i.e.*, the ratio between the light of two stars standing just one magnitude apart in the scale) is exactly $\sqrt[5]{100}$, or *the number whose logarithm is 0.4000, i.e.*, 2.512. Its reciprocal is the number whose logarithm is 9.6000, viz., 0.3981.

Equations showing relations between relative brightness of stars and their difference of magnitude.

If b_m is the brightness of a star of the mth magnitude (expressed either in candle-power or some other convenient unit), and if b_n is that of a star of the nth magnitude, the relation between their light is given by the fundamental equation

$$\log b_m - \log b_n = \frac{4}{10}(n-m), \text{ or } \log\left(\frac{b_m}{b_n}\right) = \frac{4}{10}(n-m). \quad (1)$$

From this, conversely,

$$(n-m) = \frac{10}{4}(\log b_m - \log b_n) = 2.5 \log\left(\frac{b_m}{b_n}\right). \quad (2)$$

The first gives the relation between the *brightness* of two stars having a known difference of magnitude $(n-m)$; the second the difference of *magnitude* between two objects having a known ratio of brightness.

For example, a certain variable star rises *seven* magnitudes between the minimum and maximum; how much does its brightness increase?

$$\log \frac{b_m}{b_n} = \text{log of increase of brightness} = \frac{4}{10} \times 7 = 2.8000.$$

Looking in the table of logarithms, we find 631 corresponding to this logarithm, 2.8000; *i.e.*, the star increases in brightness 631 times. (Four-place logarithms are always sufficient.)

Again, Nova Persei increased in brightness 25000 times between February 20 and 22, 1901; how many magnitudes did it rise? From equation (2) the rise in magnitude

$$(n-m)=\frac{10}{4}\log 25000=\frac{10}{4}\times 4.3979=11 \text{ magnitudes (nearly).}$$

If the star were of the eleventh magnitude on February 20, it was of the zero magnitude on the 22d.

Zero magnitude and negative magnitudes.

It is an infelicity of this scale that the numerical magnitudes decrease with the brightness of the object, so that a star which, like Arcturus, is one magnitude brighter than Aldebaran or Altair is of the *zero* magnitude, while Capella and Vega are of magnitude 0.2. In the case of the two brightest stars, Sirius and Canopus, we run past the zero into *negative* numbers, the magnitude of Sirius being − 1.43. That of Jupiter at opposition is about − 2, *i.e.*, three magnitudes brighter than Aldebaran. On this scale the sun is about the − 26.3 magnitude.

The sun's magnitude − 26.3.

559. Telescopic Power required to show Stars of a Given Magnitude. — If a good telescope just shows stars of a certain magnitude, then, since the light-gathering power of a telescope depends on the *area* of its object-glass (which varies as the square of its diameter), we must have a telescope with its aperture larger in the ratio of $\sqrt{2.512}$ (or 1.59) : 1, in order to show stars one magnitude smaller; *i.e.*, the aperture must be increased 1.6 times (nearly). *A tenfold increase in the diameter of an object-glass* theoretically carries the power of vision just *five* magnitudes lower.

Telescopic power in relation to star magnitude.

Assuming what seems to be very nearly true for normal eyes and good telescopes, that the *minimum visibile* for a 1-inch aperture is a star of the *ninth* magnitude, we obtain the following little table of apertures required to show stars of a given magnitude, the formula being, $m = 9 + \times 5 \log$ of aperture (in *inches*).

Telescopic aperture required to show certain magnitudes.

Star magnitude . . .	7th	8th	9th	10th	11th	12th
Aperture, inches . . .	0.40	0.63	1.00	11.59	2.51	3.98
Star magnitude . . .	13th	14th	15th	16th	17th	18th
Aperture, inches . . .	6.31	10.00	15.90	25.10	39.80	63.10

But large telescopes, on account of the increased thickness in their lenses, which causes considerable absorption of light, never quite equal their theoretical capacity as compared with smaller ones.

The Yerkes telescope (40 inches aperture) will barely show stars of the seventeenth magnitude, not quite one magnitude fainter than the smallest visible with the 26-inch telescope at Washington. But the *number* visible in the larger instrument is probably fully doubled.

Measurements of magnitudes and brightness.

560. Measurement of Magnitudes and Brightness. — Until within the last twenty-five years all such measures (with a few exceptions) were mere eye estimates, and such, when made by experienced observers, are still valuable and much used in observing *changes* of brightness, as in the case of variable stars.

At present, however, the brightness and magnitude of all the principal stars have been determined instrumentally by *photometers* of some kind. Still, even with photometers, the eye of the observer is the ultimate arbiter.

No satisfactory means has yet been found for a purely instrumental measurement of starlight, although certain promising experiments have been made by Professor Minchin in attempting to determine the luminosity of stars by its effect in changing the electrical resistance of selenium, and the method may ultimately develop into something valuable.

The instruments at present used are nearly all based on one of two different principles:

Extinction photometers.

(1) *The Method of Extinction*, in which the instrument, by varying the aperture of the telescope by an "iris diaphragm,"

or by sliding a wedge of dark glass before the eye, causes the star to grow fainter until it vanishes.

A graduation on the slider that carries the wedge, or a scale that measures the area of the opening in front of the object-glass, determines the star's brightness as compared with that of some standard star which has been observed with the same apparatus; but the observations are very trying to the eyes.

Pritchard's *Uranometria Oxoniensis* was made by observations with such a wedge-photometer in 1895.

Equalizing photometers.

(2) *The Method of Equalization.* The second class of instruments consists of such as equalize the brightness of the star investigated to that of some "standard star" brought into the same field of view by reflection, or, more commonly, to the light of an "artificial star" of constant brightness.

This artificial star is usually a pinhole through which shines a small lamp (usually a frosted electric lamp) fed by a storage battery. Its image is made to fall into the field of view close by that of the star to be measured, and its brightness is varied at pleasure by sliding a wedge in front of the pinhole, or by a polarization arrangement consisting of a pair of Nicol prisms, which is better but more expensive. The eye judges when the two stars, the artificial star and the other, are apparently equal.

With instruments of this class Pickering of Cambridge made his *Harvard Photometry*, and Müller and Kempf at Potsdam, somewhat later, constructed their still more elaborate and accurate catalogue of the brightness of thirty-five hundred stars above the seventh magnitude, north of the equator.

Probable error of photometric measures.

Photometric observations in many cases require large and somewhat uncertain corrections, especially for the absorption of light by the atmosphere at different altitudes, and the final results of different observers naturally fail of absolute accordance. Still, the agreement between the different recent photometric catalogues is remarkably close, — generally between one or two tenths of a magnitude, though with occasional notable exceptions.

Embarrassment due to differences of color.

Differences of color embarrass photometric measurements made by either of the methods described, because it is impossible to make a red star look identical with a blue one by any mere increase or diminution of brightness, and because different observers differ in setting the index of an extinction photometer according to the color of the star. Some eyes are abnormally sensitive to blue light, some to red.

Star colors.

Stars differ considerably in color. The majority are of a very pure white, like Sirius, but there are not a few of a yellowish hue, like Capella, or reddish, like Arcturus and Antares, and there are others, mostly small stars, which are as red as garnets and rubies. We also have, associated with larger stars in double-star systems, numerous small stars which are strongly green or blue; and a few large stars, Vega for instance, are distinctly of a bluish tinge, like an electric arc.

Photometry by photography.

561. Photographic Photometry. — It is found that, excepting certain strongly colored stars, the diameters of the star images formed upon an isochromatic plate with a given exposure may be taken as a measure of their relative brightness; the result is made more accurate if the stars are allowed to trail slightly upon the plate. But caution must be used in taking these photographic magnitudes as corresponding to the *visual;* the discrepancies are numerous, and sometimes large, because stars of different colors impress their images very differently, even upon plates of the kind referred to. In studying the *change* of brightness of a star the method is often highly advantageous, as, for instance, in the case of Eros (Sec. 428).

Starlight compared with sunlight.

562. Starlight compared with Sunlight. — Zöllner and others have endeavored to determine the amount of light received by us from certain stars, as compared with the light of the sun. The measures are very difficult and the result considerably uncertain, but, according to Zöllner, Sirius gives us about $\frac{1}{7000\,000000}$ as much light as the sun does, and Capella and Vega about $\frac{1}{40000\,000000}$. According to this, a *standard* first-magnitude star, like Altair or Aldebaran, gives us about $\frac{1}{80000\,000000}$, and it would take, therefore, about eight billions (English), *i.e.*, about eight million million, stars of the sixth magnitude to do the same. But the various determinations for Vega range all the way from $\frac{1}{60000\,000000}$ to $\frac{1}{30000\,000000}$.

Total amount of starlight.

Assuming what is roughly but by no means strictly true, that Argelander's magnitudes agree with the absolute scale, it appears, on the basis of Zöllner's measurements, that the 324000 stars of his *Durchmusterung*, all of them north of the celestial equator, give a light about equivalent to 240 or 250 first-magnitude stars.

How much light is given by stars smaller than the nine and one-half magnitude (which was his limit) is not certain. It must greatly exceed that given by the larger stars, because the total light given by the stars of each magnitude is always several times as great as that given by the stars of the preceding magnitude. As a rough guess, we may perhaps estimate that the *total* starlight of both the northern and southern hemispheres is equivalent to about 3000 stars like Vega,[1] or 1500 at any one time. According to this, the starlight on a clear night is about $\frac{1}{60}$ of the light of the full moon, or about $\frac{1}{33\,000000}$ of sunlight. But this estimate is hardly more than guesswork. It is pretty certain, however, that more than half of this light comes from stars which are entirely invisible to the naked eye.

Stellar heat.

563. Heat from the Stars. — Doubtless the stars send us heat, and very likely the proportion of their heat to solar heat is about the same as that of starlight to sunlight. Various attempts have been made to measure it, but the amount is so small that until very recently no apparatus has been found sufficiently sensitive even to detect it.

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[1] According to Newcomb, in an important paper just published (January, 1902), "*The total light of all the stars is about equal to that of 600 stars of magnitude 0, with a probable error of one fourth of its whole amount.*"

This is equivalent to about 750 Vegas instead of 3000. The discrepancy is mainly due to a much lower estimate of the light from stars below the ninth magnitude. Professor Newcomb, however, thinks the true result likely to be larger rather than smaller than the one he has given.

Recent success in its measurement.

About 1870 Huggins and Stone in England thought that they had found distinct indications of stellar heat by the thermopile, but later work showed that the results were not reliable. About 1890 Boys, with his radiometer, attempted the problem and obtained distinct indications, but could make no real measurement. In 1898 and 1900 Professor Nichols of Dartmouth College, working at the Yerkes Observatory with a radiometer from twelve to twenty times as sensitive as that of Boys, succeeded in getting distinct deflections upon his scale from the rays of Vega, Arcturus, Jupiter, and Saturn, indicating heat radiations in the ratio of 1, 2.2, 4.7, and 0.74, respectively, after correction for the different zenith-distances of the objects. That is, Arcturus appeared to give 2.2 times as much heat as Vega, Jupiter 4.7 as much, and Saturn about $\frac{3}{4}$.

The observations indicate also that Arcturus, in the zenith, sends to a square foot of the earth's surface as much heat as would come to it from a standard candle at a distance 5.8 miles, *provided none of the candle heat were absorbed in passing through the air.*

Under the same conditions the heat from Vega equals that received from a candle 8.7 miles distant.

But the correction for the absorption of the candle heat is so uncertain that these last results are subject to large errors. The scale deflection in the case of Arcturus was a little more than a millimeter.

Light emitted by certain stars compared with the sun's light.

564. Total Amount of Light emitted by Certain Stars. — When we know the parallax of a star (and therefore its distance in astronomical units or light-years) it is easy to calculate its light emission as compared with that of the sun. If l be the light *received from a star* on the earth, expressed in terms of sunlight, and L the light *it emits* as compared with the sun, $L = l \times D^2$ (astronomical units), or (nearly) $4000\,000000\ lD^2$ (light-years).

For Sirius, referring to Table IV, we have $D_y = 8.4$ light-years and $l = \frac{1}{7000\,000000}$. Hence, $L = \frac{4000\,000000}{7000\,000000} \times (8.4)^2 = 40$ (nearly).

Similarly for other stars: using the magnitudes of the *Harvard Photometry* with distances from Table IV, we get for Polaris ($p = 0''.07$), $L = 68$; Vega ($p = 0''.16$), $L = 44$; α Centauri ($p = 0''.75$), $L = 1.9$; 70 Ophiuchi ($p = 0''.25$), $L = 0.41$; 61 Cygni ($p = 0''.40$), $L = 0.10$; Ll., 21258 ($p = 0''.26$), $L = \frac{1}{113}$.

565. Why the Stars differ in Brightness. — The apparent brightness of a star, as seen from the earth, depends both on its *distance* and on the *quantity of light it emits*, and the latter depends on the *extent and the luminosity of its surface*. As a class, the bright stars doubtless *average* nearer to us than the faint ones; just as certainly they average larger in diameter and are also more intensely luminous. But when we compare a single bright star with another fainter one we can seldom say to which of the three different causes it owes its superiority; or that a particular faint star is smaller, or darker, or more distant than a particular bright star, unless we know something beyond the simple fact that it is fainter.

Three causes of difference of brightness.

566. Dimensions of the Stars. — We have very little absolute knowledge on this subject; in a single instance, that of Algol (see Sec. 582), it has been possible to obtain an indirect measure, showing that that particular star is probably somewhat more than 1 000000 miles in diameter and considerably bulkier than the sun.

Dimensions of stars.

The apparent *angular* diameter of a star is probably in no case large enough to be directly measured or even perceived by any of our present instruments. At the distance of α Centauri the sun's apparent diameter would be somewhat less than $0''.01$. In the case of binary stars of which we happen to know the parallax, we can determine their *masses*, but *diameters*, *volumes*, and *densities* are at present quite beyond our reach.

Diameter too small for micrometric measurement.

STAR SPECTRA

Fraunhofer's observations of stellar spectra.

567. As early as 1824 Fraunhofer observed the spectra of a number of bright stars by looking at them through a small telescope with a prism in front of the object-glass, using a cylindrical lens in the eyepiece to widen the spectrum, which otherwise would have been a simple streak of colored light.

Work of Huggins and Secchi.

In 1864, as soon as the spectroscope had taken its place as a recognized instrument of research, it was applied to the stars by Huggins and Secchi. The former studied comparatively few spectra (especially those of the stars *α* Tauri and *α* Orionis), but very thoroughly, with reference to the identification of their chemical constituents. He found with certainty in their spectra the lines of sodium, magnesium, calcium, iron, and hydrogen, and more or less doubtfully a number of other metals. Secchi, on the other hand, examined a great number of stars (nearly four thousand), less in detail, but with reference to a classification from the spectroscopic point of view.

Secchi's four classes of stellar spectra.

568. Secchi's Classes of Spectra. — Secchi divided the spectra into four classes, as follows:

(I) Those which have a spectrum characterized by *great intensity of the hydrogen lines*, which look very much like H and K in the solar spectrum, all other lines being comparatively feeble or absent. This class comprises considerably more than half of all the stars examined, — nearly all the white or bluish stars; Sirius and Vega are types. (See Fig. 184, Sec. 571.)

(II) Those which show *a spectrum resembling that of the sun; i.e.*, characterized by numerous fine dark lines in it, due to the presence of metallic vapors. Capella (*α* Aurigæ) and Pollux (*β* Geminorum) are conspicuous examples. The stars of this class are also numerous, the first and second classes together comprising fully seven eighths of all the stars observed. These classes shade into each other, however; certain stars, like Procyon and Altair, seeming to be intermediate between the two.

(III) About five hundred stars show spectra characterized by dark *bands, sharply defined at the upper* or more refrangible *edge* and shading out towards the red. Most of the red stars, and a large number of the variable stars, belong to this class, and some of them show also bright lines in their spectra; *α* Herculis may be taken as the type. (See also Fig. 186, Sec. 578.)

(IV) This class comprises a comparatively small number of faint stars, which show, like the preceding, dark bands, but *shading in the opposite direction*, with sometimes a few bright lines.

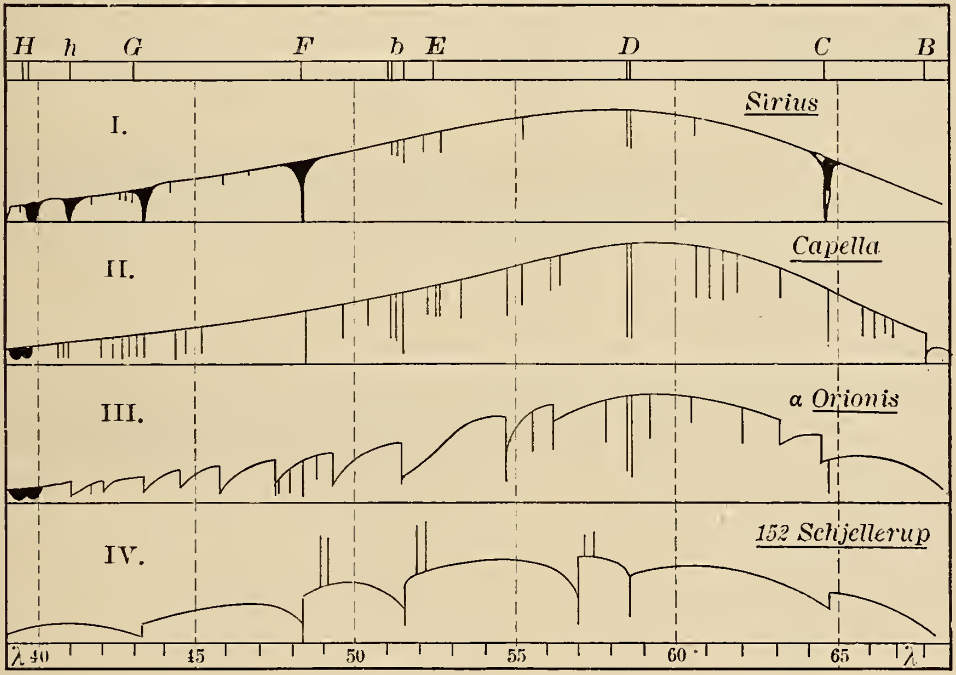

Fig. 181. — Secchi's Types of Stellar Spectra

The light curves of the different classes.

569. The typical light curves of the four classes of spectra are represented in Fig. 181, the dark lines of the spectrum being indicated by lines running downward from the contour of the curve and the bright lines by lines projecting upward. Fig. 182, from photographs by Pickering, gives the blue and violet portions of spectra of several stars ranging from the first type to the second.

Pickering has proposed a *fifth class*, — the *Wolf-Rayet stars*, so called, — containing about one hundred members, which have a peculiar spectrum, different from any of the others, and characterized by bright lines. All of them are found in or very near the Milky Way or in the two "Nubeculæ" near the south pole.

Vogel uses Secchi's classification, considerably modified, and Lockyer has proposed an entirely new one, based on his meteoritic hypothesis. We give Secchi's, however, as the best known and lying at the basis of the more recent and elaborate classifications by Pickering and others. Pickering divides Secchi's first class into five distinct groups, and the second class into six, designating the subclasses by letters of the alphabet.

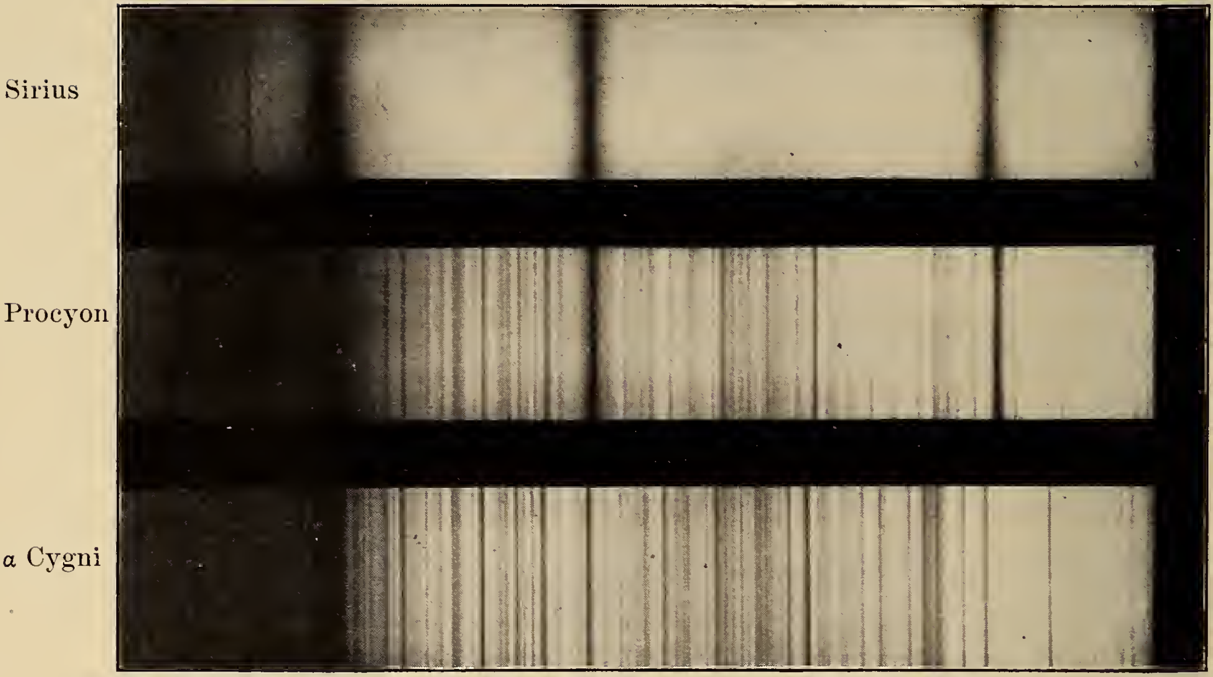

Fig. 182. — Star Spectra
Pickering

Photography of stellar spectra.

570. Photography of Stellar Spectra. — The observation of these spectra by the eye is very tedious and difficult, and photography has been brought into use most effectively. Huggins in England and Henry Draper in this country were the pioneers about 1880. At present there are numerous workers in this line both in this country and abroad, and the method has been developed to high perfection.

All but a few stars are so faint that satisfactory *visual* observation of their spectra is impracticable, but photography is nearly independent of such limitations, for with sufficient length of exposure the sensitive plate records whatever falls upon it, however feeble the light, — at least no limits are at present known.

A majority of observers use *prismatic* spectroscopes with slit and collimator substantially like that shown in Fig. 176. With this they photograph the spectra of stars separately, one by one, each on a little plate like a microscope slide; with the star spectrum is also photographed a reference spectrum, produced by an electric spark playing between electrodes of known metals.

Spectroscope with quartz prisms.

From such photographs we can measure the wave-length of lines in the star spectra and the shift of the lines, if any, due to radial motion and recognize the constituent elements of the star's atmosphere. By using prisms of *quartz*, Sir William Huggins and Lady Huggins have been able to carry their investigations with great success into the invisible ultra-violet spectra of a multitude of stars.

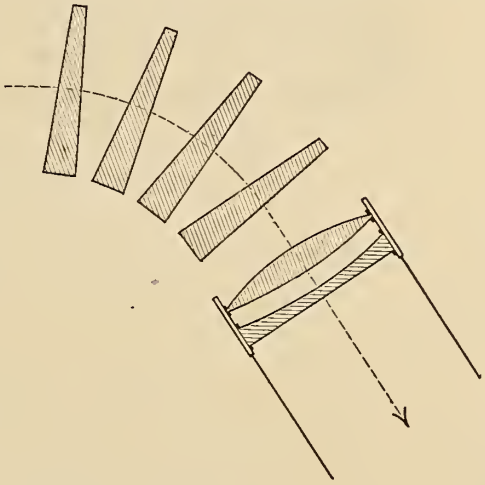

FIG. 183. — Arrangement of the Prisms in the Slitless Spectroscope

The slitless spectroscope.

571. The Slitless or Objective-Prism Spectroscope. — Professor Pickering has attained his wonderful results by reverting to the slitless spectroscope, arranged in the manner first used by Fraunhofer and later revived by Secchi. The instrument consists of a telescope with its object-glass corrected for the *photographic* instead of the visual rays, equatorially mounted and carrying outside of the object-glass one or more *objective prisms* having a refracting angle of 10° to 30°, each large enough to cover the whole lens.

Pickering uses ordinarily an 11-inch telescope, formerly belonging to Draper, or a 14-inch telescope, with a battery of four enormous prisms placed outside of the object-glass, as shown in Fig. 183. The edges of the prisms lie east and west, and the clockwork on the telescope is adjusted to go a little too fast or too slow, in order to give width to the spectrum formed upon the sensitive plate, which is placed at the focus of the object-glass; if the clockwork followed the star exactly, the spectrum would be a mere narrow streak.

With this apparatus and an exposure of from ten to fifty minutes, according to the brightness of the star, spectra are obtained which, before enlargement, are fully 3 inches long from the F line to the ultra-violet extremity. They easily bear tenfold enlargement and show several hundred lines in the spectra of the stars belonging to the second or solar class.

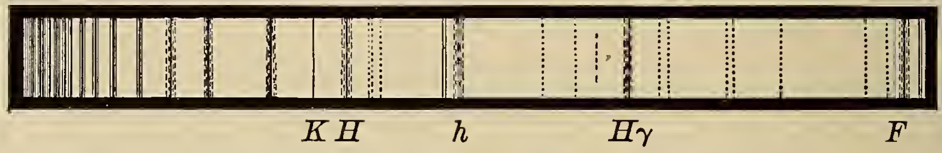

Fig. 184. — Photographic Spectrum of Vega

Fig. 184, showing the spectrum of Vega, is from one of these photographs, and the spectra of Fig. 182 are enlarged from plates made with the same instrument.

The great Bruce telescope at Arequipa (Sec. 536) has also been provided with an object-glass prism, and so has the McClean telescope at the Cape of Good Hope; with both these instruments the spectra of very faint stars can now be photographed.

Its advantages.

572. Advantages and Disadvantages of the Slitless Spectroscope. — It has three great advantages: *first*, that it utilizes all the light which comes from the star to the object-glass, much of which, in the usual form of instrument, is lost in the jaws of the slit; *second*, by taking advantage of the length of a large telescope, it produces a very high dispersion with even a single prism; *third*, and most important of all, it gives on the same plate and with a single exposure the spectra of all the many stars (sometimes more than a hundred) whose images fall upon the plate.

Its disadvantages.

On the other hand, the giving up of the slit precludes all the usual methods of identifying the lines of a spectrum by actually confronting it with comparison spectra, and makes it impossible to use the instrument for measuring the *shift* of spectrum lines and thus determining the "radial motion" of a star. Nor can it be used to study the spectrum of an object of sensible extent, like a planet or a nebula.

Moreover, it gives well-defined spectra only when the air is very steady and the star images quiescent, — a condition of comparatively little importance with a slit spectroscope, since atmospheric disturbance, with such an instrument, does not affect the distinctness of the spectrum photographed, but only makes it necessary to give a longer exposure.

Of course, too, the enormous prisms make the slitless spectroscope very costly.

Admirable for reconnaissance.

For what may be called the "reconnaissance," or classification of star spectra by the thousands, the slitless spectroscope stands unrivaled and is effective for the study of one class of spectroscopic binaries (Sec. 593), in which the lines of the spectra double and undouble themselves.

With instruments of this kind at Cambridge and Arequipa, the Cambridge observers have already obtained and stored away the photographic spectra of at least one hundred thousand stars and have issued one great catalogue of spectra (the *Draper Catalogue*), soon to be followed by others. It should be mentioned that the funds for the prosecution of these spectroscopic researches by Professor Pickering have been mainly provided by Mrs. Draper, as a memorial of her husband, Professor Henry Draper, the American pioneer in stellar spectroscopy.

It is impracticable in a text-book like the present to deal with the interesting and important details of stellar spectra as revealed by photography. For these the student must, for the present, be referred to the various publications which contain the results of the observers.

VARIABLE STARS

573. Many stars change their brightness more or less and are known as variable. They may be classified as follows:

A. *NON-PERIODIC VARIABLES*

Classes of variable stars.

(I) Stars that change their brightness slowly and continuously.

(II) Those that fluctuate irregularly.

(III) "Temporary stars," or "Novæ," which blaze out suddenly and then disappear.

B. *PERIODIC VARIABLES*

(IV) Variables of the type of *o* Ceti, usually having a period of several months.

(V) Variables of the type of β Lyræ, usually having short periods.

(VI) Variables of the "Algol type," in which the variation is like what would be produced if the star were periodically *eclipsed* by some intervening object.

NON-PERIODIC VARIABLES

574. Class I: Gradual Changes.—The number of stars positively known to be gradually changing in brightness is surprisingly small, considering that all are growing older. On the whole, the stars present, not only in position, but in brightness also, sensibly the same relations as in the catalogues of Hipparchus and Ptolemy.

Gradual changes of certain stars.

There are, however, instances in which it is almost certain that considerable change has occurred, even within the last two or three centuries. In the time of Eratosthenes the star in the claw of the Scorpion (now β Libræ) was reckoned as the brightest in the constellation. At present it is a whole magnitude below Antares, which is now much superior to any other star in the vicinity. So when the two stars, Castor and Pollux, in the constellation Gemini, were lettered by Bayer in 1610, the former, *α*, was certainly not inferior to Pollux, which was lettered *β*, but is now distinctly the brighter. Taking the whole heavens, we find a number of such cases, perhaps a dozen or more.

It is commonly believed that a considerable number of stars have disappeared since the early catalogues were made, and that some new ones have come into existence. While it is unsafe to deny absolutely that such things may have happened, we can say, on the other hand, that not a single case of the kind is certain. In numerous instances stars recorded in the catalogues are now doubtless *missing;* but in nearly every case the loss can be accounted for, either as an error of observation or printing, or by the fact that the stars observed were asteroids. There is not a single case on record of a new star appearing and *remaining permanently visible,* nor of the certain disappearance of any, except the few so-called "temporary stars."

Stars that fluctuate irregularly.

Class II: Irregular Fluctuations. — The most conspicuous variable star of this class is *η* Argûs (or *η* Carinæ), a star not visible in the United States. It varies all the way from zero magnitude (which it had in 1843, when it stood next to Sirius in brightness) down to the seventh, which has been its status since 1865, although in 1888 it was for a time reported as slightly increasing.

α Orionis and *α* Cassiopeiæ behave in a similar way, except that their range of brightness is small, not much exceeding half a magnitude. About forty or fifty other stars belong to the same class, — at least no regular periodicity has yet been found in their variations.

Temporary stars, or Novæ.

575. Class III: Temporary Stars. — There are seventeen or eighteen well-authenticated instances (and several others which are doubtful) of stars which have shone out suddenly and then gradually faded away.

Tycho's star of 1572.

The most remarkable of them was that known as "Tycho's star," which appeared in the constellation of Cassiopeia in November, 1572, was for some days as bright as Venus at her best (visible in the daytime), and then gradually waned, until at the end of sixteen months it became invisible, for there were no telescopes then. It is not certain whether it still exists as a telescopic star; so far as we can judge, it may be any one of several which are near the place determined by Tycho.

There has been a curious and utterly unfounded notion that this star was the "Star of Bethlehem," and would reappear to herald the second advent of the Lord.

Kepler's star of 1604.

Another temporary star was observed by Kepler in 1604, which for some weeks was as bright as the planet Jupiter, and remained visible for nearly two years.

Nova Coronæ of 1866.

A temporary star, which appeared in the constellation of Corona Borealis between the 10th and 12th of May, 1866, is interesting as having been the first to be studied by the spectroscope. When near its brightest (second magnitude) it was examined by Huggins, and then showed the same bright lines of hydrogen which are conspicuous in the solar prominences. Before its outburst it was an eighth-magnitude star of Argelander's catalogue, and within a few months it returned to its former state, which it still retains.

Nova Cygni, 1876.

In 1876 another second-magnitude star appeared on November 24 in Cygnus, and according to its observer, Schmidt of Athens, rose from invisibility to the second magnitude *within four hours*, remained at its maximum for only a day or two, and faded away to below the sixth magnitude within a month. It still exists as a very minute telescopic star of the fifteenth magnitude. This also was spectroscopically studied by several observers (by Vogel, especially) with the remarkable result that the spectrum, which at the maximum was nearly continuous, though marked by the bright lines of hydrogen and by bands of other unknown substances, at last became a simple spectrum of three bright lines *like that of a nebula* (Sec. 602).

In August, 1885, a sixth-magnitude star suddenly appeared in the great nebula of Andromeda, very near its nucleus. The star began to fade almost immediately and in a few months entirely disappeared. Its spectrum was sensibly continuous, without lines of any sort.

Nova Andromedæ, 1885.

576. Nova Aurigæ. — In December, 1891, a "Nova," shown in photographs, though not *seen* by any one until Jan. 30, 1892, appeared in the foot of Auriga. Early in February it was very nearly of the fourth magnitude, and remained visible to the naked eye for about a month. Its spectrum was carefully studied, both visually and photographically, and was very interesting. The bright lines were numerous, those of hydrogen and helium, with the H and K of calcium, being specially conspicuous; and each of them was accompanied by a dark line on the more refrangible side, *as if* two bodies were concerned; one of them giving *bright* lines in its spectrum and receding from us, the other with corresponding *dark* lines in its spectrum, but approaching. Fig. 185 is from a photograph made at Potsdam. According to Vogel, the relative velocity of the two masses must, if this is the true explanation, have exceeded 550 miles a second.

Nova Aurigæ, 1891–92.

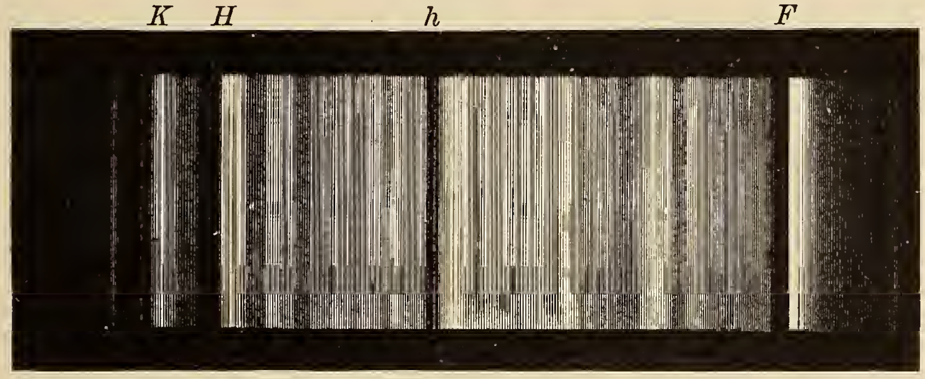

FIG. 185. — Spectrum of Nova Aurigæ

Twin lines of hydrogen bright and dark.

A slightly different explanation, suggested by Lord Kelvin, is that the high velocity indicated is not that of the large masses which collide, but of small fragments and particles, "spattered off," so to speak, by the impact.

Question whether the doubling is explained by Doppler's principle.

On the whole, it now, however, seems somewhat more probable that the displacement and widening of the lines is to be explained by violent pressure, on the principle discovered by Humphreys and Mohler (Sec. 256), rather than by the Doppler-Fizeau principle. The phenomena then would appear to be a result of *explosion* instead of *collision*, as has been very generally assumed, — something very analogous to the phenomena which accompany the eruptive prominences ejected from the sun.

In April the star became invisible, but slightly brightened again in the autumn, and then showed an entirely different spectrum, closely resembling that of a nebula. (See Fig. 203, Sec. 602.) In this figure all but the lower line are photographs of small nebulæ made with a "slitless spectroscope" (Sec. 571), so that each of the images of the nebula corresponds to what would be a "bright line" in its spectrum, if a spectroscope with a slit had been used. The lowest line is the photograph of the star made by the same instrument. Finally, however (January, 1902), its spectrum, according to photographs of Campbell, has become continuous, the nebula having apparently reverted to the original condition of a star. At present it is of the thirteenth magnitude.

Its spectrum becomes nebular and finally continuous.

The behavior of this star has led to a great deal of discussion and cannot be said to have reached as yet a wholly satisfactory explanation.

Photographic Nova.

The still more recent Novæ of 1893, 1895, and 1898, all of them in the southern hemisphere, are peculiar in that they were detected by photography, having been recognized by Mrs. Fleming of the Harvard College Observatory, both upon the chart plates and spectrum photographs taken at the Arequipa station in South America. The stars were hardly large enough to be seen by the naked eye, and there is no record of their *visual* observation, but their photographic spectra appear to be identical with that of Nova Aurigæ. It now seems rather probable that "new stars" are not really extremely rare, and it is clear that there are important physical resemblances between them.

577. Nova Persei. — The most recent instance of a new star, and one of the most remarkable, is that of the star which first appeared, probably on Feb. 20, 1901, but was first *seen* on the 21st, having then about the brightness of the pole-star.

Nova Persei of 1901.

Photographs of the region containing the star, taken at the Harvard College Observatory on several dates previous and up to the 19th, show that on the 19th the star had not yet appeared, or at least had not reached a magnitude above the twelfth.

Rapid increase of brightness.

It increased in brightness at least 25000-fold within three days, and on the 22d it was for a few hours the brightest star in the heavens, Sirius alone excepted, having attained very nearly the zero magnitude, — the most brilliant Nova since Kepler's star of 1604. Its rise was extremely rapid, and its descent was also swift as compared with that of Kepler's star, for it faded rapidly, so that by the end of March it was barely visible to the eye.

Its spectrum during the early stages.

The spectrum, as photographed at Cambridge on the 22d, was quite unlike that of Nova Aurigæ and most other new stars, resembling the spectrum of β Orionis (Rigel), — mainly continuous, but crossed by more than thirty not very conspicuous dark lines. Clouds prevented further spectrographs until the 24th, and then a great change had occurred. The bright lines of hydrogen, with their dark correlatives, were now conspicuous, just as they were in the spectrum of Nova Aurigæ (Fig. 185).

Spectrum becomes nebular.

Since then the star has followed the usual course; its spectrum has become nebular and still continues so (January, 1902), though with some non-nebular peculiarities in the breadth of the nebular lines and the presence of other conspicuous lines not found in nebulæ.

Oscillations of brightness.

During its decline its brightness oscillated capriciously more than a whole magnitude, the irregular interval between the maxima increasing from about two days in February to six or eight in the autumn, when it had fallen to the limit of unaided vision.

Nebulosity photographed.

Its enormous dimensions.

In September it became possible to photograph the invisible surrounding nebulosity with the reflecting telescopes (not with the great refractors) of the Lick and Yerkes observatories. It was found to be very extensive, roughly circular, with an apparent diameter about half that of the moon; and since the Nova shows no sensible proper motion or parallax, making it certain that its distance is greater than that of the nearer stars, the real diameter of the nebula must be at least fifteen hundred times the diameter of the earth's orbit; probably this is an extreme underestimate.

There are several pretty well marked knots of condensation in the nebula, and the comparison of photographs made at different dates shows that these are moving away from the center at various rates, averaging about 1′ in six weeks, — a motion not apparently very rapid seen from the earth, but really not less than 2000 miles a second, if the Nova is as near as α Centauri. Very probably it is ten to a hundred times more distant, or even remoter yet. Observations show no sensible parallax.

Swift motion of luminous condensations.

Kapteyn suggests that the apparent motion is simply the progressive illumination of spiral streams of nebulosity advancing along them with the velocity of light, the object being some 300 light-years distant.

PERIODIC VARIABLES

Long-period variables: type of o Ceti.

578. Class IV: Variables of the "Omicron Ceti" Type. — These objects behave almost exactly like the temporary stars in remaining most of the time faint, rapidly brightening, and then gradually fading away; but *they do it periodically.* o Ceti, or *Mira* (*i.e.*, "the wonderful"), is the type. It was discovered by Fabricius in 1596, and was the first variable recognized as such. During most of the time it is invisible to the naked eye, of about the ninth magnitude at the minimum; but at intervals of about eleven months it runs up to the fourth or third, or even second, magnitude, and then back, the rise being much more rapid than the fall. It remains at its maximum about a week or ten days.

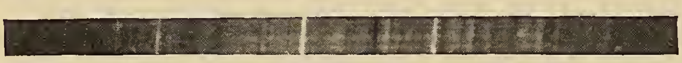

Fig. 186. — Spectrum of Mira Ceti

The maximum brightness varies very considerably; and its period, while always *about* eleven months, also varies to the extent of two or three weeks, and during the last few years seems to have shortened materially. The spectrum of the star at its maximum is very beautiful, showing a large number of intensely bright lines (some of which are certainly due to hydrogen) superposed upon a fine banded spectrum of Secchi's third class (Fig. 186, photographed by Pickering).

Its "light curve" is *A* in Fig. 187.

A large proportion of the known variables belong to this class (nearly half of the whole), and a large percentage of them have periods which do not differ very widely from one year. None so far discovered exceed two years, and none are less than two months. Most of the periods, however, are more or less irregular.

Short-period variables of β Lyræ type.

579. Class V: Short-Period Variables. — In these the periods range from about seven and three-fourths hours (that of S Antliæ, the shortest known at present) to three or four weeks, and the light of the star fluctuates continually. In many cases there are two or more maxima in a complete period, accompanied by complicated spectroscopic phenomena much like those observed in Nova Aurigæ. The light curves of β Lyræ and η Aquilæ, which are typical of this class, are given at *B* (Fig. 187).

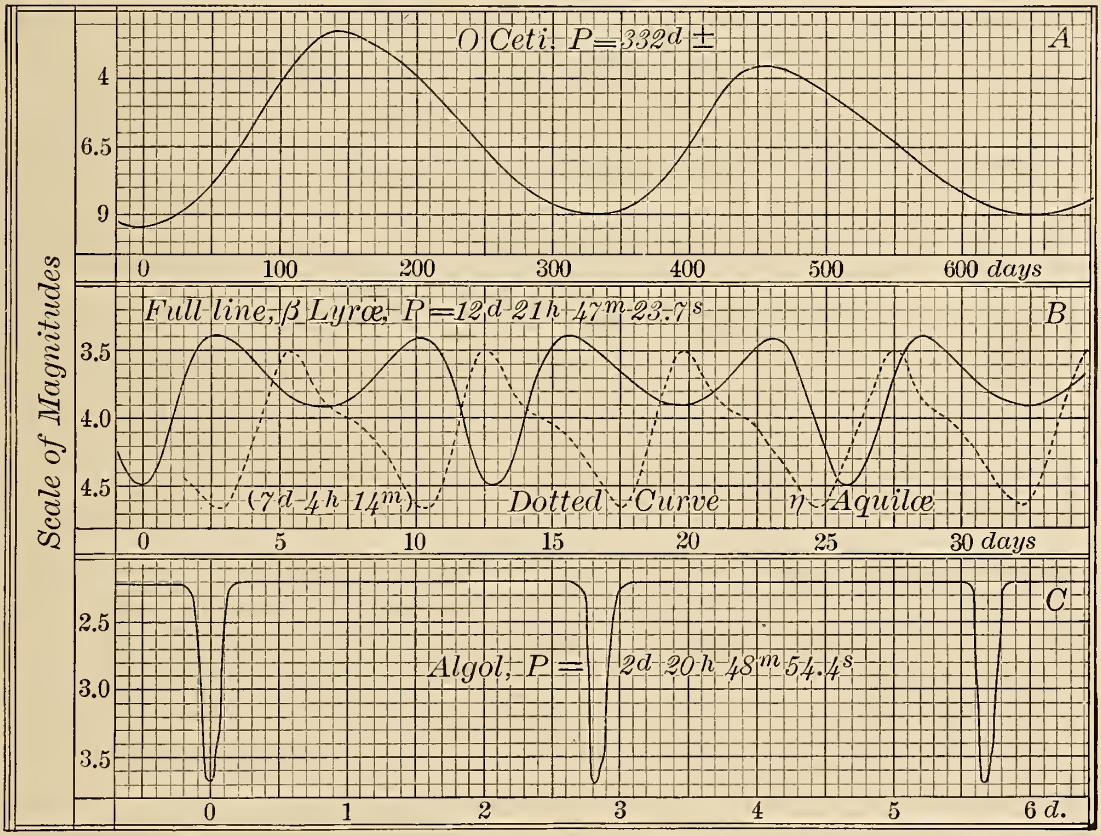

FIG. 187. — Light Curves of Periodic Variables

Eclipse variables: the Algol type.

580. Class VI: the Algol Type. — In this class the star remains bright for most of the time, but apparently suffers a periodical *eclipse*. The periods are mostly very short, ranging from ten hours to about five days.

Algol, or β Persei, is the type star. Usually it is of the second magnitude, and it loses about five sixths of its light at the time of obscuration. The fall of brightness occupies about four and one-half hours; the minimum lasts about twenty minutes, and the recovery of light takes about three and one-half hours. The period, a little less than three days, is known with great precision, — to less than a single second indeed, — and is given in connection with the light curve of the star in Fig. 187. About twenty stars of this class are known at present (1902).

581. Explanation of Variable Stars. — No single explanation will cover the whole ground. As to *progressive* changes, none need be looked for. The wonder rather is that as the stars grow old such changes are not more notable. As for *irregular* changes, no sure account can yet be given. Where the range of variation is small (as it is in most cases) one thinks of spots on the surface of the star, more or less like sun-spots; and if we suppose these spots to be much more extensive and numerous than are sun-spots, and also like them to have a regular period of frequency, and also that the star revolves upon its axis, we find in the combination a possible explanation of a large proportion of all the variable stars. For the *temporary* stars we may imagine either great eruptions of glowing matter, like solar prominences on an enormous scale, or, with Mr. Lockyer, we may imagine that most of the variable stars are only swarms of meteors, rather compact, but not yet having obtained the condensed condition of our sun. Stars of the Mira type, according to his view, owe their regular outbursts of brightness to the *collisions* due to the passage of a smaller swarm through the outer portions of a larger one, around which the smaller revolves in a long ellipse.

Explanations of periodic variability in case of long-period variables.

But the great irregularity in the periods of variables belonging to this class is hard to reconcile with a true orbital revolution, which is usually an accurate timekeeper.

Many of the spectroscopic phenomena of the temporary stars and of the periodic stars of Class IV resemble pretty closely those that appear in the solar chromosphere and prominences, suggesting in such cases a theory of explosion or eruption.

Short-period variations due to revolution of stellar system.

In the case of the short-period, "punctual" variables, as Miss Clerke calls them, of Class V, the spectroscopic phenomena in some instances seem to indicate the mutual interaction of two or more bodies revolving close together around a common center of gravity; this is certainly the case with β Lyræ. (See Sec. 592.) Others admit of simpler explanation, as due merely to the axial rotation of a body with large spots upon its surface.

Stellar eclipses.

582. Stellar Eclipses. — As to stars of the Algol type, the most natural explanation, suggested by Goodricke more than a century ago, is that the obscuration is *an eclipse* produced by the periodical interposition of some opaque body between us and the star.

Vogel's results as to the system of Algol.

The truth of this theory was substantially demonstrated in 1889 by Vogel, who found by his spectroscopic observations (see Sec. 542) that seventeen hours before the minimum Algol is receding from us at the rate of nearly 27 miles a second, while seventeen hours after the minimum it is coming toward us at practically the same rate. This is just what ought to happen if Algol had a large dark companion and the two were revolving around their common center of gravity, in an almost circular orbit, nearly edgewise towards the earth. Vogel's conclusions are that the distance of the dark star from Algol is about 3 250000 miles, and that their diameters are about 840000 and 1 060000 miles, respectively. Furthermore, their period being $2^d20^h48^m.9$, it follows (Sec. 594) that their united mass is about *two thirds* that of the sun, and their mean density only about one fifth as great as his, less even than that of Saturn, and not much above the density of cork. Fig. 188 represents the system as described by Vogel, the common center of gravity being at *C*, around which both stars revolve, always keeping opposite each other.

In the case of Y Cygni both components are about equally bright, so that *two* minima occur at each revolution, but not at equal intervals. Dunér has shown that this can be explained by the *elliptical* form of the two orbits described around the common center.

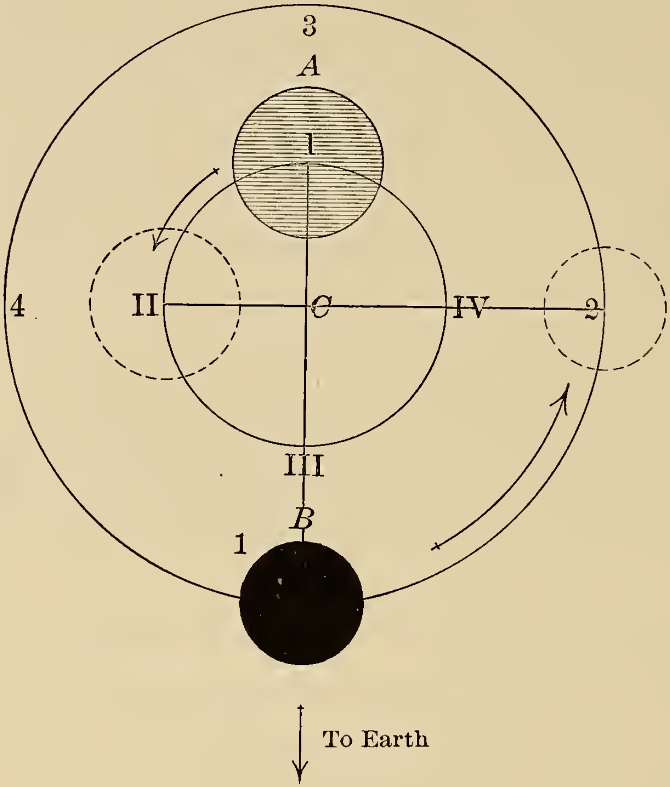

FIG. 188. — System of Algol

Number of variables.

583. Number and Designation of Variables and their Range of Variation. — Mr. Chandler's catalogue of known variables (published in 1896) included 393 objects, besides also a considerable number of suspected variables. About three hundred of them are clearly *periodic* in their variation. The rest of them are, some irregular, some temporary, and in respect to many we have not yet certain knowledge whether the variation is or is not periodic. Since 1896 the number has considerably increased and is now probably near six hundred.

Their designation.

Such variable stars as had not familiar names of their own before the discovery of their variability are generally indicated by the letters R, S, T, etc.; *i.e.*, R Sagittarii is the first discovered variable in the constellation of Sagittarius; S Sagittarii is the second, and so on.

In a considerable number of the earlier discovered variables the range of brightness is from two to eight magnitudes, *i.e.*, the maximum brightness exceeds the minimum from six to a thousand times. In the majority, however, the range is much less, — often only a fraction of a magnitude.

Range of brightness.

It is worth noting that a large proportion of the variables, especially of Classes IV and V, are *reddish* in their color. This is not true of the Algol type.

Photography has lately come to the front as a most effective method of detecting variables. A very large proportion of all those discovered within the last dozen years have been found by the comparison of the photographic star charts made at Cambridge and at their South American subsidiary stations. In most cases the photographed *spectrum* of a star has attracted attention by its bright lines and a peculiar "colonnaded" structure, marking it as "suspicious"; and the suspicion is usually soon confirmed.

Detection of variables by photography.

584. Variable-Star Clusters. — One of the most interesting recent results of stellar photography is the discovery of *variable-star clusters*, announced by Pickering in 1895, from the study of photographs made by Prof. S. I. Bailey at Arequipa. A large number of negatives of several different clusters were made, and it soon appeared that while in some no changes were apparent, in others variable stars abound. In 1898 Professor Pickering's census stood as follows: in the cluster known as 3 Messier (in the constellation of the "Hunting Dogs") no less than 132 stars had been found to be variable, out of about 900 which can be counted in the cluster; in the cluster known as ω Centauri (Fig. 195, Sec. 598) there were 122; in 5 Messier (Libræ) (Fig. 189), 85; in the cluster known as N. G. C., 7078; 51 and 47 more in three or four other clusters, — 437 in all. Since then the number has been continually increasing with further observations, and in 1902 it probably stands above

Variable stars in star-clusters.

500. In many clusters (and even in the majority) equally bright with these not a single variable has yet been found.

Rapidity and range of variation.

The periods generally range from ten to fourteen hours, the average being about twelve and one half. The range of brightness is usually from one to two magnitudes, and the light curves resemble that of *o* Ceti, the rise being more abrupt than the descent. Photographs of some of these clusters, taken at an interval of only an hour or two, show numerous cases where the change amounts to a full magnitude.

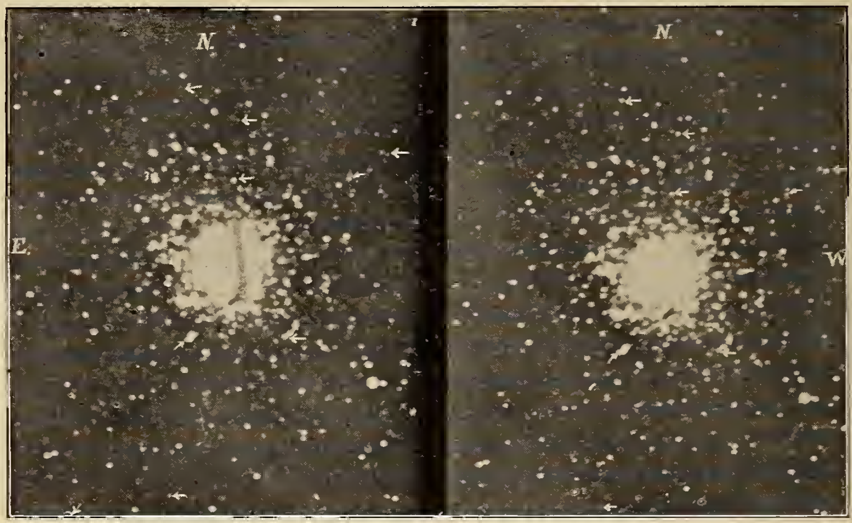

FIG. 189. — Variable-Star Cluster, 5 M Libræ

Fig. 189 is from two such Arequipa photographs of 5 Messier, taken two hours apart, and the little arrows point out some of the stars which have changed their brightness in that short time. The stars of the cluster are mostly below the eleventh or twelfth magnitude.

In Table VI of the Appendix we give from Chandler's catalogue a list of the principal naked-eye variables which can be seen in the United States.

The observation of variables is especially commended to the attention of amateurs, because, with a very scanty instrumental equipment, work of real scientific value can be done in this line. It was an amateur (the Rev. Dr. Anderson of Edinburgh) who first announced both Nova Aurigæ and Nova Persei. The observer should put himself in communication with the director of some active observatory, in order to secure the proper discussion and publication of his results.

EXERCISES

1. What is the brightness of a star of the 10.5 magnitude (on the absolute scale) compared with that of a star of the standard first magnitude?

Solution. From Sec. 558, equation (1), we have $\log b_{10.5} = \log b_1 - \frac{4}{10} \times 9.5$. If we take the brightness of the first-magnitude star as the unit of brightness, $\log b_1 = 0$, and we have $\log b_{10.5} = 0 - 0.4 \times 9.5 = -3.8000$. To bring this entirely negative logarithm into the usual tabular form, in which the characteristic only is negative while the mantissa is positive, we numerically increase the characteristic by unity, making it -4, and at the same time take for the new mantissa $1 - 0.8000$, or $.2000$; we have, therefore, $\log b_{10.5} = \bar{4}.2000$; whence, from the logarithmic table, we find $b_{10.5} = 0.000158$.

Ans. $b_{10.5} = 0.000158$.

Also $\log \frac{b_1}{b_{10.5}} = 0 - (-3.8000) = +3.8000$; whence,

Ans. $b_1 = 6309.6 \times b_{10.5}$.

(In all computations respecting stellar magnitudes four-place tables are sufficient.)

2. What is the brightness of an eleventh-magnitude star in terms of the first? *Ans.* 0.0001, or $\frac{1}{10000}$.

3. What is the brightness of a 4.8-magnitude star in terms of the first? *Ans.* 0.0302, or $\frac{1}{33.11}$.

4. What is the magnitude of a star whose brightness is $\frac{1}{100000}$ that of a first-magnitude star? (Sec. 558, equation (2).) *Ans.* 13.5 magnitude.

5. What is the magnitude of a star a millionth as bright as a first-magnitude star? *Ans.* 16th magnitude.

6. What is the magnitude, on the absolute scale, of a luminary 80000000000 times as bright as a first-magnitude star? ($\log 80000\,000000 = 10.9031$.)

Ans. -26.26 magnitude. (This is about the estimated brightness of the sun.)

7. What is the apparent magnitude of a double star whose components are of the first and second magnitudes, respectively? *Ans.* 0.64 magnitude.

8. What, if the components are of the second and fourth magnitudes? *Ans.* 1.85 magnitude.

9. If the distance of a fourth-magnitude star were diminished one half, of what magnitude would it appear? *Ans.* 2.50 magnitude.

10. If the distance of a star were increased by 40 per cent, how much would its magnitude be changed?

Ans. 0.73 of a magnitude, *numerical increase.*

11. If the distance of a star were diminished by 40 per cent, how would its magnitude be affected?

Ans. 1.11 magnitude, *numerical decrease.*

12. If a star of the ninth magnitude has a parallax of $0''.25$, how does the light emitted by it compare with that of the sun? *Ans.* $\frac{1}{186}$.

Paris Observatory, from the Garden

CHAPTER XX

STELLAR SYSTEMS, CLUSTERS, AND NEBULÆ

Double and Multiple Stars — Binaries — Spectroscopic Binaries — Clusters — Nebulæ — The Stellar Universe — Cosmogony

585. Double Stars. — The telescope shows numerous cases in which two stars lie so near each other that they can be separated only by a high magnifying power. These are *double stars*, and at present nearly twelve thousand such couples are known.

Double stars.

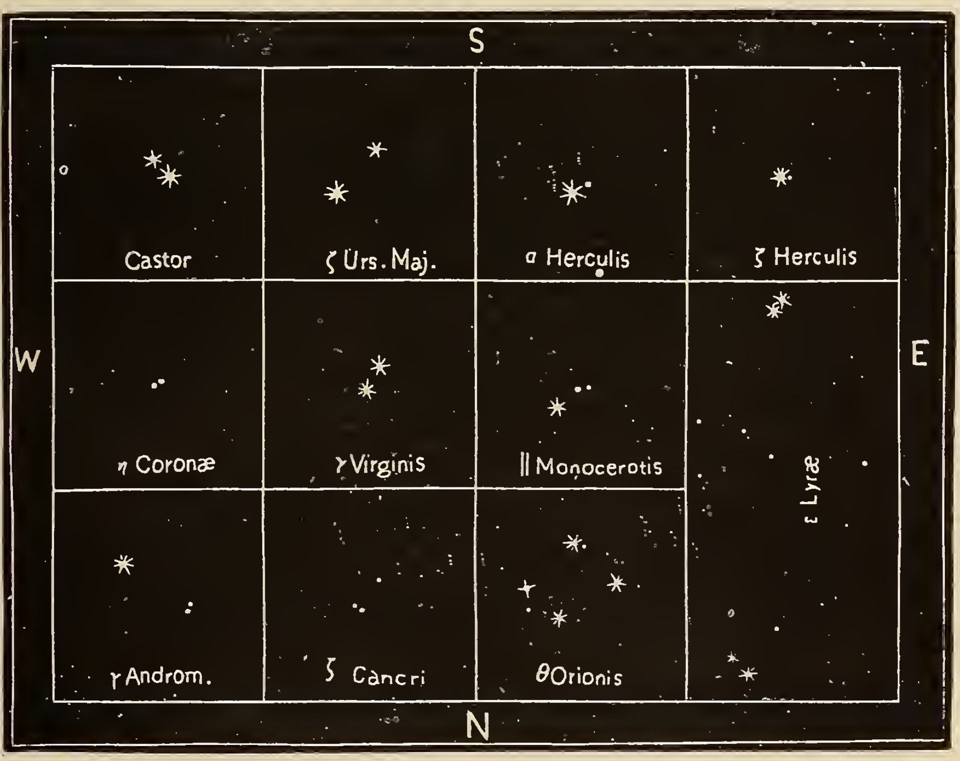
S
W
E
N
Castor
ζ Urs. Maj.
α Herculis
ζ Herculis
η Coronæ
γ Virginis
11 Monocerotis
ε Lyræ
γ Androm.
ζ Cancri
θ Orionis

Fig. 190. — Double and Multiple Stars

There is also a considerable number of triple stars and a few are quadruple. Fig. 190 represents some of the best known telescopic objects of each class.

Triple and multiple stars.

Apparent distances.

Color.

The apparent distances generally range from 30″ downwards, very few telescopes being able to separate stars closer than one fourth of a second. In a large proportion of cases, perhaps one third of all, the two components are nearly equal; in many, however, they are very unequal: in that case (never when they are equal) they often present a contrast of color, and when they do the smaller star, for some reason not yet known, almost without exception, has a tint higher in the spectrum than that of the larger, — if the larger star is reddish or yellow, the smaller is green, blue, or purple. γ Andromedæ and β Cygni are fine examples for a small telescope.

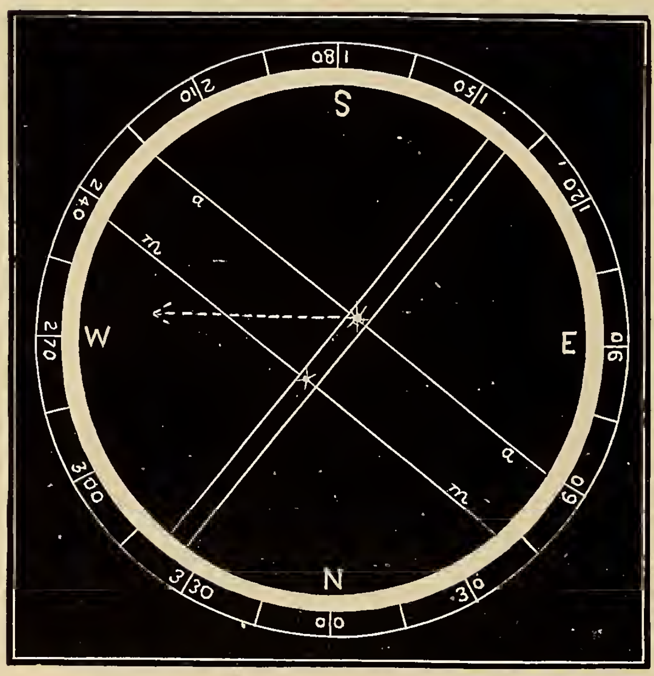

Fig. 191. — Measurement of Distance and Position Angle of a Double Star

Distance and position angle.

The *distance* and *position angle* of a double star are usually measured with the filar micrometer, the position angle being the angle made at the larger star between the north and south line and the line which joins the stars. This angle is always reckoned from the *north* through the *east*, completely around the circle; *i.e.*, if the smaller star were southeast of the larger one, its position angle would be 135°. Fig. 191 illustrates the matter. The position angle of the double star shown is about 325°.

Optical and physical doubles.

586. Stars optically and physically Double. — Stars may be double in two ways, — *optically* or *physically*. In the first case they are one far beyond the other, but nearly in line as seen from the earth. In the second case they are really near each other.

Criterion by which they may be distinguished.

In the case of stars that are only *optically double* it usually happens that we can, after some years, detect their mutual independence by the fact that their relative motion is in a straight line and uniform. This is a simple consequence of the combination of their independent rectilinear proper motions.

If they are *physically connected*, we find, on the contrary, that the relative motion is not in a straight line, but in a *concave curve; i.e.*, taking one of the two as a center, the other moves around it.

Optical pairs not numerous.

The doctrine of chances shows, what direct observation confirms, that the optical pairs must be comparatively rare and that the great majority of double stars must be physically connected, — in all probability by the same attraction of gravitation which controls the solar system.

587. Binary Stars. — Stars thus physically connected are known as "binary." They revolve in elliptical orbits around their common center of gravity in periods which range from eleven to fifteen hundred years (so far as at present known), while the apparent major axis of the oval ranges from $40''$ to $0''.3$.

Discovery of binary stars by W. Herschel.

Sir William Herschel, a little more than a century ago, first discovered this orbital motion in trying to ascertain the parallax of some of the few double stars that were known at his time. It was then supposed that they were simply optical pairs, and he expected to find an annual parallactic displacement of one of the stars with reference to the other. He failed in this, but found instead a true orbital motion.

At present the number of pairs in which this orbital motion has been *certainly* detected is over two hundred and is rapidly increasing with time. Most of the double stars have been discovered too recently to show much motion as yet, but about fifty pairs have progressed so far — either having completed an entire revolution or a large part of one — that it is possible to compute their orbits with some precision.

Their orbits.

588. Orbits of Binaries.—In the case of a binary pair the *apparent* orbit of the smaller star with reference to the larger one is always an ellipse; but this apparent orbit is only the true orbit seen more or less obliquely, and the larger star is usually not in its focus. If we assume what is probable,[1] though not *proved* as yet, that the orbital motion of the pair is under the law of gravitation, we know that the larger star must be in the focus of the *true* relative orbit described by the smaller, and, moreover, that the latter must describe around it equal areas in equal times. By the help of these principles it is possible to deduce from the apparent oval the true orbital ellipse; but the calculation is troublesome and delicate, and the result in most cases is to be regarded as at present only approximate, on account of the insufficiency of data.

Determination of the true orbit from the apparent.

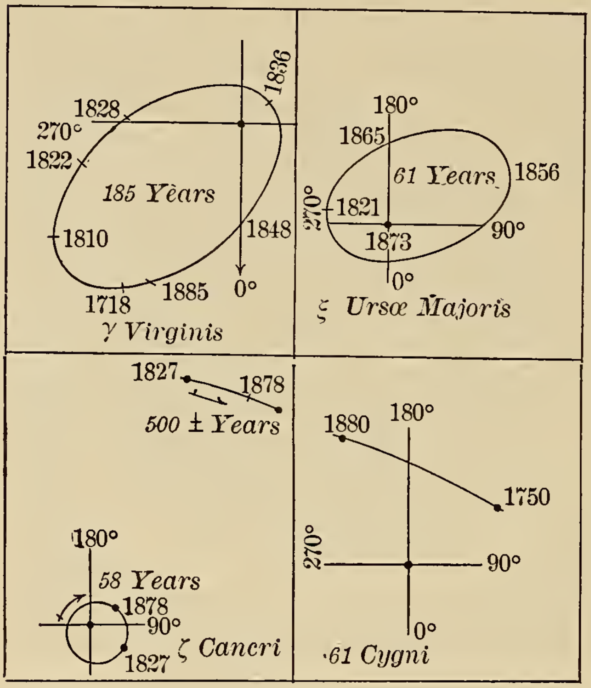
1836
1828
270°
1822
185 Years
1810
1848
1718
1885
0°
γ Virginis
180°
1865
61 Years
1856
270°
1821
1873
90°
0°
ξ Ursæ Majoris
1827
1878
500 ± Years
180°
58 Years
1878
90°
1827
ζ Cancri
1880
180°
1750
270°
90°
0°
61 Cygni

Fig. 192.—Binary Systems: Apparent Orbits

Fig. 192 represents the orbits of three of the best determined systems. The fourth, that of 61 Cygni, is still very uncertain, and the motion indicated in the diagram is very doubtful.

Micrometer observations give only the relative orbit.

589. Orbit of Sirius.—The *relative* orbit is all that can be determined from micrometer observations of the distance and position angle between the two stars of a binary pair; but in a few cases, where we have sufficient meridian-circle observations, or where the two components of the pair have had their position and distance *separately measured* from neighboring stars *not partaking of their motion*, we can deduce the absolute motion of each of the two with respect to their common center of gravity, and thus get data for determining their relative masses.

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[1] The question can be decided by spectroscopic observations whenever we become able to observe separately the two spectra of the components of a binary and so can determine the radial velocity of each at several different points in the orbit. The difficulties of observation are great, but probably will ultimately be overcome.

Orbit of the Sirius system.

The case of Sirius is in point. Before 1850 Bessel, from meridian-circle observations, had found it to be moving, for no (then) assignable reason, in a small oval orbit with a period of about fifty years. In 1862 Clark found near it a minute companion, which explained everything; only we have to admit that this faint acolyte, which does not give $\frac{1}{12000}$ part as much light as Sirius itself, has a mass more than a quarter as great; it was the first discovery of one of Bessel's "dark stars."

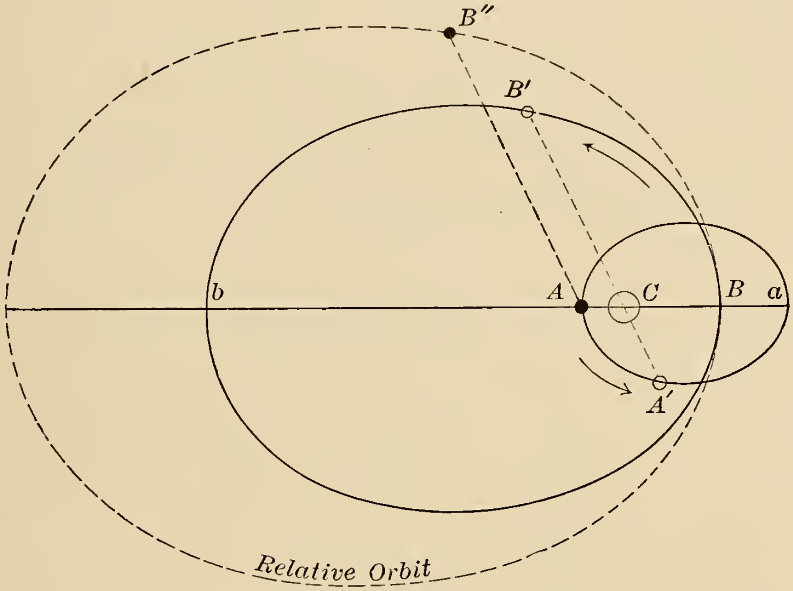

Fig. 193. — Orbits of Sirius and its Companion

The relative orbit and the two real orbits.

Fig. 193 shows the absolute and relative orbits of the system of Sirius. The smallest oval is the orbit of Sirius itself as determined by the meridian-circle, and the other full-line oval is the actual orbit of the faint companion around the common center of gravity, *C*, of the two stars. The large broken-line ellipse is the *relative* orbit of the companion with respect to

Sirius, as determined by micrometer measures of distance and position angle. When the small star is at B' and Sirius itself at A' in their actual orbits, the smaller star will be at B'' in the relative orbit, $A''B''$ being always parallel and equal to $A'B'$.

The case of Procyon is similar, but its period is not yet determined.

Size of binary orbits comparable with the larger planetary orbits.

590. Size of the Orbits.—The real dimensions of a double-star orbit can be obtained only when we know its distance from us. Fortunately, a number of the stars whose parallaxes have been ascertained are also binary, and assuming the best available data as to parallax and orbit, we find the following results, —the semi-major axis in astronomical units being always equal to the fraction $\frac{a''}{p''}$, in which a'' is the semi-major axis of the double-star orbit, and p'' its parallax, both in seconds of arc.

Name	Assumed Parallax	Angular Semi-Axis	Real Semi-Axis	Period in Years	Mass $\odot = 1$
η Cassiopeiæ . .	0″.35	8″.21	23.5	195.8	0.33
Sirius	0 .39	8 .03	20.6	52.2	3.24
α Centauri . . .	0 .75	17 .70	23.6	81.1	2.00
70 Ophiuchi . .	0 .25 (?)	4 .55 (?)	18.2 (?)	88.4	0.77 (?)

N.B. — The parallaxes here assumed differ more or less from those adopted by Kapteyn and given in Table IV of the Appendix.

These double-star orbits are evidently comparable in magnitude with the larger orbits of the planetary system, none of those given being smaller than the orbit of Uranus and none quite as large as that of Neptune. The elements of the orbits are from the data of Dr. See, given in Table VII of the Appendix. In the case of Sirius the observations made since the reappearance of the companion in 1897 indicate that the true period, distance, and mass are all a little less than here given.

Spectroscopic binaries.

591. Spectroscopic Binaries.—One of the most interesting results of spectroscopic work is the discovery, dating from 1889, of numerous pairs of double stars so close that no telescope can separate them, but proved to be double by the behavior of the lines in their spectra. From a spectroscopic point of view these fall into two classes:

Those characterized by shift of spectrum lines.

(*a*) Those in which the duplicity is exhibited by a backward and forward periodical *shift of the lines in their spectra*, as observed with an ordinary spectroscope with slit and collimator, but not observable with the slitless spectroscope. In cases of this kind only one of the stars is large or bright enough to show an observable spectrum, the other being very faint.

Those in which lines double and undouble.

(*b*) Those which not only shift their lines but also periodically *double and undouble* them, — a phenomenon observable with the slitless spectroscope as well as with an instrument of the more usual form. In this case the two stars are of not very unequal brightness.

Binaries of the first class.

592. Spectroscopic Binaries of the First Class. — Algol, already described (Sec. 582), belongs to this class, and Vogel in 1889, while he was at work upon this star, found that the star Spica (α Virginis) also shifts its spectrum lines in the same way, in a period of $4^d 19^m$, but without any observable variation in its brightness. From this he concluded that Spica also is double, having a faint or dark companion like Algol's, but with the orbital plane so much inclined that the bright star is never eclipsed; the smaller one never comes exactly between us and the larger so as to eclipse it.

Case of Spica Virginis.

From the amount by which the lines shift, Vogel, assuming the orbital inclination to be small, computes that Spica moves in an orbit about 13 000000 miles in diameter, with a velocity of about 56½ miles a second. If, however, the inclination is really considerable, the actual orbital velocity must be much higher and the orbit larger. As to the velocity of the smaller star in its orbit, we have no data (but see exercises at the end of the chapter).

Other binaries.

More recently Belopolsky of Pulkowa, Dunér, Sir Norman Lockyer, Newall, and others in Europe, and in this country Keeler and Campbell especially, at the Lick Observatory, have detected a considerable number of other binaries of this class, among which the most notable perhaps are the following: Castor, δ Cephei (its period of revolution being identical with that of its variability), Capella, with a long period of 104 days, and Polaris, which latter, like Spica, has a period of only four days and an extremely low range of apparent radial velocity, probably indicating that its orbit is nearly perpendicular to the line of sight.

Peculiar behavior of Polaris.

There are also indications that Polaris and its companion are together in orbital motion around a much more distant invisible body in a period, not yet determinable, of many years. In 1889 the star's radial velocity towards the earth was about 16 miles a second; this had dropped to 10 miles in 1896, and to a little over 7 in 1899, but in July, 1901, it had *increased* to 8¼, having apparently reversed its tendency.

Campbell finds that about one in twelve of all the stars he has thus far examined shows indications of similar orbital motion, and is now engaged in an extensive campaign for studying the motions of the smaller stars.

In the case of β Lyræ the lines of its spectrum *double* as well as *shift*.

Total number about fifty.

About fifty pairs of this first class are now known, and the number is rapidly increasing.

Binaries of the second class.

593. Spectroscopic Binaries of the Second Class.—In 1889, almost simultaneously with Vogel's discoveries relating to Algol and α Virginis, Professor Pickering announced that the lines in the spectrum of the brighter component of the well-known double star Mizar (ζ Ursæ Majoris), as photographed in the slitless spectroscope, *double* themselves at regular intervals of about fifty-two days. At these times the two components, not very unequal in brightness, are moving one towards, the other from, us, their relative velocity being about 100 miles a second. From

Mizar the first discovered.

all the observations then available he concluded that the orbit was an eccentric ellipse described in a period of 104 days, with a semi-major axis of about 140 000000 miles; but certain irregularities in the observations made some of his conclusions doubtful.

Vogel, in a paper published in 1901, announces recent photographic observations of the spectrum of the star made with the newly erected instrument figured in our frontispiece, which, while substantially confirming the relative *velocity* observed by Pickering, give a period of only 20.6 days,—just one fifth of Pickering's period.

Vogel's correction of Pickering's result as to period and size of orbit.

Assuming the orbit to be circular and its inclination small, this corresponds to a semi-major axis of about 20 000000 miles and a mass something over *four* times that of the sun.

The lines in the spectrum of β Aurigæ present the same peculiarity, but the doubling occurs once every two days, and the relative velocity of the pair is 150 miles a second; the diameter of the orbit appears to be about 16 500000 miles, and the united mass of the pair about *five and one-half* times that of the sun.

β Aurigæ: velocity 150 miles a second.

The two most remarkable objects of this class were discovered in 1896 by spectrum photographs made at Arequipa. The first is μ^1 Scorpii (fourth magnitude), in which the relative velocity of the components is nearly *300 miles a second* and the period $34^{h}42^{m}.5$. This makes the diameter of the relative orbit, if circular, about 6 050000 miles and the mass of the system about *eighteen* times that of the sun.

μ^1 Scorpii: velocity 300 miles.

The other is a little star of the fifth magnitude, known as Lacaille 3105. The relative velocity of this pair is no less than *385 miles a second* and the period $74^{h}46^{m}$, corresponding to an orbital diameter of 16 500000 miles and a mass *seventy-seven* times that of the sun.

Lacaille 3105: velocity 385 miles.

About a dozen spectroscopic binaries of this class are at present known, but the number in both classes is steadily increasing.

594. Masses of Binary Systems.—If we assume that the binary stars move under the law of gravitation, then, when we have ascertained the semi-major axis of the real orbit in astronomical units and the period of revolution in years, we can at once find the mass of the pair as compared with that of the sun by the proportion from Sec. 380, in which the third term becomes unity when the distance and period are expressed as stated,

$$(S + e) : (M + m) :: 1 : \frac{a^3}{t^2}.$$

In this proportion $(S + e)$ is the united mass of the sun and earth, e being insignificant, while $(M + m)$ is the united mass of the two stars. This gives

Formula for mass of a binary system.

$$(M + m) = S\frac{a^3}{t^2}.$$

Results liable to a large error.

The final column of the little table of Sec. 590 gives the masses of the star pairs resulting from the data there presented; but the reader must bear in mind that they are liable to large error because of the uncertainty of the parallaxes, — a slight error in the parallax produces a vastly greater error in the computed mass.

Density of spectroscopic binaries very low.

The reader is also again reminded of the fact that the *mass* of a pair gives no clue to the *diameter or density* of the stars; though what has been ascertained in the case of Algol and other stars of the same class of variable indicates that generally their densities are small compared with that of the sun. Russell of Princeton and Roberts of South Africa have recently shown, simultaneously and independently, that in the case of the Algol variables it is possible from the period, the duration of obscuration, and the peculiarities of the light curve during the "eclipse" to determine the *maximum* value possible for the mean density of the system. From eight or ten of these variables — all for which observation furnishes the necessary data — Russell found values of the limiting density in terms of that of water, ranging from 0.035 in the case of S Cancri to 0.728 for Z Herculis. The density of the sun is 1.41.

Results of Russell for limits of density.

Evolution of binary systems.

595. Evolution of Binary Systems. — As already remarked, the theory of probabilities indicates that the great majority of double stars must be physically connected, but our observations have not yet continued long enough to give us any accurate knowledge of the orbits of more than a very few. Table VII (Appendix) presents a list of twenty, mostly computed by Dr. See, which may be regarded as fairly known. Two others of long period are added, not yet, however, to be accepted as trustworthy, the data being still insufficient.

It will be noticed that these orbits are very eccentric as compared with those of the planets, their average eccentricity being nearly 0.50. Dr. See has investigated the probable origin of these binary systems and finds that the peculiarities of their orbits can be accounted for by the theory of "tidal evolution." It is supposed that in such cases the primitive nebula in whirling assumed the dumb-bell form known as the "apioid"; that the two parts then separated into a spectroscopic double, and as they revolved around their common center of gravity great tides were raised upon them, which, as mathematically proved, must tend to push the spinning globes apart into eccentric orbits. (See Sec. 346.)

Eccentricity of stellar orbits large.

Peculiarities explained by tidal evolution.

596. Planetary Systems attending Stars. — It is a natural question whether some, at least, of the stars have not planetary systems of their own, and whether some of the small "companions" that we see may not be the Jupiters of such systems.

Question of planets attending stars.

While it is entirely possible that many of the stars do have such attendant planets, we can only say that no telescope ever yet constructed could even come near to making visible a planet bearing to its primary the relations of size, distance, and brightness which Jupiter bears to the sun.

Invisible if they exist.

As viewed from our nearest neighbor among the stars, Jupiter would be a star of about the *twenty-first* magnitude and not quite 5″ distant from the sun, which itself would be a star of the second magnitude. To render a star of the twenty-first magnitude barely visible (apart from all the difficulties raised by the nearness of an immensely brighter star) would require a telescope of more than 20 feet in diameter.

597. Multiple Stars. — There are a considerable number of cases where we find three or more stars connected in a single system. ζ Cancri (Figs. 190 and 192) consists of a close pair revolving in a nearly circular orbit, with a period of somewhat less than sixty years, while a third star revolves in the same direction around them at a much greater distance and with a period that must be at least five hundred years. Moreover, the

Multiple stars, triple and quadruple.

third star appears to be subject to a peculiar irregularity in its motion, which seems to indicate that it has near it a companion unseen as yet, the system probably being really quadruple.

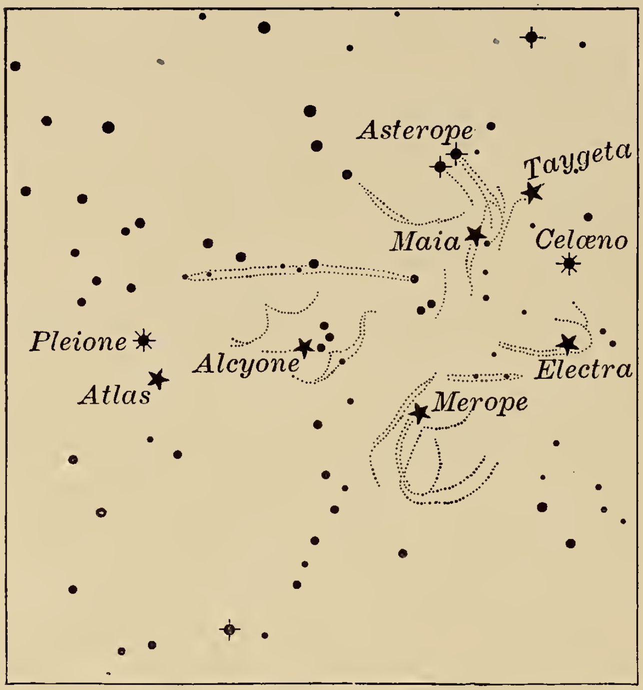

FIG. 194. — Map of the Pleiades

In ε Lyræ we have a most beautiful quadruple system, composed of two pairs, each pair making its own slow revolution with a period not yet determined, but probably exceeding two hundred years; moreover, since they have an identical proper motion, the two pairs probably revolve around each other in a period to be reckoned only by thousands of years.

In θ Orionis we have a multiple star in which the six components are not arranged in pairs, but are at not very unequal distances from each other (Fig. 190).

STAR-CLUSTERS

Star-clusters.

598. There are in the sky numerous groups of stars, containing from a hundred to many thousand members. A few are resolvable by the naked eye, as, for instance, the Pleiades (Fig. 194); some, like "Præsepe" (in Cancer), break up under the power of even an opera-glass; but most of them require a large telescope to show the separate stars. To the naked eye or small telescopes, if visible at all, they look merely like faint globular clouds of shining haze, but in a large instrument they are among the most magnificent objects the heavens afford.

Naked-eye groups.

Telescopic clusters.

The cluster known as "13 Messier" (Herculis), not far from the "apex of the sun's way," is the finest in the northern heavens, containing several thousand stars within a space one fourth of the diameter of the moon. ω Centauri, in the southern hemisphere, is perhaps even finer. (See Fig. 195, from an Arequipa photograph.)

Stars in telescopic clusters probably really small.

The question at once arises whether the stars in such a cluster are comparable with our own sun in magnitude and separated from each other by distances like that between the sun and α Centauri, or whether they are really small and closely packed, — mere sparks of stellar matter; whether the swarm is about the same distance from us as the stars or far beyond them.

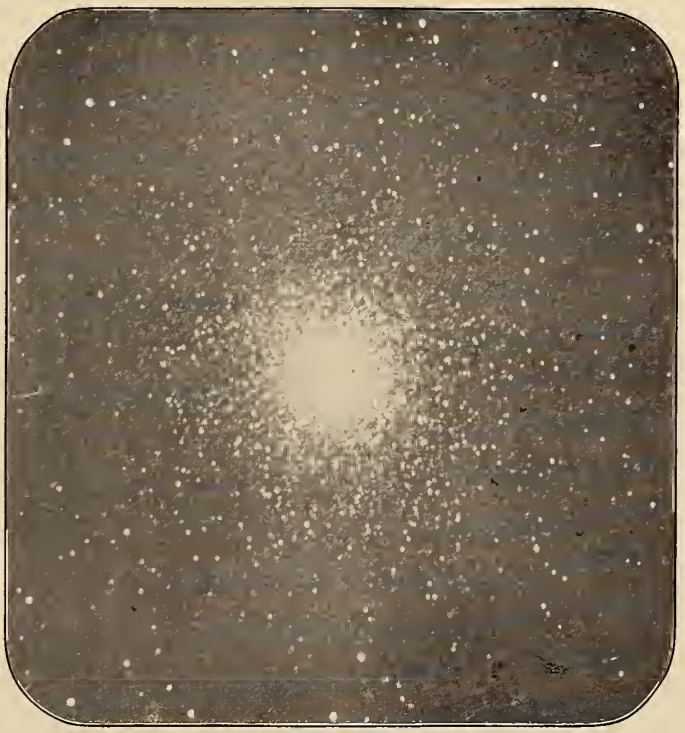

Fig. 195. — Cluster of ω Centauri

Star-clusters belong to our stellar system: not remote Galaxies.

Half a century ago the prevalent view was that they and the nebulæ (the nebulæ being then supposed to be really clusters) are, in fact, "stellar universes" so remote that the separate stars can be made out only with the telescope, — "Galaxies," like the group of stars to which it was supposed the sun belongs, but so inconceivably distant that in appearance they dwindle to mere shreds of luminous cloud. It is now, however, quite certain that the opposite view is correct. These objects are *among our stars* and *form a part of our own stellar universe.* Large and small stars are so associated in the same clusters and nebulæ (see Fig. 196) as to leave no doubt on the point, although it has never yet been possible to determine the actual parallax and distance of any cluster or nebula.

NEBULÆ

The nebulæ.

599. Besides the clusters there are other luminous clouds which no telescopic power resolves into stars, and among them some which are brighter than many of the clusters. These irresolvable objects are *the nebulæ* (*clouds*), of which something like ten thousand are now catalogued, with probably hundreds of thousands as yet uncatalogued, but discoverable by photography. Half a dozen of them are visible to the naked eye, — one, the brightest of all and the one in which the temporary star of 1885 appeared, is the Great Nebula of Andromeda (Fig. 200). Another, next in brightness and the most beautiful nebula of all, is that in the sword of Orion. (See Fig. 197, from Keeler's magnificent photograph of the nebula, made in 1900.)

The larger and brighter mostly irregular in form.

The larger and brighter nebulæ are mostly irregular in form, sending out sprays and streams in all directions and containing dark openings and "lanes." They are of enormous volume. The nebula of Orion (which includes within itself the multiple star θ Orionis) covers several square degrees, and since we know with certainty that it is far more remote than α Centauri, its cross-section as seen from the earth must exceed the area of Neptune's orbit by many thousand times. The nebula of Andromeda is not quite so extensive, and it is rather more regular in form, though it shows curious dark streaks within it.

Their immense magnitude.

Smaller nebulæ mostly oval with brighter center.

The smaller nebulæ are usually more or less nearly oval and brighter in the center. In the so-called *nebulous stars* the central nucleus looks like a star shining through a fog.

Planetary and annular nebulæ.

The *planetary* nebulæ are nearly circular and of about uniform brightness throughout, and the rare *annular* or *ring nebulæ* are darker in the center; the finest of these is the one

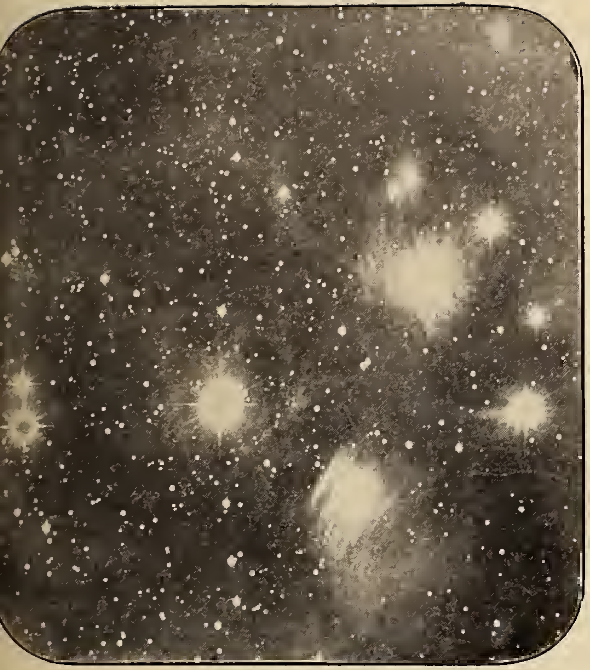

FIG. 196. — Pleiades and Enveloping Nebulæ
Roberts

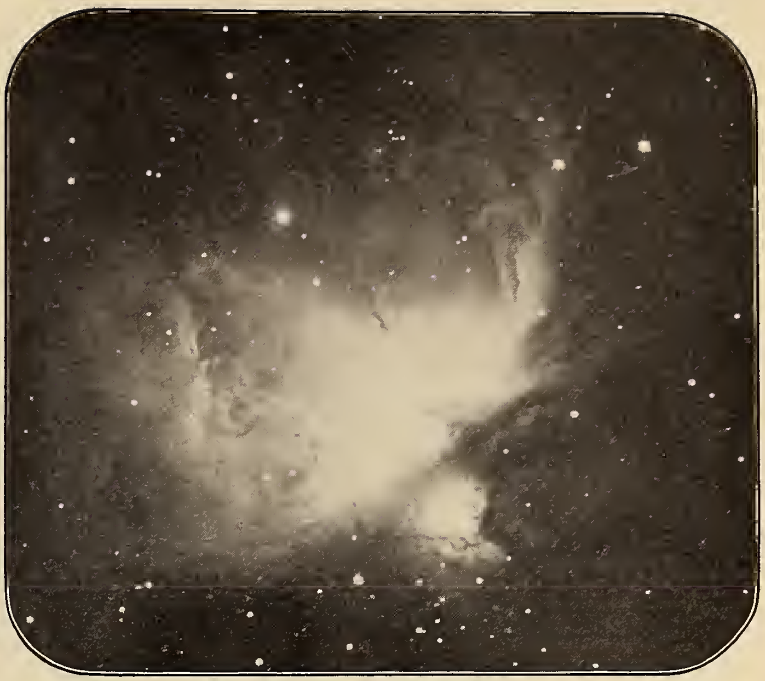

FIG. 197. — Great Nebula of Orion
Keeler

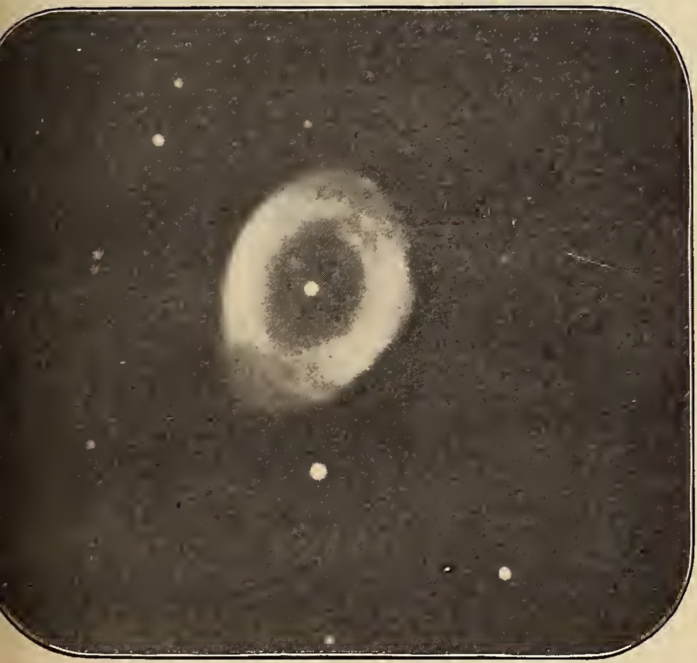

FIG. 198. — Annular Nebula in Lyra
Keeler

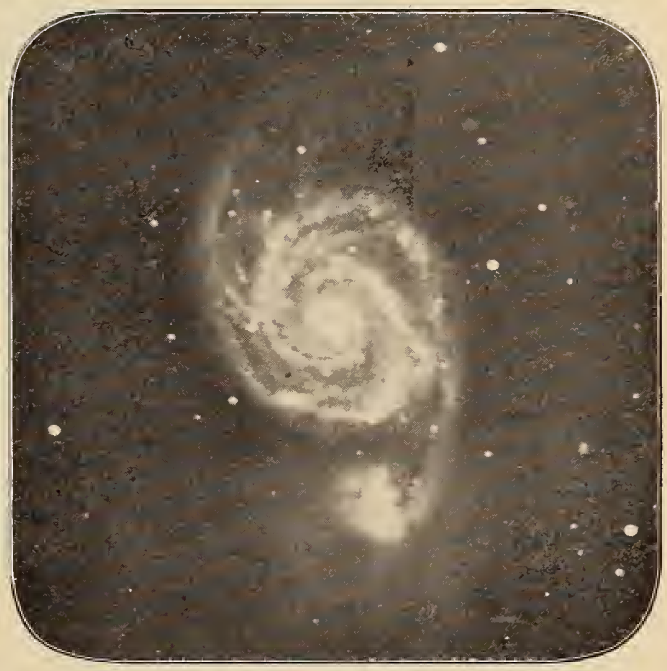

FIG. 199. — The Whirlpool Nebula
Keeler

in the constellation of Lyra (Fig. 198). Many of the nebulæ exhibit a remarkable *spiral* or whirlpool-like structure in large telescopes. Indeed, the photographic work of Keeler shows that this spiral structure is perhaps predominant in the great majority of nebulæ. Fig. 199 is from his photograph of the so-called "whirlpool nebula," 51 Messier. There are numerous *double* nebulæ, perhaps double stars in process of making, and a few that are *variable in brightness*, though no periodicity has yet been ascertained in their variations.

Spiral nebulæ

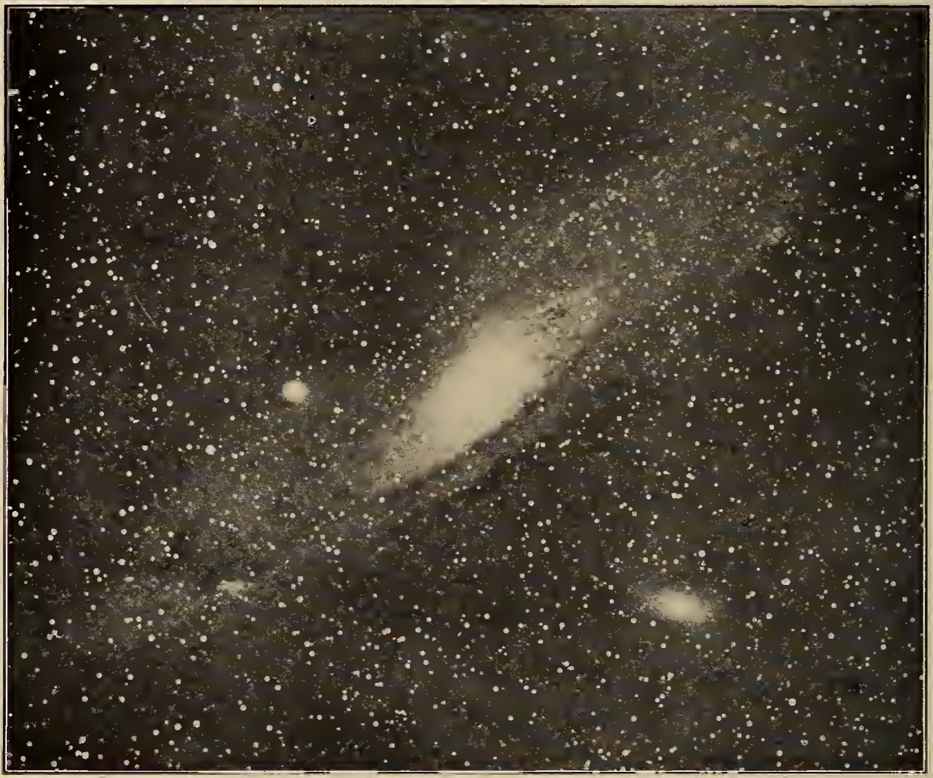

Fig. 200. — Nebula of Andromeda
Roberts

The great majority of the nebulæ are extremely faint, but the few that are reasonably bright are very interesting objects with large telescopes.

Drawings and engravings of nebulæ.

600. Drawings and Photographs of Nebulæ. — Not very long ago the correct representation of a nebula was an extremely difficult task. A few more or less elaborate engravings exist of perhaps fifty of the most conspicuous, but photography has recently taken possession of the field. The first success in this line was by Henry Draper of New York, in 1880, in photographing the nebula of Orion.

Superseded by photographs which reveal features not visible in telescope.

Since his death in 1882 great progress has been made, both in Europe and this country, and at present photographs have quite superseded drawings. The photographs are continually bringing out new and before unsuspected features not visible in any telescope. Fig. 200, for instance, is a half-tone reproduction of a photograph of the nebula of Andromeda, taken by Mr. Roberts of Liverpool in 1888, which was a revelation to astronomers. It shows that the so-called "dark lanes," which up to that time had been seen only as straight and wholly inexplicable markings, are really curved ovals, like the divisions in Saturn's rings, and brings out clearly a distinct annular or spiral structure pervading the whole nebula, though never yet seen by the eye.

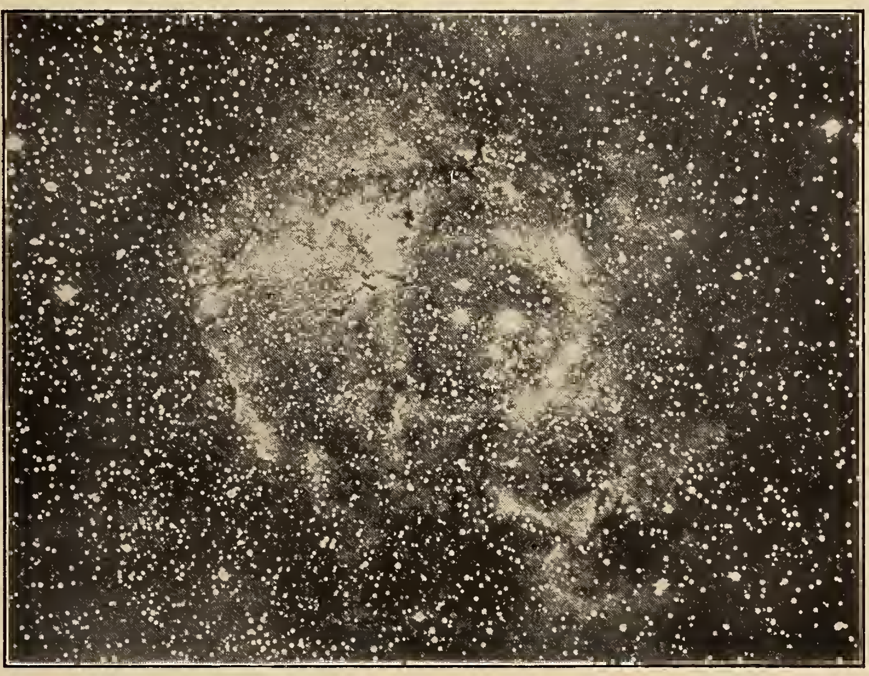

FIG. 201. — Great Nebula in Monoceros

Roberts

Fig. 201 is a photograph by Roberts of a faint but enormous nebula which covers an area more than a degree in diameter in the constellation of Monoceros, — apparently a chaos in the initial stages of evolution.

Certain disadvantages of photography.

The photograph has its drawbacks, however; *stars* present in the nebulæ are not properly shown, nor is the relative brightness of different portions fairly given on any single negative. The exposure necessary to bring out faint details is far too great for the brighter parts of the nebula and wholly destroys the stars. Moreover, the nebula is very rich in ultra-violet light, so that the relative *photographic* brightness of different parts differs from the visual. In Fig. 203 (Sec. 602), made with a slitless spectroscope used as a "prismatic camera," the brightest images of the annular nebula in Lyra are ultra-violet, made by light invisible to the eye.

Photography multiplies greatly the number of known nebulæ.

The photographs not only show new features in old nebulæ, but they reveal immense numbers of nebulæ invisible to the eye with any telescope. Thus, in the Pleiades, it has been found that nearly all the larger stars have wisps of nebulous matter attached to them, as shown by Fig. 196, from a photograph by Roberts. In a small territory in and near the constellation of Orion, Pickering, with an 8-inch photographic telescope, found upon his negatives nearly as large a number of *new* nebulæ as those that were previously known within the same boundary, and in 1892 Wolf of Heidelberg found 130 small planetary nebulæ in a circle of 1° radius around the star η Virginis. Keeler also concluded that the number that could be photographed with his 3-foot reflector must be "*many times* greater" than those that could be *seen* with it.

Very recently Wolf has begun a systematic campaign for the purpose of discovering and cataloguing objects of this class, using the magnificent twin cameras of 16-inch aperture, with lenses made by Brashear, provided by the liberality of the late Miss Bruce.

The photographs of nebulæ require generally an exposure of from one to four or five hours, or even more, and it may be noted in passing that thus far the finest nebular photographs have been made with *reflecting telescopes.*

601. Changes in Nebulæ. — It cannot, perhaps, be stated with certainty that sensible changes have occurred in any of the nebulæ since they first began to be observed, — the early instruments were so inferior to the modern ones that the older drawings cannot be trusted very far; still, some of the differences between them and more recent representations and photographs make it extremely probable that real changes are going on.

Question of changes in the nebulæ.

At present the best authenticated instance of such a change, according to Professor Holden, is in the so-called "trifid" nebula in Sagittarius. In this object there is a peculiar three-armed area of darkness which divides the nebula into three lobes. A bright triple star, which in the early part of the century was described and figured by Herschel and other observers as in the *middle* of one of these dark lanes, is now certainly in the edge of the nebula itself. The star does not seem to have moved with reference to the neighboring stars, and it seems, therefore, that the nebula itself must have drifted and changed its form.

602. Spectra of Nebulæ. — One of the most important of the early achievements of the spectroscope was the proof that the light of the nebulæ proceeds not from aggregations of stars, but from glowing gas in a condition of no great density; Sir William Huggins, in 1864, first made the decisive observation *by finding bright lines in their spectra.*

Spectrum first observed by Huggins in 1864.

Thus far the spectra of all the nebulæ that show lines at all appear to be substantially the same. Four bright lines are usually easily observed: two of them are due to hydrogen; but the other two, *in the bluish green (which are much brighter than the hydrogen lines), are not yet identified and are almost certainly due to some element not yet detected on the earth or sun and apparently peculiar to the nebulæ.*

A bright-lined gaseous spectrum.

Brightest lines due to an unknown element: others to hydrogen and helium.

At one time the brightest of the four lines (λ 5007) was thought to be due to *nitrogen*, and even yet the statement is found in some books; but it is now certain that, whatever it may, be nitrogen is not the substance.

Mr. Lockyer later ascribed this line to *magnesium*, in connection with his "meteoritic hypothesis"; but subsequent observations show conclusively that this identification also is incorrect. The line and its neighbor (λ 4959) still remain a mystery.

About seventy lines have been photographed.

Fig. 202 shows the position of the principal lines so far visually observed; in the brighter nebulæ a number of others are also sometimes *seen* and over seventy have been *photographed* in the spectra of different nebulæ; the lines of *helium*, as well as hydrogen, are generally found to be present.

Fig. 203 is from a photograph by Gothard of the ring nebula and a number of planetary nebulæ made with a slitless spectroscope. In this case each bright line of the nebular spectrum is replaced by an image of the object, though the blue-violet and ultra-violet rays alone have impressed themselves upon the plate, and, as already mentioned, the brightest photographic image of the ring nebula is due to invisible ultra-violet light.

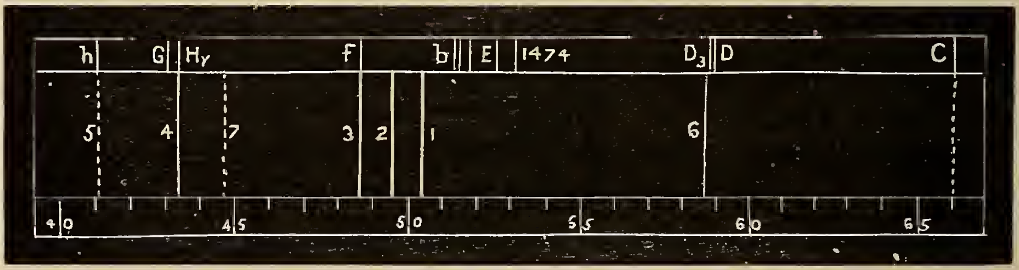

Fig. 202. — Visual Spectrum of the Gaseous Nebulæ

At the bottom of the figure is the photograph of Nova Aurigæ, made in the same way, and showing the identity of the spectrum of the star at that time with the nebular spectrum.

Spectrum of nebula of Orion.

One of Huggins' photographs of the spectrum of the nebula of Orion shows, in addition to the bright lines that are visible to the eye, a considerable number in the ultra-violet; and what is interesting, these lines seem to pertain also to the spectrum of the *stars* in the so-called "Trapezium" (θ Orionis); as if, which is very likely, the stars themselves were mere condensations of the nebulous matter. Indeed, the telescope shows that close around each star the nebulous matter has partially disappeared, as if it had been absorbed by it.

Some nebulæ give a continuous spectrum.

Not all the nebulæ show the bright-line spectrum. Those which do — about half the whole number — are of a *greenish* tint, at once recognizable in a large telescope.

The white nebulæ, with the nebula of Andromeda, the brightest of all, at their head, present only a plain, continuous spectrum, unmarked by lines of any kind. This, however, does not necessarily indicate that the luminous matter is not gaseous, for a gas *under pressure* gives a continuous spectrum, like an incandescent solid or liquid.

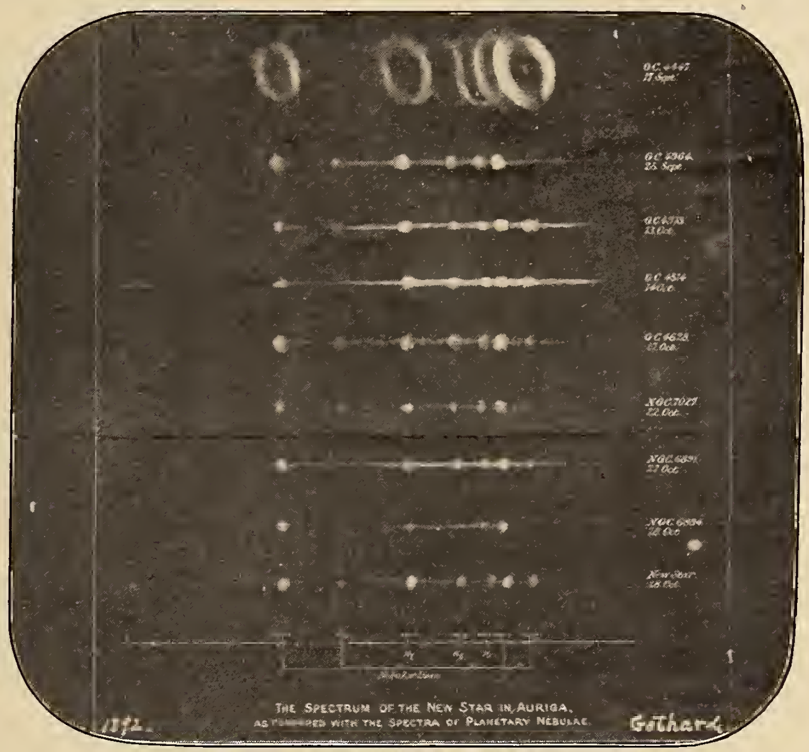
G.C. 4447
17 Sept.
G.C. 4964
25. Sept.
G.C. 4373
13. Oct.
G.C. 4514
14 Oct.
G.C. 4628
17. Oct.
NGC 7027
22. Oct.
NGC 6891
23 Oct.
NGC 6826
28 Oct
New Star
26 Oct.
The Spectrum of the New Star in Auriga,
as compared with the Spectra of Planetary Nebulae.
1892.
Gothard

Fig. 203. — Nebulæ and Nova Aurigæ
Gothard

The telescopic evidence as to the non-stellar constitution of nebulæ is the same for all; no nebula resists all attempts at resolution more stubbornly than that of Andromeda.

Radial motion of nebulæ determined by Keeler.

Keeler, at the Lick Observatory, with a powerful spectroscope has been able to detect and to measure the radial motion of several of the brighter nebulæ. It appears to be of the same order as that of the stars, the nebulæ observed giving results ranging from 0 up to about 40 miles a second,— some approaching and others receding.

Nature of nebulæ: clouds of matter which may contain solid and liquid particles mixed with the gases.

603. Nature of the Nebulæ. — As to the real constitution and temperature of these bodies we can only speculate.

The fact that the matter which shines is mainly gaseous does not indicate that they do not also contain dark matter, either liquid or solid, nor even that this dark matter may not constitute the main portion of the nebulous mass. In the green nebulæ we can say with confidence that hydrogen, helium, and some unknown gas are certainly present, and that these gases *emit most of the light* that reaches us from them. But how much other less luminous matter in the form of grains and drops may be included within the gaseous cloud we cannot tell.

The idea of Mr. Lockyer (a part of his wide induction as to what may be called the "meteoritic constitution of the universe") is that they are clouds of "sparse meteorites, the collisions of which bring about a rise of temperature sufficient to render luminous one of their chief constituents," which some years ago, when the sentence was written, he imagined to be magnesium, though that is no longer maintained.

How far this theory will stand the test of time and future investigations remains to be seen. It is very doubtful, however, whether the *collisions* in such a body could be frequent or violent enough to account for its luminosity, and one is tempted to look to other causes for the source of light. "Luminescence" does not require a high temperature.

They are among the stars.

604. Distance and Distribution of Nebulæ. — As to their distance, we can only repeat that like the star-clusters they are within the star universe and not beyond its boundaries; this is clearly shown by the "nebulous stars," first discussed and pointed out by the older Herschel, and by such peculiar associations of stars and nebulæ as we find in the Pleiades (Fig. 196). Moreover, in certain luminous masses known as the "nubeculæ" (near the south pole of the heavens), we have stars, star-clusters, and nebulæ promiscuously intermingled. In the sky generally, however, the distribution of the nebulæ is in *contrast* to that of the stars. The stars crowd together near the Milky Way; the nebulæ, on the other hand, are most numerous near its poles, just where the stars are fewest, as if the stars had somehow consumed in their formation the substance of which the nebulæ are made, or as if, possibly, on the other hand, the nebulæ had been formed by the disintegration of stars, — as a few astronomers have maintained, in opposition to the more common view.

Their distribution in the heavens. They mostly avoid regions where stars abound.

THE CONSTITUTION OF THE SIDEREAL HEAVENS

605. The Galaxy, or Milky Way. — This is a luminous belt of irregular width and outline which surrounds the heavens nearly in a great circle. It is very different in brightness in its different parts, and in several constellations is marked by dark bars and patches which make the impression of overlying clouds; the most notable of these is the so-called "coal-sack," near the southern pole. For about a third of its length (from Cygnus to Scorpio) the Milky Way is divided into two nearly parallel streams.

The Galaxy

The telescope shows it to be made up almost wholly of small stars from the eighth magnitude down; it contains also numerous star-clusters, but very few true nebulæ.

A belt of small stars.

Fig. 204 is from one of Barnard's exquisite small-scale photographs of the edge of the Milky Way in the constellation of Sagittarius. With a powerful telescope the star clouds would be resolved into points of light.

The poles of the Galaxy: its plane the fundamental plane of stellar astronomy.

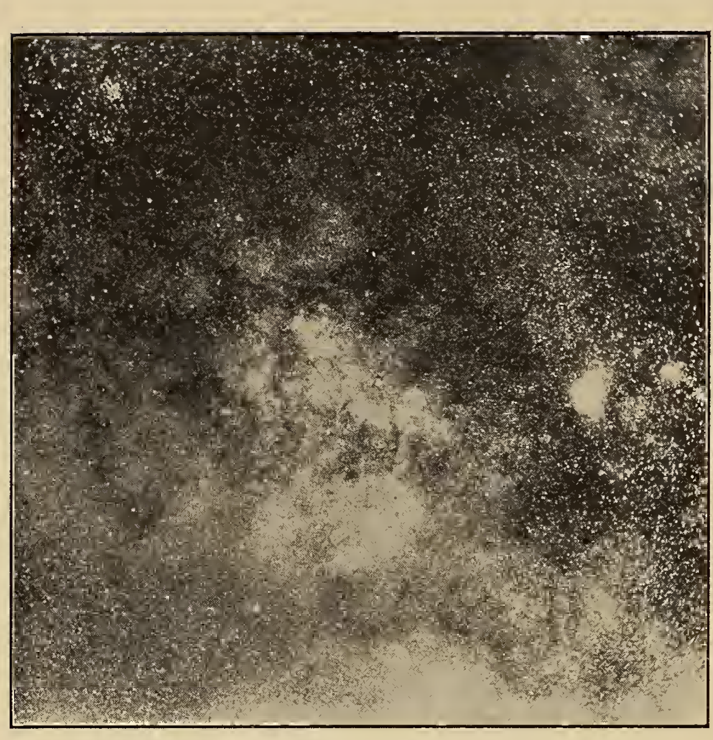

Fig. 204. — Star Clouds in Edge of Milky Way

The Galaxy intersects the ecliptic at two opposite points not far from the solstices and at an angle of nearly 60°, the northern "pole of the Galaxy" being, according to Herschel, in the constellation of Coma Berenicis.

As Herschel remarks, "the 'galactic plane' is to the sidereal universe much what the plane of the ecliptic is to the solar system, — a plane of ultimate reference, and the ground-plan of the stellar system."

Distribution of stars on celestial sphere.

606. Distribution of the Stars in the Heavens. — It is obvious that the distribution of the stars is not even approximately uniform; they gather everywhere in groups and streams.

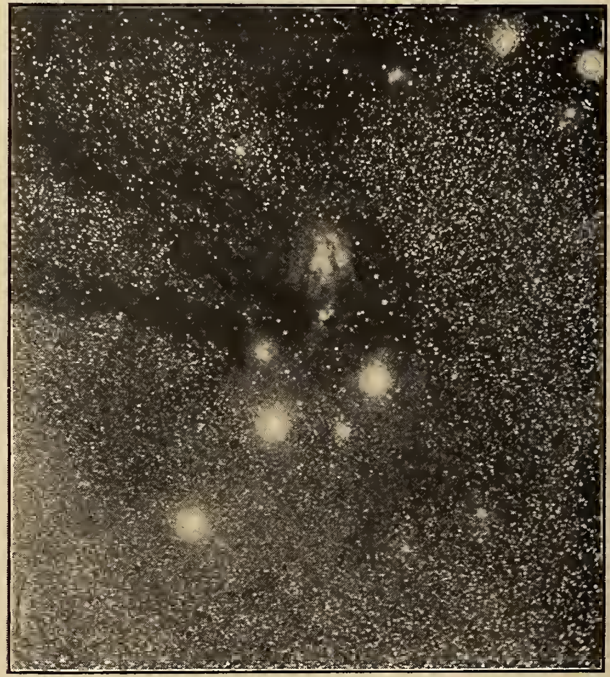

Fig. 205. — Nebula in Ophiuchus
Barnard

Fig. 205 is from another of Barnard's photographs, covering parts of Scorpio and Ophiuchus. The object at the intersection of lines indicated at the edge of the figure is the little star ρ Ophiuchi, barely visible to the naked eye, but surrounded by nebulosity which, in the photograph, makes it as conspicuous as Antares itself, below and to the left, — the middle one of the three. The dark "lanes" and other starless spaces in this region are very remarkable.

But besides this the examination of any of the great star-catalogues shows that the average number to a square degree increases rapidly and pretty regularly from the galactic pole to the galactic circle itself, where they are most thickly packed. This is best shown by the "star-gauges" of the elder Herschel, each of which consisted merely in an enumeration of the stars visible in a single field of view of his 20-foot reflector, the field being 15′ in diameter.

Most crowded near the plane of the galactic circle.

He made 3400 of these "gauges," and his son Sir John, using the same telescope, followed up the work at the Cape of Good Hope with 2300 more in the south circumpolar regions. From the data of these star-gauges, Struve has deduced the following figures for the number of stars visible in one field of view:

DISTANCE FROM GALACTIC CIRCLE	AVERAGE NUMBER OF STARS IN FIELD
90°	4.15
60°	6.52
30°	17.68
0	122.00

Structure of the stellar universe.

607. Structure of the Stellar Universe. — Herschel, starting from the unsound assumption that the stars are all of about the same size and brightness and separated by approximately equal distances, drew from his observations certain untenable conclusions as to the form and structure of the "galactic cluster," to which the sun was supposed to belong, — theories for a time widely accepted and even yet more or less current, though in many points certainly incorrect.

But although the apparent brightness of the stars does not thus depend entirely or even mainly upon their distance, it is certain that, *as a class*, the faint stars are smaller, darker, and more remote than the brighter ones; we may, therefore, safely draw a few conclusions, which, *so far as they go*, in the main agree with those of Herschel and are formulated by Newcomb, in his *Popular Astronomy*, substantially as follows:

Distribution of stars in space; mostly within a disklike space.

608. (I) The great majority of the stars we see are contained within a space having roughly the form of a rather thin flat disk, like a thin watch, with a diameter eight or ten times as great as its thickness, our sun being not far from the center. In other words, the stars which compose the star system are spread out on all sides in or near a widely extended plane, passing through the Milky Way.

(II) Within this space the *naked-eye* stars are distributed rather uniformly, but with some tendency to cluster, as shown in the Pleiades. The *smaller* stars, on the other hand, are strongly "gregarious" and are largely gathered in groups and streams, leaving comparatively vacant spaces between them.

(III) At right angles to the "galactic plane" the stars are scattered more thinly and evenly than in it, and we find here on the sides of the disk the comparatively starless region of the nebulæ.

(IV) As to the Milky Way itself, it is not certain whether the stars which compose it form a sort of thin, flat, continuous sheet, or whether they are ranged in a kind of *ring* or in *spires*, with a comparatively empty space in the middle where the sun is placed.

(V) The disk described above does not represent the form of the stellar system, but only the limits within which it is mostly contained. The circumstances are such as to prevent our assigning any more definite form to the system itself than we could assign to a cloud of dust.

Dimensions of the disklike space.

As to the size of the disklike space, very little can be said positively, but it seems quite certain that its diameter must be at least as great as from ten thousand to twenty thousand light-years, — how much greater we cannot even guess, and as to what is beyond we are still more ignorant. If, however, there are other stellar systems of the same order as our own, they are neither the nebulæ nor the clusters which the telescope reveals, but are far beyond the reach of any instrument at present existing.

Question as to whether the stars form a system.

609. Do the Stars form a System? — It is probable, though not yet absolutely proved, that gravitation operates between the stars (as indicated by the motions of the binaries), and they are certainly moving very swiftly in various directions. The question is whether these motions are governed by *gravitation* and are *orbital* in the common sense of the word.

Exploded idea of a central sun.

There has been a very persistent belief that somewhere there is "a great central sun," around which the stars are all circling. As to this, there is no longer any question; the "central sun" speculation is certainly unfounded, though we have not space here for the demonstration of its fallacy.

Possible revolution of the stars around the center of gravity of the whole.

Another less improbable doctrine is that there is a general revolution of all the stars *around the center of gravity of the whole*, — a revolution nearly in the plane of the Milky Way. Half a century ago Maedler, in his speculations already mentioned, concluded that this center of gravity of the stellar universe was near Alcyone, the brightest of the Pleiades, and that therefore this star was in a sense the "central sun." Very recently (1900) André, in his admirable work on *Stellar Astronomy*, has brought this theory again into favorable notice, showing that it agrees with several statistical facts relating to the proper motions of the stars. He even goes so far as to deduce from the data (*as they stand at present*), with the help of certain assumptions, that our distance from the "central sun" is about 715 light-years, the period of revolution about twenty-two million years, and the velocity of motion about 36 miles a second. But the evidence of any such general revolution of the stars is very far from conclusive, and the data are so insufficient that the numerical results are not entitled to much confidence.

André's conclusion.

Probable that the stellar motions are not orbital.

610. Indeed, on the whole, the most probable view still seems to be that the stars are moving much as bees do in a swarm, each star mainly under the control of the attraction of its nearest neighbors, though influenced more or less, of course, by that of the general mass. If this is so, the paths of the stars are not "orbits" in any *periodic* sense; *i.e.*, they are not paths which *return into themselves.* The forces which at any moment act upon a given star are so nearly balanced that its motion must be sensibly in a straight line for thousands of years at a time, except in cases where two stars are near together.

Question of cosmogony.

611. Cosmogony. — One of the most interesting and one of the most baffling topics of speculation relates to the process by which the present state of things has come about.

In a forest, to use an old comparison of Herschel's, we see around us trees in all stages of their life-history, from the sprouting seedlings to the prostrate and decaying trunks of the dead. Is the analogy applicable to the heavens, and if so, which of the heavenly bodies are in their infancy and which decrepit with age?

General principles which must lie at the basis of all speculations.

At present many of these questions seem to be absolutely beyond the reach of investigation. Others, though at present unsolved, appear approachable, and a few we can already answer. In a general way we may say that the condensation of diffuse, cloudlike masses of matter under the force of gravitation, the conversion into heat of the energy of motion and of position (the "kinetic" and "potential" energy — *Physics*, p. 121) of the particles thus concentrated, the effect of this heat upon the mass itself, and the effect of its radiation upon surrounding bodies, — these principles cover nearly all the explanations that can thus far be given of the present condition of the heavenly bodies.

612. Genesis of the Planetary System. — Our planetary system is clearly no accidental aggregation of bodies. Masses of matter coming haphazard to the sun would move (as the comets actually do move) in orbits which, though always conic sections, would have every degree of eccentricity and inclination. In the planetary system this is not so. Numerous relations exist for which the mind demands an explanation and for which gravitation does not account.

We note the following as the principal:

(1) The orbits of the planets are all *nearly circular.*

(2) They are all nearly *in one plane* (excepting those of some of the asteroids).

Facts which indicate a process of evolution in the solar system.

(3) The revolution of all, without exception, is *in the same direction.*

(4) There is a curious and *regular progression of distances* (expressed by Bode's Law, which, however, breaks down with Neptune).

As regards the planets themselves:

(5) The plane of the planet's rotation nearly coincides with that of the orbit (probably excepting Uranus).

(6) The direction of rotation is the same as that of the orbital revolution (excepting probably Uranus and Neptune).

(7) The plane of the orbital revolution of the planet's satellites coincides nearly with that of the planet's rotation, wherever this can be ascertained.

(8) The direction of the satellites' revolution also coincides with that of the planet's rotation (with the same limitation).

(9) The largest planets rotate most swiftly.

Now this arrangement is certainly an admirable one for a planetary system, and therefore some have argued that the Deity constructed the system in that way, perfect from the first. But to one who considers the way in which other perfect works of nature usually attain to their perfection — their processes of growth and development — this explanation seems improbable. It appears far more likely that the planetary system was *formed by growth* than that it was *built outright.*

The nebular hypothesis.

613. The Nebular Hypothesis. — The theory which in its main features has been very generally accepted, as supplying an intelligible explanation of the facts, is that known as the "nebular hypothesis." In a more or less crude and unscientific form it was first suggested by Swedenborg and Kant, and afterwards, about the beginning of the present century, was worked out in mathematical detail by Laplace. He maintained:

Its principles as laid down by Laplace.

(*a*) That at some time in the past the matter which is now gathered into the sun and planets was in the form of a nebula. But there was no assumption, as is often supposed, that matter was first *created* in the nebulous condition. It was only assumed that, as the egg may be taken as the starting-point for the life-history of an animal, so the nebula is to be regarded as the starting-point of the life-history of the planetary system.

(*b*) This nebula, according to him, was a cloud of *intensely heated gas.* (This postulate is more than questionable.)

(*c*) Under the action of its own gravitation the nebula assumed *a form approximately globular, with a motion of rotation*, the rotational motion depending upon accidental differences in the original velocities and densities of different parts of the nebula. As the contraction proceeded the swiftness of the rotation would necessarily increase for mechanical reasons, since every shrinkage of a revolving mass implies a shortening of its rotation period.

(*d*) In consequence of the rotation the globe would necessarily become flattened at the poles and ultimately, as the contraction went on, the centrifugal force at the equator would become equal to gravity and *rings of nebulous matter*, like the rings of Saturn, *would be detached* (*not "thrown off"*) *from the central mass.* In fact, Saturn's rings suggested this feature of the theory.

(*e*) The ring thus formed would for a time revolve as a whole, but would ultimately break, *and the material would collect into a globe revolving around the central nebula as a planet.* Laplace supposed that the ring would revolve as if solid, the particles at the outer edge moving more swiftly than those at the inner. If this were always so, the planet formed would necessarily *rotate in the same direction as the ring had revolved.*

(*f*) The planet thus formed might throw off rings of its own and so form for itself a system of *satellites.*

The theory obviously explains most of the facts of the solar system, which were enumerated in the preceding article, though some of the exceptional facts, such as the short periods of the satellites of Mars and the retrograde motions of those of Uranus and Neptune, cannot be explained by it *alone* in its original form; other considerations must be introduced. Even they, however, do not *contradict* it, as is sometimes supposed.

Explains the obvious facts, and is not contradicted by some which seem unfavorable.

Many things also make it questionable whether the outer planets are so much older than the inner ones, as the theory would indicate. It is not impossible that they may even be younger.

614. On the whole, we may say that while in its main outlines the theory probably is true, it also probably needs serious modifications in details. It is rather more likely, for instance, that in the early stages the nebula was a cloud of *ice-cold* meteoric dust than an incandescent gas, or a "fire mist," to use a favorite expression; and it is likely that planets and satellites were usually separated from the mother orb otherwise than in the form of rings. Nor is it possible that a thin wide ring could revolve in the same way as a solid coherent mass; the particles near the inner edge must make their revolution in periods much shorter than those upon the circumference.

Probable modifications needed.

No original fire mist.

Rings could not revolve in manner assumed by Laplace.

A most serious difficulty arises also from the apparently irreconcilable conflict between the conclusions as to the age and duration of the system, which are based on the theory of heat, and the length of time which would seem to be required by the nebular hypothesis for the evolution of our system.

Conflict of conclusions as to age of system.

Our limits do not permit us to enter into a discussion of Darwin's "tidal theory" of satellite formation, which may be regarded as, in a sense, supplementary to the nebular hypothesis; nor can we more than mention Faye's proposed modification of it. According to him, the *inner* planets are the *oldest*.

615. Lockyer's Meteoritic Hypothesis. — Within the last fifteen or twenty years Sir Norman Lockyer has vigorously revived a theory which has been from time to time suggested

Lockyer's meteoritic hypothesis.

before, viz., that all the heavenly bodies in their present state are mere *clouds of meteors*, or have been formed by the aggregation of such clouds; and it is an interesting fact, as G. H. Darwin has shown, that a large swarm of meteors in which the individuals move swiftly in all directions would, *in the long run and as a whole*, behave almost exactly, from a mechanical point of view, in the same way as one of Laplace's "gaseous nebulæ."

This is not very strange, after all. According to the modern "kinetic theory of gases" (*Physics*, p. 157), a meteor-cloud is mechanically just the same thing as a mass of gas *magnified*. The kinetic theory asserts that a gas is only a swarm of minute molecules, the peculiar gaseous properties depending upon the collisions of these molecules with each other and with the walls of the inclosing vessel. Magnify sufficiently the molecules and the distances between them, and you have a meteoric cloud.

The spectroscopic basis doubtful.

The spectroscopic "facts" upon which Lockyer rests his attempted demonstration are, indeed, many of them rather doubtful, but that does not really invalidate the main idea, except in so far as the question of the origin and nature of the light produced is concerned. He makes the light in all cases depend upon the *collisions* between the meteors, and finds in the spectra of the heavenly bodies evidences of the presence of materials with which we are familiar in the meteorites that fall upon the earth's surface. These identifications are in many cases discredited, and it seems much more probable that the luminosity depends to a great degree upon other than mere mechanical actions.

Genetic connection between stars, star-clusters, and nebulæ.

616. Stars, Star-Clusters, and Nebulæ. — It is obvious that the nebular hypothesis in all of its forms applies to the explanation of the relations of these different classes of bodies to each other. In fact, Herschel, appealing only to the "law of continuity," had concluded, before Laplace formulated his theory, that the nebulæ develop sometimes into clusters, sometimes into double or multiple stars, and sometimes into single stars. He showed the existence in the sky of all the intermediate forms between the nebula and the finished star. For a time, about forty years ago, while it was generally believed that all the nebulæ were nothing but star-clusters, only too remote to be resolved by existing telescopes, his views fell rather into abeyance; but they regained acceptance in their essential features when the spectroscope demonstrated the substantial difference between gaseous nebulæ and the star-clusters.

Conclusions drawn from the theory of heat.

617. Conclusions from the Theory of Heat. — Kant and Laplace, as Newcomb says, seem to have reached their results by reasoning *forward.* Modern science comes to very similar conclusions by working *backward* from the present state of things.

Many circumstances go to show that the *earth* was once much hotter than it now is. As we penetrate below the surface the temperature rises nearly a degree (Fahrenheit) for every 60 feet, indicating a white heat at the depth of a few miles only; the earth at present, as Sir William Thomson says, "is in the condition of a stone that has been in the fire and has cooled at the surface."

The earth and planets once hot.

The *moon* apparently bears on its surface the marks of the most intense igneous action, but seems now to be entirely chilled.

The *planets*, so far as we can make out with the telescope, exhibit nothing at variance with the view that they were once intensely heated, while many things go to establish it. Jupiter and Saturn, Uranus and Neptune, do not seem yet to have cooled off to anything like the earth's condition.

Age and duration of the solar system.

618. Age and Duration of the Solar System. — In the *sun* we have a body continuously pouring forth an absolutely inconceivable quantity of heat without any visible source of supply. As has been explained already (Sec. 275), the only rational explanation of the facts thus far presented is that which makes it a huge cloud-mantled ball of elastic substance slowly shrinking under its own central gravity, and thus converting into the *kinetic* energy of heat[1] the *potential* energy of its particles as they gradually settle towards the center. A shrinkage of 300 feet a year in the sun's diameter (150 feet in its *radius*) will account for the whole annual output of radiant heat and light. Looking *backward*, then, and trying to imagine the course of time and of events *reversed*, we see the sun growing larger and larger, until at last it has expanded to a huge cloud that fills the largest orbit of our system. How long ago this may have been we cannot state with certainty. If we could assume that the amount of heat yearly radiated by the solar surface had remained constantly the same through all those ages, and, moreover, that all the radiated heat came only from the slow contraction of the solar mass, apart from any considerable original capital in the form of a high initial temperature, and without any reinforcement of energy from outside sources, — *IF we could assume these premises*, it is easy to show that the sun's past history must cover about fifteen or twenty million years. But such assumptions are at least doubtful; and if we discard them, all that can be said is that the sun's age must be *greater*, and probably many times greater, than the limit we have named.

Assuming continuity, the sun and planets once formed a cloud with diameter equal to Neptune's orbit.

Assumptions doubtful.

Prospect for the future.

Looking *forward*, on the other hand, from the present towards the future, it is easy to conclude with certainty that if the sun continues its present rate of radiation and contraction and receives no subsidies of energy from without, it must within five or ten million years become so dense that its constitution will be radically changed. Its temperature will fall, and its function as a sun will end. Life on the earth, as we know life, will be no longer possible when the sun has become a dark, rigid, frozen globe. At least this is the inevitable consequence of what now seems to be the true account of the sun's *present* activity and *the story of its life.*

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[1] So far we have no decisive evidence whether the sun has passed its maximum of temperature or not. Mr. Lockyer thinks its spectrum (resembling as it does that of Capella and the stars of the second class) proves that it is now on the *downward grade* and growing cooler, and the fact that its density is apparently much higher than that of other stars — at least than that of the variables of the Algol type (see Sec. 582) — certainly falls in well with the idea that it has reached an advanced stage of development and perhaps passed its culmination. But the evidence can hardly, as yet, be considered conclusive.

Possibility of a different outcome.

At the same time it is by no means certain that the processes now observed have been going on steadily through all the past, or will continue to do so in the future, without break or interruption. Catastrophes and paroxysms, sudden changes and reversals of the course of events at critical moments, are certainly possible and actually occur, as the phenomena of the solar surface and temporary stars abundantly make evident.

The system not eternal.

General aggregation of scattered masses: but exceptions.

619. The Present System not Eternal. — One lesson seems to to be clearly taught: that the present system of stars and worlds is not an *eternal* one. We have before us everywhere evidence of continuous, irreversible progress from a definite beginning towards a definite end. Scattered particles and masses are gathering together and condensing, so that the great grow continually larger by capturing and absorbing the smaller. And yet, on the other hand, the phenomena of the coronal streamers, of comets' tails, and those presented by the swiftly expanding nebulosity of Nova Persei, seem to indicate in certain cases a process exactly the reverse, — a repulsion and dissipation in space of finer grained materials, possibly the "ions" of the most modern physicists.

Dissipation of energy.

At the same time the hot bodies are losing their heat and distributing it to the colder ones, so that there is an unremitting tendency towards a uniform, and therefore *useless*, temperature throughout our whole universe; for heat is available as energy (*i.e.*, *it can do work*) only when it can pass from a warmer body to a colder one. The continual warming up of cooler bodies at the expense of hotter ones always means a loss, therefore, not of energy, for that is indestructible, but of *available* energy.

To use the ordinary technical term, energy is continually "dissipated" by the processes which constitute and maintain life on the universe. This "dissipation of energy" can have but one ultimate result, that of absolute stagnation when the temperature has become everywhere the same.

The beginning and the end beyond our knowledge.

If we carry our imagination backward, we reach "a beginning of things," which has no intelligible antecedent; if forward, we come to an end of things in dead stagnation. That in some way this end of things will result in a "new heavens and a new earth" is, of course, very probable, but science as yet presents no explanation of the method.

EXERCISES

1. Find the mass of the system of α Centauri from the data given in Tables IV and VII; namely, parallax (p) = $0''.75$, semi-major axis of orbit (a'') = $17''.70$, and period (t) = 81.1 years. (See Secs. 590 and 594.)
Ans. Mass of system = 1.99 × mass of the sun.

2. Find the mass of the system of Sirius from the tabular data.
Ans. 3.07 × mass of the sun.

3. Find the mass of the system of η Cassiopeiæ from the tabular data.
Ans. 2.10 × mass of the sun.

4. Find the mass of the system of 70 Ophiuchi from the tabular data.
Ans. 2.92 × mass of the sun.

NOTE. — The results obtained by the solution of problems 1, 2, 3, and 4 will not agree with those given in Sec. 590, because of the different values of parallax employed. The discrepancies fairly illustrate the uncertainties of our present knowledge.

5. Find the radius of the apparent orbit of the spectroscopic binary Lacaille 3105, the relative velocity of the components being 385 miles a second and the period $3^d 2^h 46^m$, as indicated by the doubling of the lines in the spectrum. Assume that the orbit is circular, that its plane is directed towards the sun, and that the two components are equal.
Ans. Radius of orbit = 16 493000 miles.

6. Compute the mass of the system on the same assumptions as above, remembering that the radius of this apparent orbit is also the radius of the *relative* orbit which each component describes around the other regarded as at rest.
Ans. 76.75 × mass of the sun.

7. Carry out similar computations for the systems of ζ Ursæ Majoris, β Aurigæ, and μ Scorpii, using the data of Sec. 593.

8. Determine the radius of the orbit described by Spica Virginis, as shown by the shift of the lines in its spectrum. Velocity = 56.6 miles a second; period = $4^{d}19^{m}$. Orbits assumed circular and in plane of the sun.
Ans. Radius = 3 123500 miles.

9. From this determine the mass of the system, assuming that the mass of the bright star is infinitesimal as compared with that of the dark star, *i.e.*, that it is a small planet revolving around a dark central sun. (A very improbable hypothesis, of course.) *Ans.* 0.315 × mass of the sun.

10. What is the mass of the system if the dark star is equal to the bright one? (In this case the radius of the *relative* orbit is the diameter of the apparent orbit of Spica, or *double* its value in the last example.)
Ans. $2^3 \times 0.315$, or 2.520, × mass of the sun.

11. What is the mass if the dark star has a mass only one fourth that of the bright one? (In this case the orbit of the dark star has a radius 4 times as great as that of Spica, and the radius of the *relative* orbit is 5 times as great as that of the apparent orbit of Spica.)
Ans. $5^3 \times 0.315$, or 39.37, × mass of the sun; the mass of the bright star being 31.50 and that of the dark star being one fourth as great, or 7.87.

NOTE. — The assumption that the bright star is a small planet, revolving around a dark central body vastly more massive than itself, gives us a minor limit to the possible mass of the system, but the major limit cannot be fixed without knowledge as to the relative mass of the dark body.

If the dark body is larger than the bright one, the mass of the system cannot exceed eight times that minor limit.

The general formula is easily obtained: let n be the ratio between the masses of the bright and dark stars, so that if r is the radius of the circle described by the *bright* star around the common center, the radius of the circle described by the other will be nr, and the radius of the *relative* orbit will be $(n+1)\,r$. Also let μ be the united mass of the two stars. Then, expressing the period, t, in years, r in astronomical units, and μ in terms of the sun's mass, we have

$$\mu = (n+1)^3 \frac{r^3}{t^2}.$$

The factor $(n+1)^3$ becomes unity when $n = o$, — *i.e.*, when the *bright star* is a particle; and infinity when n becomes infinite, — *i.e.*, when the *dark star* is a particle revolving at the infinite distance, $r\,(n+1)$. It becomes 8 when $n = 1$, the two stars being equal.

It may be added that the assumption that the orbit is circular and that its plane passes through the solar system is entirely gratuitous and not likely to be correct. But the general character of the results would not be seriously changed unless the inclination and eccentricity of the orbit were great, as, for instance (probably), in the case of Polaris.

APPENDIX

700. Transformation of Astronomical Coördinates. — It is often necessary to change one set of coördinates into another; to convert, for instance, right ascension and declination into altitude and azimuth or into latitude and longitude, and *vice versa*. The process is a simple trigonometrical calculation, in which we have to deal with spherical triangles having (usually) given two sides and the included angle. There are various methods of solution: we may drop a perpendicular from one of the unknown angles upon the opposite side and carry out the solution by Napier's rules for right-angled triangles; or we may apply Napier's analogies to compute the two unknown angles; or, finally, we may use an auxiliary angle, as indicated in Campbell's *Practical Astronomy.*

701. To convert Right Ascension (α) and Declination (δ) into Altitude (h) and Azimuth (A). — The observer's latitude, ϕ, must also be known and the sidereal time, θ. Referring to Fig. 206, we see that the triangle to be solved is OPZ, in which we have $PZ = 90° - \phi$, $PO = 90° - \delta$, and t, the hour angle ZPO, $= (\alpha - \theta)$. Required ZO or z (which $= 90° - h$), and the angle PZO (which is the supplement of SZO, the azimuth A).

The formulæ most used are the following (Campbell's *Practical Astronomy*, Sec. 5):

$$\left.\begin{aligned} n \sin \mathrm{N} &= \sin \delta \qquad (a) \\ n \cos \mathrm{N} &= \cos \delta \sin t \qquad (b) \end{aligned}\right\},$$

whence,

$$\tan \mathrm{N} = \tan \delta \sin t.$$

N is the auxiliary angle, and its quadrant is determined by equations (a) and (b), which give the signs of its sine and cosine; n is always positive, but is not used.

Then

$$\tan A = \frac{\tan t \cos \mathrm{N}}{\sin (\phi - \mathrm{N})}, \text{ and } \tan z = \frac{\tan (\phi - \mathrm{N})}{\cos A}.$$

Care must be taken to observe the algebraic signs throughout.

702. To convert Right Ascension and Declination into Latitude and Longitude. — The triangle to be employed is OPP' (Fig. 206), in which we have given $PP' = \epsilon$, the obliquity of the ecliptic (23° 28′),

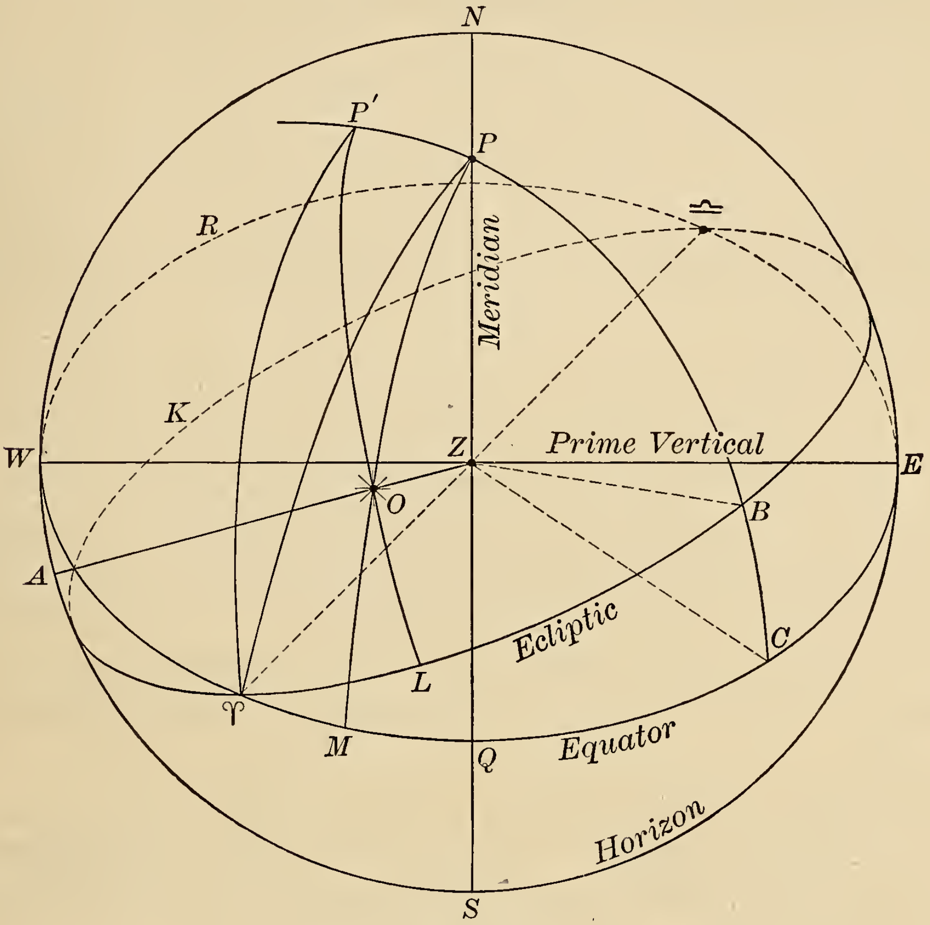

FIG. 206

$PO = (90^\circ - \delta)$, and $P'PO = (90^\circ + \alpha)$. $P'P♈ = 90^\circ$, and $♈ PM =$ arc $♈ M =$ right ascension of O. $PP'O = (90^\circ - \lambda)$, ($\lambda =$ angle $♈ P'O$), and $P'O = (90^\circ - \beta)$. The formulæ then are:

$$\left.\begin{array}{ll} f \sin F = \sin \delta & (c) \\ f \cos F = \cos \delta \sin \alpha & (d) \end{array}\right\},$$

whence $$\tan F = \frac{\tan \delta}{\sin \alpha}.$$

Then $$\tan \lambda = \frac{\cos (F - \epsilon) \tan \alpha}{\cos F}, \text{ and } \tan \beta = \tan (F - \epsilon) \sin \lambda.$$

For other cases, see Campbell's *Practical Astronomy* or Wentworth's *Spherical Trigonometry*.

703. Projection of a Lunar Eclipse (supplementary to Sec. 288). — We take as an example the eclipse of Oct. 16, 1902, which will be generally visible in the United States.

The data, as given in the American Ephemeris, are:

	SUN	MOON
Declination	8° 55′ 20″.5 S.	9° 08′ 52″.7 N.
Hourly motion in right ascension .	$9^s.33$	$138^s.31$
Hourly motion in declination . . .	0′ 55″.2 S.	10′ 06″.4 N.
Semidiameter	16′ 03″.1	16′ 08″.3
Horizontal parallax	8″.8	59′ 13″.2

Greenwich mean time of opposition in right ascension, Oct. 16, $18^h 10^m 12^s.7$.

A convenient scale is 1000″ to the inch; this will bring the figure within the limits of an 8 × 10 sheet, and is large enough to give all the accuracy required. Fractions of a second of arc are neglected.

The work is made easier by the use of squared paper, but the results are seldom quite as precise, because of the inaccuracies of the ruling.

704. I. The first step is to *lay off the "relative orbit" of the moon with respect to the shadow.* Draw two lines accurately perpendicular to each other, their crossing point O (Fig. 207) being the place of the moon's center at the moment of opposition.

(*a*) On the horizontal line EW lay off the difference of the hourly motions of the sun and moon in right ascension, expressed in seconds of arc *and reduced to seconds of a great circle, by multiplying by the cosine* of the moon's declination. In this case we have $(138.31 - 9.33) \times 15 \times \cos 9° 08' 53''$, which equals 1910″. Ob and Od are each laid off with this value, while Oa and Oe are made twice as great.

(*b*) At b and d lay off perpendicular to the line EW the difference of the hourly motions in declination (the *shadow* moves *north* when the sun moves *south*). We have in this case $(10' 06''.4 - 0' 55''.2)$, which equals 551″. We lay off, therefore, at b and d ordinates each equal to 551, and at a and e ordinates twice as great.

Since the moon is moving *northwards*, the ordinates west of O must be laid off downwards, and east of O upwards.

If the work has been carefully done, the four points thus plotted will lie accurately on a straight line with each other and with O, and will be the points occupied by the moon's center one and two hours before and after opposition. The "hours" on this line are to be divided into halves and quarters, and wherever necessary the fifteen-minute spaces can be divided into five-minute portions.

II. *Mark the center of the shadow.* Lay off north or south of O a distance equal to the difference of declination of the moon and the shadow. (The declination of the shadow is the same as the sun's *with its sign changed.*) In this case we have $OC = (9° 08' 52''.7 - 8° 55' 20''.5) = 812''$, — to be laid off to the *south* because the moon is north of the center of the shadow.

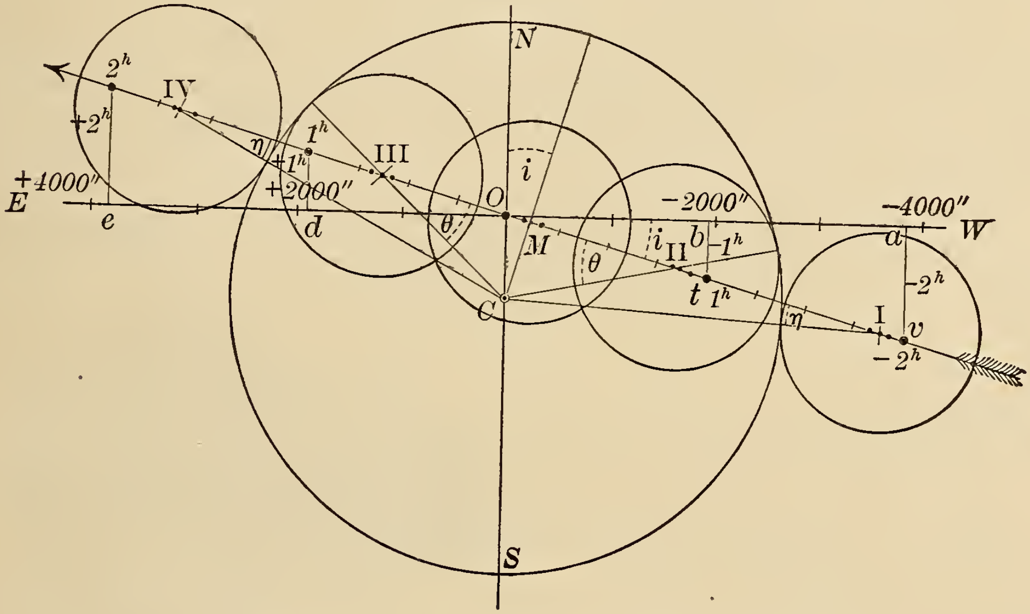

Fig. 207. — Projection of a Lunar Eclipse

III. *Find the radius of the shadow and draw the shadow.* From Fig. 208 we see that MEN (the angular semidiameter, or "radius," of the shadow) $= aNE - ACO$ or BCO. But $BCO = OEB - EBb$, so that $MEN = aNE + EBb - OEB$. aNE is the horizontal parallax, P, of the moon; EBb is the horizontal parallax,

p, of the sun; and OEB is the sun's semidiameter, S, as seen from the earth. As a formula this is written $\rho = \frac{61}{60}(P + p - S)$ (the $\frac{61}{60}$ being applied to take account of the slight apparent enlargement of the shadow by the earth's atmosphere). In this case we have, therefore,

$$\rho = \tfrac{61}{60}(59' \, 13''.2 + 8''.8 - 16' \, 03''.1) = 2642''.$$

With this radius and C as a center, the outline of the shadow may be drawn, but it is not necessary.

IV. *Mark the points on the relative orbit occupied by the moon's center at the moments of contact with the shadow.*

(*a*) To the radius of the shadow *add* the semidiameter of the moon (in this case $2642'' + 968'' = 3610''$), and with this distance as a radius, from C as a center, strike two arcs cutting the relative orbit at I and II, which give the position of the moon at the two external contacts.

(*b*) Subtract the moon's semidiameter from the radius of the shadow, and with this difference ($2642'' - 968'' = 1674''$) as a radius, from C as a center, find the points II and III of internal contact. The figure may be completed by drawing the circles to represent the moon, using I, II, M, III, and IV as centers. M, the middle of the eclipse, is, of course, half-way between II and III.

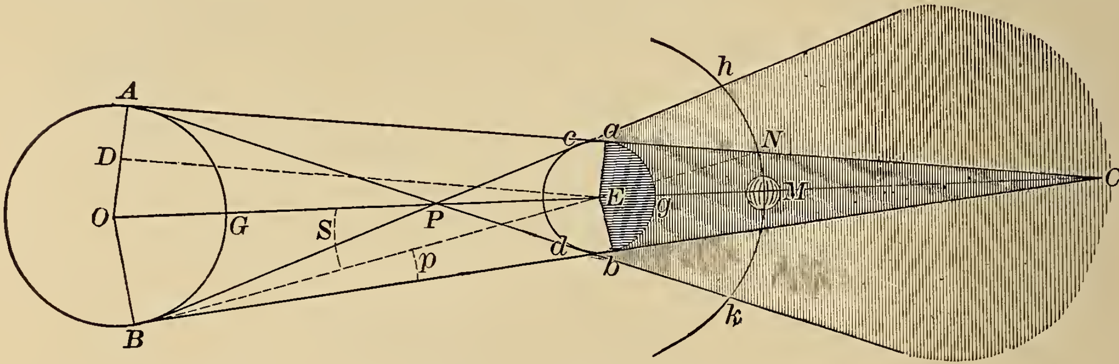

FIG. 208. — The Earth's Shadow

V. Finally, *read off the times of contact on the relative orbit regarded as a scale of time.* For points west of O subtract the time readings from the time of opposition ($18^h 10^m.2$ in this case), and for points east add them.

In the present case the projection as figured gives the following results :

I	II	M	III	IV
$-\ 1^h53^m.0$	$-\ 0^h51^m.0$	$-\ 0^h\ 6^m.7$	$+\ 0^h38^m.0$	$+\ 1^h39^m.5$
18 10 .2	18 10 .2	18 10 .2	18 10 .2	18 10 .2
16 17 .2	17 19 .2	18 03 .5	18 48 .2	19 49 .7
(17.3)	(19.0)	(03.4)	(47.9)	(49.7)

The figures in parentheses are the calculated results as given in the American Ephemeris.

To get eastern standard time subtract 5 hours; *i.e.*, the eclipse begins at $11^h17^m.2$, E. S. T., its middle is at $1^h03^m.5$ A.M., and it ends at $2^h49^m.7$ A.M., being total from $12^h19^m.2$ until $1^h48^m.2$ A.M.

705. Calculation of the Eclipse. — This is very simple, the triangles concerned being all right-angled plane triangles. Five-place logarithms are sufficient.

I. From the elements given in the Almanac, form the following quantities :

(*a*) The relative hourly motion in right ascension in seconds of arc, *reduced to arc of a great circle*, as in I (*a*) of the preceding section (Ob in Fig. 207). It comes out $1910''.2$ in this case. (*Beginners are very apt to forget the multiplication by cosine of moon's declination.*)

(*b*) The relative motion in declination (bt in the figure), $551''.2$ in this case.

(*c*) The distance of the center of the shadow from the point of opposition (the line OC in the figure), $812''.2$ in this case.

(*d*) The radius of the shadow, ρ (the line CN in the figure) $= \frac{61}{60}(P + p - S_{\odot})$, in this case $2642''.2$.

(*e*) The distance C, I, and C, IV $= (\rho + S_{☾}) = 3610''.5$ in this case.

(*f*) The distance C, II, and C, III $= (\rho - S_{☾}) = 1673''.9$ in this case.

II. In triangle Obt, given Ob and bt, compute the angle bOt $(= i)$, and the hypotenuse Ot, the orbital relative hourly motion. i comes out $16°\ 5'.8$ and Ot, $1988''.2$ (but only its *logarithm* is needed).

III. In triangle OCM, given OC ($812''.2$) and angle OCM ($i = 16°\ 5'.8$), compute CM ($780''.4$) and log OM. $\frac{OM}{Ot} =$ time (in hours) by which the *middle of the eclipse* differs (earlier in this case) from the time of opposition. $\frac{OM}{Ot}$ comes out $6^m.78$, giving **$18^h03^m.4$** for the middle of the eclipse.

IV. In the triangle CM, I (or CM, IV), having given CM ($780''.4$) and C, I ($\rho + S_{☾}$, *i.e.*, $2642''.2 + 968''.3 = 3610''.5$), compute log M, I. $\frac{M, \text{I}}{Ot}$ is the time (in hours) by which I precedes or IV follows the middle of the eclipse; it comes out $1^h.773$ or $1^h46^s.4$, so that contact I occurs at **$16^h17^m.0$**, and IV at **$19^h49^m.8$**.

V. In the triangle CM, II (or CM, III), having given CM and C, II $= \rho - S_{☾}$ ($= 1673''.9$), compute log M, II. $\frac{M, \text{II}}{Ot} =$ time by which contacts II and III precede and follow the middle. It comes out $44^m.70$, so that contact II occurs at **$17^h18^m.7$**, and III at **$18^h48^m.1$**.

The slight differences between these results and those given in the Ephemeris are probably due to the fact that the latter uses for ρ, $\frac{76}{75}(P + p - S_{☾})$, instead of $\frac{61}{60}$ of the same. But the times cannot be observed within half a minute.

It should be noted also that the motion of the moon is not quite uniform, either in right ascension or declination during the three and a half hours of the eclipse, as assumed in the projection and calculation. It would greatly complicate the matter to take the variations into account, and is unnecessary, considering that the tenths of a minute (which alone would be affected) are quite below the uncertainties of observation. In the computation of a *solar* eclipse the variations would have to be included; but that subject lies quite beyond our scope.

THE GREEK ALPHABET

Letters	Name	Letters	Name	Letters	Name
Α, α,	Alpha.	Ι, ι,	Iota.	Ρ, ρ ϱ,	Rho.
Β, β,	Beta.	Κ, κ,	Kappa.	Σ, σ ς,	Sigma.
Γ, γ,	Gamma.	Λ, λ,	Lambda.	Τ, τ,	Tau.
Δ, δ,	Delta.	Μ, μ,	Mu.	Υ, υ,	Upsilon.
Ε, ε,	Epsilon.	Ν, ν,	Nu.	Φ, φ,	Phi.
Ζ, ζ,	Zeta.	Ξ, ξ,	Xi.	Χ, χ,	Chi.
Η, η,	Eta.	Ο, ο,	Omicron.	Ψ, ψ,	Psi.
Θ, θ,	Theta.	Π, π ϖ,	Pi.	Ω, ω	Omega.

MISCELLANEOUS SYMBOLS

☌, Conjunction.
□, Quadrature.
☍, Opposition.
☊, Ascending Node.
☋, Descending Node.

A.R., or α, Right Ascension.
Decl., or δ, Declination.
λ, Longitude (Celestial).
β, Latitude (Celestial).
φ, Latitude (Terrestrial).

ω, angle between line of nodes and line of apsides.
ε, obliquity of ecliptic.

DIMENSIONS OF THE TERRESTRIAL SPHEROID

(According to Clarke's Spheroid of 1878. For the spheroid of 1866, see Sec. 134.)

Equatorial semidiameter, —

20 926202 feet = 3963.296 miles = 6 378190 meters.

Polar semidiameter, —

20 854895 feet = 3949.790 miles = 6 356456 meters.

Mean semidiameter, *i.e.*, $\frac{1}{3}(2a + b)$, —

20 902433 feet = 3958.794 miles = 6 370945 meters.

Oblateness (Clarke), $\frac{1}{293.46}$; (Harkness), $\frac{1}{300}$.

Length (in meters) of 1° of meridian in lat. $\phi = 111132.09 - 556.05 \cos 2\phi + 1.20 \cos 4\phi$.

Length (in meters) of 1° of parallel in lat. $\phi = 111415.10 \cos\phi - 94.54 \cos 3\phi$.
1° of lat. at pole = 111699.3 meters = 69.407 miles.
1° of lat. at equator = 110567.2 meters = 68.704 miles.

These formulæ correspond to the Clarke Spheroid of 1866, used by the United States Coast and Geodetic Survey.

TIME CONSTANTS

The sidereal day = $23^h\ 56^m\ 4^s.090$ of mean solar time.
The mean solar day = $24^h\ 3^m\ 56^s.556$ of sidereal time.

To reduce a time-interval expressed in units of *mean solar time to units of sidereal time,* multiply by 1.00273791; log of 0.00273791 = [7.4374191].

To reduce a time-interval expressed in units of *sidereal time* to units of *mean solar time,* multiply by 0.99726957 = (1 − 0.00273043); log 0.00273043 = [7.4362316].

Tropical year (Leverrier, reduced to 1900)	365^d	5^h	48^m	$45^s.51$.
Sidereal year " " "	365	6	9	8 .97.
Anomalistic year " " "	365	6	13	48 .09.

Mean synodical month (Neison)	29^d	12^h	44^m	$2^s.864$.
Sidereal month	27	7	43	11 .545.
Tropical month (equinox to equinox) . .	27	7	43	4 .68.
Anomalistic month (perigee to perigee) . .	27	13	18	37 .44.
Nodical or draconitic month (node to node)	27	5	5	35 .81.

Obliquity of the ecliptic (Newcomb),
$23^\circ\ 27'\ 8''.26 - 0''.468\,(t - 1900)$.
Constant of precession (Newcomb), $50''.248 + 0.000222\,(t - 1900)$.
Constant of nutation (Paris Conference, 1896), 9″.21.
Constant of aberration (Paris Conference, 1896), 20″.47.
Solar parallax (Paris Conference, 1896), 8″.80.
Velocity of light (Michelson and Newcomb),
186330 miles, 299860 km.

TABLE I — PRINCIPAL ELEMENTS OF THE SOLAR SYSTEM

	NAME	SYMBOL	Semi-Major Axis of Orbit	Mean Dist. Millions of Miles	Sidereal Period (mean solar days)	Period in Years	Orbit Velocity (miles per second)	Eccen-tricity	Inclination to Ecliptic	Longitude of Ascending Node	Longitude of Perihelion	Longitude at Epoch, Jan. 1, 1850
Ter'strial Planets	Mercury . . .	☿	0 387099	36.0	87.96926	0.24	23 to 35	.20560	7° 00′ 8″	46° 33′ 9″	75° 7′ 14″	327° 15′ 20′
	Venus	♀	0.723332	67.2	224.7008	0.62	21.9	.00684	3 23 35	75 19 52	129 27 15	245 33 15
	Earth	⊕	1.000000	92.9	365.2564	1.00	18.5	.01677	0 00 00	0 00 00	100 21 22	100 46 44
	Mars	♂	1.523691	141.5	686.9505	1.88	15.0	.09326	1 51 2	48 23 53	333 17 54	83 40 31
	Ceres	(1)	2.767265	257.1	1681.414	4.60	11.1	.07631	10 37 10	80 46 39	149 37 49	103 25 3
	Eros	(433)	1.4581	135.5	643.1	1.76	12 to 19	.2228	10 49 30	303 29 50	121 8 12	
Major Planets	Jupiter . . .	♃	5.202800	483.3	4332.580	11.86	8.1	.04825	1 18 41	98 56 17	11 54 58	160 1 10
	Saturn . . .	♄	9.538861	886.0	10759.22	29.46	6.0	.05607	2 29 40	112 20 53	90 6 38	14 52 28
	Uranus . . .	⛢	19.18329	1781.9	30686.82	84.02	4.2	.04634	0 46 20	73 13 54	170 50 7	29 17 51
	Neptune . . .	♆	30.05508	2791.6	60181.11	164.78	3.4	.00896	1 47 2	130 6 25	45 59 43	334 33 29

	NAME	SYMBOL	Apparent Angular Diameter	Mean Diameter		Mass		Volume ⊕=1	Mean Density		Axial Rotation	Inclination of Equator to Orbit	Oblate-ness	Surface Gravity ⊕=1	Albedo (Müller)
				In Miles	⊕=1	⊙=1	⊕=1		⊕=1	Water=1					
	Sun . .	⊙	32′ 04″ (mean)	866 400	109.4	1.000	332 000	1 300 000	0.25	1.39	$25^d\ 7^h\ 48^m \pm$	7°15′ (to ecliptic)	?	27.65	
	Moon .	☾	31 07 "	2 163	0.273	$\frac{1}{27\,058\,000}$	$\frac{1}{81\cdot5}$	0.0204	0.61	3.39	27 7 43	6 33 (mean)	?	$\frac{1}{6}$	0.17
Ter'strial Planets	Mercury	☿	5″ to 13″	3 030	0.382	$\frac{1}{7\,000\,000}$?	$\frac{1}{21}$	0.056	0.85?	4.7?	88^d	?	?	0.43	0.14
	Venus .	♀	11 to 67	7 700	0.972	$\frac{1}{407\,000}$	0.82	0.92	0.89	4.94	225^d	small	?	0.82	0.76
	Earth .	⊕		7 917.6	1.000	$\frac{1}{332\,000}$	1.000	1.000	1.000	5.55	$23^h\ 56^m\ 4^s.09$	23° 27 08″	$\frac{1}{295}$	1.00	0.20?
	Mars .	♂	3.6 to 25	4 230	0.534	$\frac{1}{3\,093\,500}$	$\frac{1}{9.32}$	0.152	0.71	3.92	24 37 22 .67	24 50	$\frac{1}{200}$	0.38	0.22
	Ceres .	(1)	0.25 to 0.5	488 ?	$\frac{1}{16.22}$?		$\frac{1}{7000}$?	$\frac{1}{4270}$?	Assumed same as moon?		?	?	?	$\frac{1}{265}$?	(low)
Major Planets	Jupiter	♃	32 to 50	86 500	10.92	$\frac{1}{1047.35}$	317.7	1309	0.24	1.32	$9^h\ 55^m \pm$	3 05	$\frac{1}{17}$	2.65	0.62
	Saturn	♄	14 to 20	73 000	9.17	$\frac{1}{3502}$	94.8	760	0.13	0.72	10 14 ±	26 49	$\frac{1}{9.2}$	1.18	0.72
	Uranus	⛢	3.8 to 4.1	31 900	4.03	$\frac{1}{22\,760}$	14.6	65	0.22	1.22	?	?	$\frac{1}{11}$	0.90	0.60
	Neptune	♆	2.7 to 2.9	34 800	4.39	$\frac{1}{19\,500}$	17.0	85	0.20	1.11	?	?	?	0.89	0.52

The masses given are substantially those adopted by Newcomb in his *Astronomical Constants* (Washington, 1895).

TABLE II — THE SATELLITES

	NAME	Discovery		Dist. in Equatorial Radii of Planet	Mean Distance in Miles	Sidereal Period
	Moon	. . . .		60.27035	238 840	27^d 7^h 43^m $11^s.5$

SATELLITES OF

1	Phobos	Hall,	1877	2.771	5 850	7^h 39^m $15^s.1$
2	Deimos	"	"	6.921	14 650	1^d 6 17 54.0

SATELLITES OF

5	Nameless . . .	Barnard,	1892	2.551	112 500	11^h 57^m $22^s.6$
1	Io	Galileo,	1610	5.933	261 000	1^d 18 27 33.5
2	Europa	"	"	9.439	415 000	3 13 13 42.1
3	Ganymede . . .	"	"	15.057	664 000	7 3 42 33.4
4	Callisto	"	"	26.486	1 167 000	16 16 32 11.2

SATELLITES OF

1	Mimas	W. Herschel,	1789	3.11	117 000	22^h 37^m $5^s.7$
2	Enceladus . . .	"	"	3.98	157 000	1^d 8 53 6.9
3	Tethys	J. D. Cassini,	1684	4.95	186 000	1 21 18 25.6
4	Dione	"	"	6.34	238 000	2 17 41 9.3
5	Rhea	"	1672	8.86	332 000	4 12 25 11.6
6	Titan	Huyghens,	1655	20.48	771 000	15 22 41 23.2
7	Hyperion . . .	G. P. Bond,	1848	25.07	934 000	21 6 39 27.0
8	Iapetus	J. D. Cassini,	1671	59.58	2 225 000	79 7 54 17.1
9	Phœbe (?) . . .	W. Pickering,	1898	Existence still doubtful		

SATELLITES OF

1	Ariel	Lassell,	1851	7.52	120 000	2^d 12^h 29^m $21^s.1$
2	Umbriel	"	"	10.46	167 000	4 3 27 37.2
3	Titania	W. Herschel,	1787	17.12	273 000	8 16 56 29.5
4	Oberon	"	"	22.90	365 000	13 11 7 6.4

SATELLITE OF

1	Nameless . . .	Lassell,	1846	12.93	221 500	5^d 21^h 2^m $44^s.2$

OF THE SOLAR SYSTEM

Synodic Period	Inc. of Orbit to Ecliptic	Inc. to Plane of Planet's Orbit	Eccentricity	Diam'r in Miles	Mass in Terms of Primary	Remarks
$29^d\ 12^h\ 44^m\ 2^s.7$	5° 08 40″	— —	0.05491	2162	$\frac{1}{81.5}$	Specific gravity 3.44
MARS						
— —	26° 17′.2	28° ±	0	35 ?	?	Orbits sensibly coincident with planet's equator
— —	25 47.2	28° ±	0	10 ?	?	
JUPITER						
	?	— —	?	100 ?	?	
$1^d\ 18^h\ 28^m\ 35^s.9$	2° 08′ 3″	— —	0	2500	.00001688	The diameters are Engelmann's. The rest of the data are from Damoiseau
3 13 17 53.7	1 38 57	— —	0	2100	.00002323	
7 3 59 35.9	1 59 53	— —	.0013	3550	.00008844	
16 18 5 6.9	1 57 00	— —	.0072	2960	.00004248	
SATURN						
Long. of Ascend.	28° 10′ 10″	About 27°	0	600 ?	?	The planes of the 5 inner orbits sensibly coincide with the plane of the ring
Node of orbits on	"	Inclination of	0	800 ?	?	
ecliptic for 1900,	"	the 5 inner satel-	0	1200 ?	?	
168° 10′ 35″. (5 in-	"	lites to plane of	0	1100 ?	?	
ner satellites and	"	celestial equator	0	1500 ?	?	
ring)	27 38 49	= 6° 57′ 43″ (1900)	0.0299	3500 ?	$\frac{1}{4600}$	
	27 4.8	— —	0.1189	500 ?	?	Discovered independently by Lassell
	18 31.5	— —	0.0296	2000 ?	?	
?	?		?	?	?	On Arequipa photographs
URANUS						
Long. of Ascend.	97° 51′		0	500 ?	?	Retrograde
Node of orbits on	"	Inc. to celestial	0	400 ?	?	
plane of ecliptic	"	equator 75° 18′	0	1000 ?	?	
165° 32′ (1900)	"	(1900)	0	800 ?	?	
NEPTUNE						
Long. Asc. Node, 184° 25′ (1900)	145° 12′	120° 05′ (1900)	0	2000 ?	?	Retrograde

TABLE III — PERIODIC COMETS OBSERVED AT MORE THAN ONE PERIHELION PASSAGE

(From *Annuaire du Bureau des Longitudes*, 1901. Abridged.)

No.	NAME	Last Perihelion Passage	Period (years)	Perihelion Distance	Aphelion Distance	Eccentricity	Long. of Perihelion	Long. of Node	Inclination	No.
1	Encke	1901, Sept. 15	3.303	0.3407	4.0949	0.84639	158° 45′.6	334° 46′.7	12° 54′.6	1
2	Tempel I	1899, July 29	5.281	1.3885	4.6764	0.54211	306 34 .3	120 57 .1	12 38 .9	2
3	Brorsen	1890, Feb. 25	5.456	0.5878	5.6104	0.81034	116 23 .2	101 27 .6	29 23 .8	3
4	Tempel-Swift . .	1897, June 5	5.547	1.0897	5.1771	0.65223	43 26 .7	296 27 .2	5 23 .4	4
5	Winnecke . . .	1898, Mar. 21	5.831	0.9241	5.5548	0.71472	274 14 .6	100 53 .2	16 59 .6	5
6	De Vico-Swift . .	1901, Apr. 27	6.400	1.6696	5.2248	0.51566	348 56 .9	24 50 .6	3 35 .3	6
7	Tempel II . . .	1898, Oct. 4	6.538	2.0911	4.9020	0.40194	241 16 .1	72 36 .1	10 47 .2	7
8	Finlay	1900, Feb. 17	6.556	0.9694	6.0362	0.72324	8 04 .0	52 23 .1	3 02 .9	8
9	D'Arrest	1890, Sept. 18	6.675	1.3212	5.7690	0.62731	319 19 .6	146 15 .6	15 43 .5	9
10	Biela (1)	1866, Jan. 26	6.692	0.8792	6.2229	0.75242	109 40 .3	245 46 .2	12 22 .0	10
11	Wolf	1898, July 5	6.845	1.6030	5.6071	0.55534	19 21 .6	206 29 .1	25 12 .3	11
12	Holmes	1899, Apr. 29	6.874	2.1281	5.1023	0.41135	345 47 .9	331 43 .5	20 48 .2	12
13	Brooks	1896, Nov. 5	7.097	1.9592	5.4267	0.46947	1 52 .3	18 04 .3	6 03 .6	13
14	Faye	1881, Jan. 23	7.566	1.7381	5.9701	0.54901	50 48 .8	209 35 .4	11 19 .7	14
15	Tuttle	1899, May 5	13.667	1.0191	10.4133	0.82171	116 29 .1	269 49 .9	54 29 .3	15
16	Pons-Brooks . . .	1884, Jan. 26	71.56	0.7757	33.6981	0.95500	93 17 .3	254 05 .7	74 02 .6	16
17	Olbers	1887, Oct. 19	72.65	1.1991	33.6234	0.93113	149 52 .5	84 32 .3	44 34 .3	17
18	Halley . . . due	1910, May 17	76.08	0.6871	35.2238	0.96173	168 42 .9	57 10 .5	162 13 .1	18

TABLE IV — STELLAR PARALLAXES AND PROPER MOTIONS

(Kapteyn, 1901.)

No.	Name	α (1900)	δ (1900)	Mag.	Parallax (p)	Weight	Dist. (light-years) (y)	Proper Motion (μ)	Cross Motion $\frac{9}{10}\mu \times y$ (miles persec.)
1	Groombridge 34 . . .	$0^h12^m.7$	+43° 27′	7.9	0″.30	2	11.6	2″.80	29.4
2	ζ Tucanæ	0 14 .9	−65 28	4.1	0 .15	2	21.7	2 .05	40.0
3	β Hydræ	0 20 .5	−77 49	2.7	0 .14	2	23.3	2 .28	47.9
4	η Cassiopeiæ	0 43 .1	+57 17	3.8	0 .19	2	17.2	1 .20	18.5
5	μ Cassiopeiæ	1 01 .6	+54 26	5.4	0 .11	2	29.6	3 .75	100.3
6	τ Ceti	1 39 .4	−16 28	3.7	0 .32	1	10.2	1 .95	17.9
7	e Eridani	3 15 .9	−43 27	4.4	0 .16	2	20.4	3 .03	55.7
8	o_2 Eridani	4 10 .7	− 7 49	4.7	0 .18	2	18.1	4 .05	66.2
9	α Tauri (Aldebaran) .	4 30 .2	+16 18	1.2	0 .11?	1	29.6	0 .19	5.1
10	*Cordova Z.V. 243* . .	5 07 .7	−44 59	8.5	0 .32?	1	10.2	**8 .70**	80.0
11	Sirius	6 40 .8	−16 35	−1.4	0 .38	3	8.6	1 .31	10.1
12	Procyon	7 34 .1	+ 5 29	0.7	0 .30	2	10.9	1 .25	12.2
13	10 Ursæ Majoris . . .	8 54 .2	+42 11	4.2	0 .20	2	16.3	0 .50	7.3
14	Ll. 18115	9 07 .6	+53 07	7.5	0 .14	1	23.3	1 .69	35.6
15	Arg.-Oeltzen 10603 . .	10 05 .3	+49 58	7.0	0 .18	1	18.1	1 .43	23.4
16	Groombridge 1646 . .	10 21 .9	+49 19	6.3	0 .11	1	29.6	0 .89	23.8
17	Ll. 21185	10 57 .9	+36 38	7.5	0 .47	2	6.9	4 .75	29.7
18	Ll. 21258	11 00 .5	+44 02	8.5	0 .24	3	13.6	4 .40	53.9
19	*Groombridge 1830* . .	11 47 .2	+38 26	6.6	0 .15	1	21.7	7 .05	**138.2**
20	Ll. 22954	12 10 .0	− 9 44	6.0	0 .14	1	23.3	1 .02	21.5
21	α *Centauri*	14 32 .8	−60 25	0.9	**0 .76**	4	**4.3**	3 .67	14.2
22	ν' Draconis	17 30 .2	+55 15	4.9	0 .32	1	10.2	0 .16	1.5
23	Arg.-Oeltzen 17415 . .	17 37 .0	+68 26	9.0	0 .25	2	13.1	1 .27	14.9
24	70, p, Ophiuchi . . .	18 00 .4	+ 2 31	4.2	0 .16	2	20.4	1 .13	20.8
25	α Lyræ (Vega)	18 33 .6	+38 41	0.4	0 .15	2	21.7	0 .36	7.1
26	α Aquilæ	19 45 .9	+ 8 36	1.1	0 .24	2	13.6	0 .65	8.0
27	61 Cygni	21 02 .4	+38 15	6.1	0 .41	4	8.0	5 .16	37.0
28	ϵ Indi	21 55 .7	−57 12	4.8	0 .28	3	11.6	4 .68	49.1
29	Fomalhaut	22 52 .1	−30 09	1.4	0 .14	1	23.3	0 .35	7.3
30	Lacaille 9352	22 59 .4	−36 26	7.1	0 .29	2	11.1	7 .00	71.0
	Polaris	1 18 .5	+88 43	2.1	0 .074	3	44.0	0 .045	1.8

Arcturus, Canopus, α Orionis, β Orionis, α Cygni, β Centauri, and γ Cassiopeiæ, all of them stars of the first or second magnitude, have also been carefully observed and have yielded no parallax exceeding 0″.05.

In the table the column headed "weight" indicates roughly the probable reliability of the parallax given, — the estimate depending on the character, number, and accordance of the different determinations for the star in question. The average "probable error" for the parallaxes of the table may be taken as about 0″.04, *i.e.*, *it is just as likely as not* that an average parallax, weighted 2, may be wrong by that amount.

With several of the stars the data are very discordant and unsatisfactory, so that it is to be expected that ultimately some of the results tabulated above will prove seriously incorrect; most of them indeed are to be regarded as only approximations to the truth.

TABLE V — RADIAL VELOCITY OF STARS (VOGEL)

(The velocities are given in English miles per second. The sign, +, indicates recession.)

NAME	Mag.	Class	Velocity	NAME	Mag.	Class	Velocity
α Andromedæ	2	Ia	+ 2.8 m.	γ Leonis	2	IIa	– 24.1 m.
β Cassiopeiæ	2.1	Ia–IIa	+ 3.2	β Ursæ Majoris	2.3	Ia	– 18.5
α Cassiopeiæ	var.	IIa	– 9.7	α Ursæ Majoris	2	IIa	– 12.0
γ Cassiopeiæ	2	Ic	– 2.3	δ Leonis	2.3	Ia	– 8.8
β Andromedæ	2.3	IIa	+ 6.9	β Leonis	2	Ia	– 12.0
α Ursæ Minoris	2	IIa	– 16.1	γ Ursæ Majoris	2.3	Ia	– 16.6
γ Andromedæ	2.4	IIa	– 7.8	ε Ursæ Majoris	2	Ia	– 19.0
α Arietis	2	IIa	– 9.2	α Virginis	1	Ia	– 9.2
β Persei	var.	Ia	– 0.9	ζ Ursæ Majoris*	2.1	Ia	– 19.5
α Persei	2	IIa	– 6.5	η Ursæ Majoris	2	Ia	– 16.1
α Tauri	1	IIa	+ 30.1	α Boötis	1	IIa	– 4.6
α Aurigæ	1	IIa	+ 15.2	ε Boötis	2	IIa	– 9.7
β Orionis	1	Ib	+ 10.1	β Ursæ Minoris	2	IIa	+ 8.8
γ Orionis	2	Ia	+ 5.5	β Libræ	2	Ia	– 6.0
β Tauri	2	Ia	+ 5.1	α Coronæ Borealis	2	Ia	+ 19.9
δ Orionis	2.5	Ia	+ 0.5	α Serpentis	2.3	IIa	+ 13.8
ε Orionis	2	Ib	+ 16.6	β Herculis	2.3	IIIa	– 22.2
ζ Orionis	2	Ia	+ 9.2	α Ophiuchi	2	Ia	– 12.0
α Orionis	var.	IIIa	+ 10.6	α Lyræ	1	Ia	– 9.7
β Aurigæ	2	Ia	– 17.5	α Aquilæ	1.3	Ia	– 23.7
γ Geminorum	2.3	Ia	– 10.1	γ Cygni	2.4	IIa	– 4.1
α Canis Majoris	1	Ia	– 9.7	α Cygni	1.6	Ib	– 5.1
α Geminorum*	2.3	Ia	– 18.4	ε Pegasi	2.3	IIa	+ 5.1
α Canis Minoris	1	Ia–IIa	– 5.5	β Pegasi	var.	IIIa	+ 4.1
β Geminorum	1.3	IIa	+ 0.9	α Pegasi	2	Ia	+ 0.9
α Leonis	1.3	Ia	– 5.5	ζ Herculis* (Belopolsky)	3.1	IIa	– 43.8

* The brighter component of the double star. Campbell has since measured several velocities of about 60 miles a second. (See Sec. 542.)

TABLE VI — VARIABLE STARS

(A selection from Dr. S. C. Chandler's third catalogue (July, 1896) containing such as are visible to the naked eye, have a range of variation exceeding half a magnitude, and can be seen in the United States.)

No.	Name	Place, 1900		Range of Variation (mag.)	Period (days)	Remarks
		α	δ			
1	T Ceti	$0^{h}\ 16^{m}.7$	−20° 37′	5.1– 7.0	65 ±	Very irreg.
2	R Andromedæ	0 18 .8	+38 1	5.6–12.8	410.7	
3	α Cassiopeiæ	0 34 .5	+55 59	2.2– 2.8		Not periodic
4	o Ceti (*Mira*)	2 14 .3	− 3 26	1.7– 9.5	331.6	Large irregularities in date and brightness
5	ρ Persei	2 58 .8	+38 27	3.4– 4.2	33?	Very irreg.
6	β Persei (*Algol*)	3 1 .7	+40 34	2.3– 3.5	$2^{d}\ 20^{h}\ 48^{m}\ 55^{s}.43$	Period now shortening
7	λ Tauri	3 55 .1	+12 12	3.4– 4.2	3 22 52 12	Algol type irregular
8	ε Aurigæ	4 54 .8	+43 41	3.0– 4.5		Not periodic
9	α Orionis	5 49 .7	+ 7 23	0.7– 1.5		Not periodic
10	η Geminorum	6 8 .8	+22 32	3.2– 4.2	231.4	
11	ζ Geminorum	6 58 .2	+20 43	3.7– 4.5	$10^{d}\ 3^{h}\ 41^{m}\ 30^{s}.6$	
12	R Canis Majoris	7 14 .9	−16 12	5.9– 6.7	1 3 15 46	Algol type
13	R Leonis Minoris	9 39 .6	+34 58	6.0–13.0	370.5	
14	R Leonis	9 42 .2	+11 54	5.2–10.0	312.8	
15	U Hydræ	10 32 .6	−12 52	4.5– 6.3	195 ± ?	Very irreg.
16	R Ursæ Majoris	10 37 .6	+69 18	6.0–13.2	302.1	
17	R Hydræ	13 24 .2	−22 46	3.5– 5.5	425.15	Period shortening
18	S Virginis	13 27 .8	− 6 41	5.7–12.5	376.4	
19	R Boötis	14 32 .8	+27 10	5.9–12.2	223.4	
20	δ Libræ	14 55 .6	− 8 7	5.0– 6.2	$2^{d}\ 7^{h}\ 51^{m}\ 22^{s}.8$	Algol type
21	R Coronæ	15 44 .4	+28 28	5.8–13.0		Not periodic
22	R Serpentis	15 46 .1	+15 26	5.6–13.0	357.0	
23	α Herculis	17 10 .1	+14 30	3.1– 3.9	60^{d} to 90^{d}	Not periodic
24	U Ophiuchi	17 11 .5	+ 1 19	6.0– 6.7	$20^{h}\ 7^{m}\ 42^{s}.56$	
25	u Herculis	17 13 .6	+33 12	4.6– 5.4		Irreg. periodic
26	X Sagittarii	17 41 .3	−27 48	4.0– 6.0	$7^{d}\ 0^{h}\ 17^{m}\ 57^{s}$	
27	W Sagittarii	17 58 .6	−29 35	4.8– 5.8	7 14 16 13	
28	Y Sagittarii	18 15 .5	−18 54	5.8– 6.6	5 18 33 24 .5	
29	R Scuti	18 42 .1	− 5 49	4.7– 9.0	71.1	Very irreg.
30	β Lyræ	18 46 .4	+33 15	3.4– 4.5	$12^{d}\ 21^{h}\ 47^{m}\ 23^{s}.72$	
31	R Lyræ	18 52 .3	+43 49	4.0– 4.7	46.4	
32	χ Cygni	19 46 .7	+32 40	4.0–13.5	406.02	Period lengthening
33	η Aquilæ	19 47 .4	+ 0 45	3.5– 4.7	$7^{d}\ 4^{h}\ 11^{m}\ 59^{s}$	
34	S Sagittæ	19 51 .4	+16 22	5.6– 6.4	8 9 11 48 .5	
35	X Cygni	20 39 .5	+35 14	6.4– 7.7	16 9 15 7	
36	T Vulpeculæ	20 47 .2	+27 53	5.5– 6.5	4 10 27 50 .4	
37	T Cephei	21 8 .2	+68 5	5.2–10.7	387	
38	μ Cephei	21 40 .4	+58 19	4.0– 5.5	430 ±	Irreg. periodic
39	δ Cephei	22 25 .4	+57 54	3.7– 4.9	$5^{d}\ 8^{h}\ 47^{m}\ 39^{s}.3$	
40	β Pegasi	22 58 .9	+27 32	2.2– 2.7		Not periodic
41	R Aquarii	23 38 .6	−15 50	5.8–11?	387.16	
42	R Cassiopeiæ	23 53 .3	+50 50	4.8–12	429.5	

TABLE VII — ORBITS OF BINARY STARS

(Mostly from Dr. See's List of Orbits, *Astronomical Journal*, Vol. XVI, 1896.)

No.	NAME	α (1900)	δ (1900)	Period (years)	a″	Eccentricity	Periastron	Magnitudes	Authority
1	Ll. 9091 . . .	$4^h 45^m.7$	+ 10° 54′	5.05 ± 0.1	$0''.62$	0.760	1896.40	8.0 : 8.0	See
2	κ Pegasi . . .	21 40 .1	+ 25 11	11.42 ± 0.4	0 .42	0.490	1896.03	4.3 : 5	"
3	δ Equulei, A.B. .	21 9 .6	+ 9 37	11.45 ± 0.2	0 .45	0.165	1892.80	4.5 : 5	"
4	ζ Sagittarii . .	18 56 .3	− 30 1	18.85 ± 1	0 .69	0.279	1878.80	3.9 : 4.4	"
5	42 Comæ Ber. .	13 5 .1	+ 18 4	25.56 ± 0.1	0 .64	0.461	1885.69	6 : 6	"
6	ζ Herculis . .	16 37 .6	+ 31 47	35.09 ± 0.3	1 .43	0.497	1864.80	3 : 6	"
7	η Coronæ Bor. .	15 19 .1	+ 30 39	41.60 ± 0.1	0 .91	0.267	1892.50	5.5 : 6	"
8	Sirius	6 40 .4	− 16 34	51.8 ± 0.2	7 .62	0.600	1893.77	−1.4 : 9	Burnham
9	γ Androm., B.C.	1 57 .8	+ 41 51	54.0 ± 1	0 .37	0.857	1892.1	5.5 : 7	See
10	ξ Ursæ Majoris .	11 12 .9	+ 32 6	60.00 ± 0.1	2 .50	0.397	1875.20	4 : 5	"
11	ζ Cancri, A.B. .	8 06 .2	+ 17 58	60.0 ± 0.5	0 .85	0.340	1870.40	5.5 : 6.2	"
12	γ Coronæ Bor. .	15 38 .5	+ 26 36	73.0 ± 2	0 .73	0.482	1841.0	4 : 7	"
13	α Centauri . .	14 32 .6	− 60 25	81.10 ± 0.3	17 .70	0.528	1875.70	1 : 2	"
14	70 Ophiuchi . .	18 0 .4	+ 2 33	88.40 ± 1	4 .54	0.500	1896.47	4.5 : 6	"
15	ϕ Ursæ Majoris .	9 45 .3	+ 54 33	97.0 ± 5	0 .34	0.440	1884.0	5.5 : 5.5	"
16	ω Leonis . . .	9 23 .1	+ 9 30	116.2 ± 1	0 .88	0.537	1842.1	6 : 7	"
17	ξ Boötis . . .	14 46 .8	+ 19 31	128.0 ± 1	5 .56	0.721	1903.9	4.5 : 6.5	"
18	γ Virginis . . .	12 36 .6	− 0 54	194.0 ± 4	3 .99	0.897	1836.5	3 : 3.2	"
19	η Cassiopeiæ . .	0 42 .9	+ 57 18	195.8 ± 10	8 .21	0.514	1907.8	4 : 7	"
20	σ Coronæ Bor. .	16 11 .0	+ 34 7	370.0 ± 25	3 .82	0.540	1821.8	6 : 7	"
21	36 Andromedæ .	0 49 .6	+ 23 5	349.1 ?	1 .54	0.634	1798.8	6 : 7	Doberck
22	α Geminorum .	7 28 .2	+ 32 8	997 ? ?	7 .54	0.344	1750.3	2.5 : 3	Thiele

TABLE VIII — MEAN REFRACTION

(Corresponding to temperature of 50° F., and to a barometric pressure of 29.6 inches.)

Altitude	Refraction	Altitude	Refraction	Altitude	Refraction
0°	34′ 50″	11°	4′ 47″.7	30°	1′ 39″.5
1	24 22	12	4 24 .5	35	1 22 .1
2	18 06	13	4 04 .4	40	1 08 .6
3	14 13	14	3 47 .0	45	57 .6
4	11 37	16	3 18 .2	50	48 .3
5	9 45	18	2 55 .5	55	40 .3
6	8 23	20	2 37 .0	60	33 .2
7	7 19	22	2 21 .6	65	26 .8
8	6 29	24	2 08 .6	70	20 .9
9	5 49	26	1 57 .6	80	10 .2
10	5 16	28	1 48 .0	90	0 .0

For every 5° F. by which the temperature is *less* than 50° F., *add one per cent* to the tabular refraction, and decrease it in the same ratio for temperatures above 50° F.

Increase the tabular refraction by *three and a half per cent* for every inch of barometric pressure above 29.6 inches, and decrease it in the same ratio below that point. These corrections for temperature and pressure, though only approximate, will give a result correct within 2″, except in extreme cases.

Comstock's Formula for Refraction

$$r'' = \frac{983\, b}{460 + t} \tan \zeta.$$

b is the barometer reading in *inches;* t, the temperature in degrees *Fahrenheit;* ζ, the apparent zenith-distance. The error of the formula is less than 1″ for zenith-distances under 75°, except in extreme conditions of temperature and pressure.

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